



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024 1 EXAMINATION

Course Code: MTH402

Credit Unit: 3

Total: 70 Marks

Course Title: GENERAL TOPOLOGY II

Time Allowed: 3 Hours

Instruction: Answer Question One (1) and Any Other 3 Questions

1. (a) If $x_0, x_1 \in X = \mathbb{R}^2$ is the plane and $f, f': [0, 1] \rightarrow \mathbb{R}^2$ are paths from x_0 to x_1 , explain the straight-line homotopy and path homotopic. **(8marks)**
(b) Define each of the following terms:
 - (i) Normal Space **(2marks)**
 - (ii) Topological embedding **(2marks)**
 - (iii) Isolated point **(2marks)**(c) Let $f: Z \rightarrow X \times Y$ be given by $f(z) = (f_1(x), f_2(y))$. Then show that f is continuous if and only if the functions $f_1: Z \rightarrow X$ and $f_2: Z \rightarrow Y$ are continuous. **(11marks)**
2. (a) If A is a compact subset of a Hausdorff topological space (X, τ) , then prove that A is closed. **(7marks)**
(b) Prove that any closed and bounded interval in \mathbb{R} is compact. **(8marks)**
3. (a) When does a topological space X said to satisfy:
 - (i) The fourth separation axiom **(4marks)**
 - (ii) Third separation axiom **(4marks)**(b) Let X and Y be topological spaces, and let $f: X \rightarrow Y$ be a function. If X is compact and f is continuous, then *show* that $f(X)$ is compact. **(7marks)**
4. (a) When is a topological space X said to be first countable? **(2marks)**
(b) Show that if L is a linear continuum in the order topology, then L is connected, and so are the intervals and rays in L . **(13marks)**
5. (a) Prove that $\mathbb{R}_+^* = (0, +\infty)$ is not compact. **(9marks)**
(b) Let X, Y and Z be topological spaces. If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are continuous, then *show* that the map $g \circ f: X \rightarrow Z$ is continuous. **(6marks)**