NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

SEPTEMBER Examination 2020_1

Course Code: MTH 302

Course Title: Elementary Differential Equations

Credit Unit: 3

Time Allowed: 3 Hours

Instruction: Answer Question Number One and Any Other Four Questions

1. (a) i. Define Power Series **(2 marks)**

ii. When is a Power Series said to be convergent? **(2 marks)**

(b) i. Define Radius of Convergence **(2 marks)**

ii. Determine the Radius of Convergence of the following Power Series

i. $f(x) = \sum_{n=0}^{\infty} \frac{2^n}{2^{n+1}}$ **(2 marks)**

ii. $f(x) = \sum_{n=0}^{\infty} \frac{2^n(n+1)^2}{n^2 2^{n+1}}$ **(2 marks)**

(c). (i) Define Orthogonality with respect to the weight function **(2 marks)**

(ii) When is a function said to be even? Give an example **(2 marks)**

(iii) When is a function said to be odd? Give an example **(2 marks)**

(d) If $P(x)\frac{d^2y}{dx^2} + Q(x)\frac{dy}{dx} + R(x)y = 0$

Where $P(x)$, $Q(x)$, and $R(x)$ are polynomials.

i. Define the ordinary point of equation (1) **(2 marks)**

ii. Define the singular point of equation (1) **(2 marks)**

iii. Define the regular singular point of equation (1) **(2 marks)**

2. Given the equation $(1 - x^2)y'' - 6xy' - 4y = 0$,near the ordinary point $x = 0$.

(i) Show that the solution is invalid in $|x| < 1$. **(5 marks)**

(ii) Obtain the recurrence relation for the equation and hence solve for y **(7 marks)**

3. Solve the equation $y'' + (x-1)^2 y' - 4(x-1)y = 0$ **(12 marks)**

4. (i) State the general form of a linear homogeneous second order boundary value problem

(ii) Given that $\mathrm{y}'' + \gamma \mathrm{y} = 0, \mathrm{y}(0) = 0, \mathrm{y}'(0) = 0$ **(3 marks)**

Show that if $\emptyset_{\mathrm{m}}$ and $\emptyset_{\mathrm{n}}$ are Eigen functions corresponding to the Eigen values γ_{m} and γ_{n} respectively, then $\int_0^1 \emptyset_{\mathrm{m}}(x)\emptyset_{\mathrm{n}}(x)\mathrm{d}x = 0$, provided $\gamma_{\mathrm{m}} \neq \gamma_{\mathrm{n}}$ **(9 marks)**

5. (i) Solve the equation $x^2 y'' + \alpha x y' + \beta y = 0$

With the transform $x = e^z$ or $z = logx$, and $x > 0$. **(6 marks)**

(ii) Describe the solution for different type of roots. **(6 marks)**

6. (i) Obtain the Fourier series over the interval $0 \leq x \leq 2$ for the function $f(x) = 2.\ x = 2$

(6 marks)

(ii) Prove that all the Eigen values of the Sturm-Liouville problem

$$L(y) = \gamma r(x) y$$

with boundary condition
$a_1 y(0) + a_2 y'(0) = 0, b_1 y(I) + b_2 y'(I) = 0,$ are real **(6 marks)**