



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja  
**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
**2024 1 EXAMINATION**

**Course Code: MTH 422**

**Course Title: Partial Differential Equations**

**Credit Unit: 3**

**Time Allowed: 3 Hours**

**Total: 70 Marks**

**Instruction: Answer Question One and any Other Three Questions**

1a. Solve the following Cauchy problems

(i)  $u_{tt} - 4u_{xx} = 6t, u(x, 0) = x, u_t(x, 0) = 0$  (5 marks)

(ii)  $u_{tt} = c^2 u_{xx}, u(x, 0) = e^x, u_t(x, 0) = \sin x$  (5 marks)

b. Evaluate  $u\left(1, \frac{1}{2}\right)$  given the Cauchy problem:

$$u_{tt} - u_{xx} = 0, -\infty < x < \infty, t > 0$$

$$u(x, 0) = f(x) = \begin{cases} 0, & -\infty < x < -1 \\ x+1, & -1 \leq x < 0 \\ 1-x, & 0 \leq x \leq 1 \\ 0, & 1 < x < \infty \end{cases} \quad (15 \text{ marks})$$

$$u_t(x, 0) = g(x) = \begin{cases} 0, & -\infty < x < -1 \\ 1, & -1 \leq x < 1 \\ 0, & 1 < x < \infty \end{cases}$$

2a. Solve the heat equation:

$$u_t - u_{xx} = 0; 0 < x < \pi, t > 0$$

$$u(0, t) = u(\pi, t) = 0, t \geq 0$$

$$u(x, 0) = f(x) = \begin{cases} x, & 0 \leq x \leq \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x \leq \pi \end{cases} \quad (15 \text{ marks})$$

3. Consider the equation  $u_{tt} - c^2 u_{xx} = 0; -\infty \leq a < x < b \leq \infty, t > 0$

Give a canonical form and the general solution of the wave equation. (15 marks)

4 (a) Verify that  $u(x, y) = f(x)g(y)$  is a solution of the PDE  $uu_{xy} = u_x u_y$  for all pairs of (differentiable) functions  $f$  and  $g$  of one variable (7 marks)

(b) Show that  $\cos(x - ct)$  is a solution of  $u_t + cu_x = 0$ . (8 marks)

5a. Solve the first order equation  $2u_t + 3u_x = 0$  with the auxiliary condition  $u = \sin x$  when  $t = 0$ . **(7marks)**

b. i) Find the general solution (in terms of arbitrary function) of the first order PDE:

$$u_t(x, y) + u_y(x, y) = 1$$

**(4marks)**

(ii) Discuss the particular solution of the above PDE corresponding to  $u(x, 0) = \phi(x)$  on the  $x$ -axis **(4marks)**

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