NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
SEPTEMBER 2020_1 EXAMINATION

Course Code: MTH 311
Course Title: Calculus of Several Variables
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Question Number One and Any other Four Questions

1 a) Investigate $\lim_{h \to 0} \frac{\sin(3h)}{h}$ **[3 Marks]**

b) Find the extrema value of $f(x,y) = x^2 - 8\ln x$ at $[1,4]$ **[5 Marks]**

c) Check that $\frac{\partial^2 f}{\partial u \, \partial t} = \frac{\partial^2 f}{\partial t \, \partial u}$ for $f = e^{\frac{u}{t}}$ **[4 Marks]**

d) Write out the Langrage equation for $f(x,y) = 2x + 5y$ on the ellipse $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$ **[6 Marks]**

e) Write $h(x,y) = e^{-x^3 y}$ as a composite function and evaluate $\lim_{(x,y) \to (1,2)} h(x,y)$ **[4 Marks]**

2. a) Calculate the second – order partial of $f(x,y) = x^3 + y^2 e^x$ **[5 Marks]**

b) The altitude of a mountain at (x,y) is $f(x,y) = 2500 + 100(x + y^2)e^{-0.3y^2}$. Find the directional derivative of f at $P = (-1,-1)$ in the direction of unit vector u making an angle of $\frac{\theta}{4}$ with the gradient. **[7 Marks]**

3a) Find the critical points of $f(x,y) = (x^2 + y^2)e^{-x}$ and analyze them using the second derivative test. **[8 Marks]**

b) Let $f(x,y)$ be a function of two variables, and let (r,θ) be polar coordinate, express $\frac{\partial f}{\partial \theta}$ in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ **[4 Marks]**

4. a) Find the maximum and minimum value of $f(x, y) = 81x^2 + y^2$ subject to the constraint

$4x^2 + y^2 = 9$ **[6 Marks]**

b) Find the dimension of the box with largest volume if the total surface area is $32cm^2$. **[6 Marks]**

5. a) If $x = u - v + w$, $y = u^2 - v^2 - w^2$ and $z = u^3 + v$, Find Jacobian $\dfrac{\partial(x, y, z)}{\partial(u, v, w)}$ **[4 Marks]**

b) If $f(x, y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$

(i) Write out the Langrage equation **[2 Marks]**

(ii) Find the maximum and minimum value of $f(x, y)$ **[6 Marks]**

6. a) If $f(x, y) = x^3 y^2$, find $\dfrac{df}{dt}$ if $x^5 + y = t$ and $x^2 + y^3 = t^3$ **[4 Marks]**

b) Find the maximum and minimum of $f(x, y, z) = 4y - 2z$ subject to the constraints $2x - y - z = 2$ and $x^2 + y^2 = 1$ **[8 Marks]**