



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024 1 EXAMINATION

Course Code: MTH421

Course Title: Ordinary Differential Equations

Credit Unit: 3

Time Allowed: 3 Hours

Total Marks: 70 Marks

Instruction: Attempt Question One (1) and Any Other Three (3) Questions

1(a) Define the following with an examples each:

- (i) Ordinary Differential Equation (3marks)
- (ii) Partial Differential Equation (3marks)
- (iii) First Order Differential Equation (3marks)

b. i) State the general form of the Bernoulli differential equations (3marks)

ii) Obtain the general solution of the Bernoulli equation: $yy' = y^2 + e^x$ (13marks)

2 a) When is a smooth function $\varphi(t)$ said to be

- (i) positive definite (1.5marks)
- (ii) positive semi-definite (1.5marks)
- (iii) negative definite (1.5marks)
- (iv) negative semi-definite (1.5marks)

b) Explain the terms:

- (i) Initial value problem (1.5marks)
- (ii) Boundary value problem (1.5marks)

c) Obtain the solutions to the following initial value problems

$$\frac{d^2y}{dt^2} - \frac{dy}{dx} - 2y(t) = 0; y(0) = 2, y'(0) = 7 \quad (6marks)$$

3 a) When is a first-order differential equation $M(x, y)dy + N(x, y)dx = 0$ is called exact, or an exact differential? (2marks)

Test for the exactness of the following differential equations:

- i. $(y\cos x + 2xe^y) + (\sin x + x^2e^y)y' = 0$ (3marks)
- ii. $(e^{x+y} + e^y)dx + (ye^y - e^y)dy = 0$ (3marks)

b) Solve the initial value problem $(e^{x+y} + e^y)dx + (ye^y - e^y)dy = 0; y(0) = -1$ by finding the integrating factor (7marks)

4) a) Solve the initial value problem $y'' + 3y' + \frac{9}{4}y = -10e^{\frac{3}{2}x} = 0; y(0) = 1, y'(0) = 0$

(15marks)

- 5) a) Show that $y_1 = e^t$ and $y_2 = e^{-2t}$ are fundamental solution to the equation
 $y'' + y' - 2y = 0$ **(7marks)**
- b) Find the solution the boundary value problem $y''(x) + 4y'(x) = 0$; $y(0) = 1$, $y'(\frac{\pi}{2}) = 0$ **(8marks)**
- 6) a) Define a principal fundamental matrix for the system $x' = A(t)x$ **(3marks)**
- b) i) State the Existence Theorem **(5marks)**
- ii) List the four important steps to prove the theorem in i) **(7marks)**

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