NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS
September, Examination 2020_1

COURSE CODE: MTH312
COURSE TITLE: Abstract Algebra II
CREDIT UNIT: 3
TIME ALLOWED: 3 Hours
INSTRUCTION: Answer Question Number One and Any Other Four Questions.

1.(a). Define the following:
(i). Ring homomorphism (ii). Group Isomorphism (ii). Automorphism **(6 marks)**
(b). (i). If $\emptyset: G \rightarrow H$ and $\theta: H \rightarrow K$ are two isomorphisms of groups, show that $\theta \circ \emptyset$ is an isomorphism of G onto K. **(6 marks)**
(ii). Prove that any cyclic group is isomorphic to $(Z, +)$ or $(Z_{n}, +)$. **(6 marks)**
(c). Show that every subgroup of Z is normal in Z. **(4 marks)**

2. Consider the groups $(R, +)$ and $(C, +)$ and define $f: (C, +) \rightarrow (R, +)$ by $f(x + iy) = x$, the real part of $x + iy$.
   (i) Show that f is a homomorphism. **(8 marks)**
   (ii) Hence, find the Im f and Ker f. **(4 marks)**

3. (a). Show that $(S_n, \circ)$ is non-commutative group for $n \geq 3$. **(6 marks)**
   (b) Do the cycles $(1\ 3)$ and $(1\ 5\ 4)$ commute? Give reason for your answer. **(6 marks)**

4**.** (a). Define the following:
   i. External direct product **(3 marks)**
   ii. Internal direct product **(3 marks)**
   (b). Let a group G be internal direct product of its subgroups H and K. Prove that:
   i. Each $x \in G$ can be uniquely expressed as $x = hk$, where $h \in H, k \in K$; **(3 marks)**
   ii. $hk = kh\ \forall\ h \in H, k \in K$. **(3 marks)**

4. (a). Define a ring for a non-empty set R. **(4 marks)**

(b). Consider the set $Z + iZ = \{m + in\text{:}\ \ m\ and\ n$ are integers$\}$, where $i^2 = -1$. Verify that $Z + iZ$ is a ring under addition and multiplication of complex number. **(8 marks)**

5. (a). Define an ideal I of a ring R. **(2 marks)**

(b). (i). Let R be a ring and $a_1, a_2 \ \in R$, show that $Ra_1 + Ra_2 = \{x_1a_1 + x_1a_1\text{:}\, x_{1,}x_2 \in R\}$ is an ideal of R. **(6 marks)**

(ii). Show that $\{\bar{0}, \bar{3}\}$ and $\{\bar{0}\ \bar{2}\ \bar{4}\}$ are proper ideals of Z_6. **(4 marks)**

6. (a). Define the following:

i. Sylow p-subgroup of G. **(3 marks)**

ii. Simple group. **(3 marks)**

(b).Show that every group of order 20 has a proper normal non-trivial subgroup. **(6 marks)**