



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.  
**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
**September, Examination 2020\_1**

**Course Code: MTH401**

**Course Title: General Topology I**

**Credit Unit: 3**

**Time Allowed: 3 Hours**

**Instruction: Attempt Number One (1) and Any Other Four (4) Questions**

1. (a) Show that  $d(x,y) = |x-y|^3$  does not define a metric on  $\mathbb{R}$ . **(4 marks)**  
(b) Let  $\mathbb{R}^2$  be the set of all ordered pairs of real numbers endowed with the metric  $d_1(x, y) = \sum_{i=1}^2 |x_i - y_i|$  for arbitrary  $x = (x_1, x_2)$  and  $y = (y_1, y_2)$  in  $\mathbb{R}^2$ . Describe the open ball  $B((0, 0); 1)$ . **(3 marks)**  
(c) Why is it that any open ball centered at  $0 \in [0, 1)$  with the usual metric on  $\mathbb{R}$ , not open in  $\mathbb{R}$ ? **(2 marks)**  
(d) Let  $E = \mathbb{R}$  and  $A = [0, 1)$ . Now, endow  $\mathbb{R}$  with the following metric: for all  $x, y \in \mathbb{R}$   $d(x, y) = 1$ , if  $x \neq y$  and  $d(x, y) = 0$  if  $x = y$  with the usual metric. Show that  $A = [0, 1)$  is open in  $\mathbb{R}$ . **(9 marks)**  
(e) Explain function space with the metric defined on it. **(4 marks)**
2. Show that in any metric space  $(E, d)$ , each open ball is an open set in  $E$ . **(12 marks)**
3. Show that the interval  $[a, b]$  is closed in  $\mathbb{R}$ . Illustrate with an example. **(12 marks)**
4. Let  $E = \mathbb{R}$  (the reals) and let  $d_0$  be defined by  $d_0(x, y) = 1$ , if  $x \neq y$ ;  $d_0(x, y) = 0$  if  $x = y$ , and let  $F = [0, 1)$ . Show that:  
(i)  $F$  has no limit points. **(6 marks)**  
(ii)  $F$  is closed. **(4 marks)**  
(iii)  $F$  is open. **(2 marks)**
5. Let  $\{F_i\}_{i \in I}$  be a nonempty family of closed sets of a metric space  $(E, d)$ . Show that (i)  $\bigcap_{i \in I} F_i$  is closed in  $E$ . **(6 marks)**  
(ii)  $\bigcup_{i=1}^k F_i$  is closed in  $E$ . **(6 marks)**
6. (a) Show that a subset  $F$  of a metric space  $(E, d)$  is closed in  $E$  if and only if its complement is open in  $E$ . **(6 marks)**  
(b) Show that every singleton subset of any metric space is closed. Hence, every finite set is closed. **(6 marks)**