



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024 1 EXAMINATION

Course Code: MTH412

Credit Unit: 3

Total: 70 Marks

Course Title: FUNCTIONAL ANALYSIS II

Time Allowed: 3 Hours

Instruction: Answer Question One (1) and Any Other 3 Questions

- 1(a) Let $T \in B(H)$ be a bounded operator. Define each of the following terms on T :
- (i) Diagonalizable (2marks)
 - (ii) Eigenvector (2marks)
 - (iii) Eigenvalue (1mark)
- (b) Prove that any norm $\|\cdot\|_\alpha$ is continuous under $\|\cdot\|_1$. (12marks)
- (c) Suppose E be an inner product space. Prove that the inner product $(\cdot, \cdot) : E \times E \rightarrow K$ is continuous with respect to the induced norm. (8marks)
- 2(a) Let $1 \leq p \leq \infty$, and p' the exponent dual of p . Prove that $\sum_{i=1}^N |x_i| |y_i| \leq |x|_p |y|_{p'}$ for all $x, y \in K^N$. (11marks)
- (b) Suppose E a vector space and F a subspace. When is E/F called the quotient space of E modulo F ? (4marks)
- 3(a) Let E be an inner product space with inner product (\cdot, \cdot) . Prove that $|(u|v)| \leq \|u\| \|v\|$ for all $u, v \in E$ with equality if u and v are linearly dependent. (11marks)
- (b) Let X be a Banach space and Y a normed linear space. Let $\{T_n\}$ be a sequence of terms from $\beta(X, Y)$ converging strongly to T . Explain that there exists a positive constant M such that $\|T_n\| < M$ for all n . (4marks)
- 4(a) Let E be a normed space. Prove that E/F is a normed space with norm if and only if F is a closed subspace of E . (13marks)
- (b) For an arbitrary non-empty subset M of an inner product space, when is M^\perp said to be the orthogonal complement of M . (2marks)
5. (a) State the closed graph theorem. (2marks)
- (b) Let $X = R^2$. For arbitrary $\bar{x} = (\bar{x}_1, \bar{x}_2) \in R^2$, define $\|\cdot\|_2 = R^2 \rightarrow [0, \infty)$;
 $\|\bar{x}\|_2 = (x_1^2 + x_2^2)^{\frac{1}{2}}$. Prove that $\|\cdot\|_2$ is a norm on X . (13marks)