



NATIONAL OPEN UNIVERSITY OF NIGERIA
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2020_1 EXAMINATION

Course Code: MTH 423
Course Title: Integral Equations
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Number One (1) and Any Other Four (4) Questions

Q1. (a) Solve the integral equation $A(x) = x + 1 + \int_0^x (1 + 2(x - y))d(y)dy$. (8 marks)

(b) Transform $\frac{d^2y}{dx^2} + \lambda y = 0$ into an integral equation with $y = 0$ at $x = 0$ and

$y' = 0$ at $x = 1$ (8 marks)

(c) State Hilbert-Schmidt theorem (6 marks)

Q2. (a) Solve the equation

$y'' + 2xy' + y = 0, y(0) = 1, y'(0) = 0$. (4 marks)

(b) If $Q(x) = 3 \int_0^x \cos(x - y) Q(y)dy + e^x$. Find the possible solution to the equation (8 marks)

Q3. (a) State the system of Laplace transform (4 marks)

(b) Define positive Kernel. (3 marks)

(c) Show by using Laplace transform the statement of kernel of the convolution type. (5 marks)

Q4. (a) If $K(x, y)$ is symmetric and continuous. Show that K has at least one Eigenvalue. (10 marks)

(b) Define half-plane of convergence. (2 marks)

Q5. (a) Solve the equation $\phi(x) = \lambda \int_0^1 (1 + xt) \phi(t) dt$, $0 \leq x \leq 1$. **(6 marks)**

(b) Write the general method of solution for Fredholm equation with degenerate kernel **(6 marks)**

Q6. (a) Use Laplace transform to solve the integral equation

$$f(x) = \int_0^x K(x-y) f(y) dy + g(x) \quad \textbf{(6 marks)}$$

(b) If $K(x, y)$ is symmetric and continuous. Write the three (3) necessary conditions for K **(3 marks)**

(c) State Convolution system of integral equation **(3 marks)**