



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikiwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024 1 EXAMINATION

Course Code: MTH401

Course Title: GENERAL TOPOLOGY 1

Credit Unit: 3

Time Allowed: 3 Hours

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other 3 Questions

- 1a. Let (E, d) be an arbitrary metric space and let $\{x_n\}$ be a Cauchy sequence in E . Then, $\{x_n\}$ is bounded. **(8 marks)**
- b. Define the following:
- i. Derived Set **(2 marks)**
 - ii. Minkowski Inequality **(2 marks)**
 - iii. Discrete Topology **(2 marks)**
- c. Let (E, d) be a metric space and let $a \in E$ be fixed. Let $f: E \rightarrow \mathbb{R}$ be defined by $f(x) = d(x, a)$, for all $x \in E$. Prove that f is uniformly continuous on E . **(5 marks)**
- d. Prove that every compact subset of a metric space is closed and bounded. **(6 marks)**
- 2a. When is the pair (E, d) said to be a metric space? **(5 marks)**
- b. Let (X, d_X) and (Y, d_Y) be metric spaces and $f: X \rightarrow Y$ be a map. Let $x_0 \in X$ be arbitrary but fixed. Then prove that the following are equivalent.
- i. f is continuous at x_0 .
 - ii. if $\{x_n\}$ is a convergent sequence in X such that $x_n \rightarrow x_0 \in X$, then $f(x_n) \rightarrow f(x_0) \in Y$ as $n \rightarrow \infty$. **(8 marks)**
- c. Define the convergence of limit of a sequence. **(2 marks)**
- 3a. Let F be the subset of a metric space (E, d) . Prove that F is closed in E if and only if its complement is open in E . **(5 marks)**
- b. i. Define an interior point x on a metric (E, d) which has a subset O . **(2 marks)**
- ii. Let (E, d) be a metric space and F be the subset of E . Define the Limit point of F on E .

- (3 marks)**
- c. Prove that the image of compact set under a continuous map is compact. **(5 marks)**
- 4a. Prove that the cartesian product of two compact sets is compact. **(6 marks)**
- b. Define the following:
- i. Condensing Point **(2 marks)**
- ii. Diameter of a Set. **(2 marks)**
- c. Prove that every convergent sequence in any metric space is Cauchy. **(5 marks)**
- 5a. Prove that a closed subset of a complete metric space is complete. **(5 marks)**
- b. Let $(R, ||)$ be a metric space and R denote the set of real numbers with $d: R \times R \rightarrow R$ be defined by $d(x, y) := |x - y|$ for all $x, y \in R$. Then verify that d is a metric on R . **(5 marks)**
- c. Let $E = R^2$ be endowed with the Euclidean metric $d_2(x, y) = \sum_{k=1}^2 [(x_k - y_k)^2]^{\frac{1}{2}}$ for all $x = (x_1, x_2), y = (y_1, y_2) \in R^2$. Describe the sets:
- i. $B((0, 0), 1)$
- ii. $B_r(x_0)$ for arbitrary $x_0 \in R^2$ and $r > 0$. **(5 marks)**