



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024 1 EXAMINATION

Course Code: MTH382

Credit Unit: 3

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other Three (3) Questions

Course Title: Calculus of Several Variables

Time Allowed: 3 Hours

QUESTION ONE

- (a) i. What are ordinary differential equations? [2 Marks]
ii. Given that $\frac{dy}{dx} + y \tan x = e^{2x} \cos x$, $y(0) = 2$. Show that the solution of the above differential equation is $y = \frac{1}{2}(e^{2x} + 3) \cos x$. [7 Marks]
- (b) Consider the initial value problem (IVP) $\frac{dy}{dt} = 2ty$, $y(0) = 1$, and apply the Method of Successive Approximations. [5 Marks]
- (c) Show that $y' = F[t, y]$, $y(t_0) = y_0$, has a unique solution defined in the interval $(t_0 - r, t_0 + r)$, where $r < \min\left(a, \frac{b}{M}, \frac{1}{K}\right)$. [11 Marks]

QUESTION TWO

- (a) Describe the Bessel equation. [2 Marks]
(b) Using Rodrigues's formula, determine the Legendre polynomial $P_2(x)$. [5 Marks]
(c) Starting from the generating function of the Bessel function of the first kind $e^{\frac{1}{2}x\left(t - \frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$, $n \in \mathbb{Z}$, show that $J_n(x) = (-1)^n J_{-n}(x)$. [8 Marks]

QUESTION THREE

- (a) What are factorial notations? [2 Marks]
(b) Show that $(\alpha)_{2n} = 2^{2n} \left(\frac{\alpha}{2}\right)_n \left(\frac{\alpha+1}{2}\right)_n$. [5 Marks]
(c) Prove that $\Gamma(2z) = \frac{2^{2z-1}}{\sqrt{\pi}} \Gamma(z) \Gamma\left(z + \frac{1}{2}\right)$, $(2z = 0, -1, -2, \dots)$. [8 Marks]

QUESTION FOUR

- (a) Define the Laplace differential equation? [2 Marks]
(b) Provide the Fourier series conforming to $f(x)$, a_n and b_n . [2 Marks]
(c) A square plate is bounded by the lines $x=0$, $y=0$, $x=1$ and $y=1$.
Apply the Laplace equation to determine the potential distribution $u(x, y)$ over the plate, subject to the following boundary conditions:

- (i) $u = 0$ when $x = 0$, $0 \leq y \leq 1$,
- (ii) $u = 0$ when $x = 1$, $0 \leq y \leq 1$,
- (iii) $u = 0$ when $y = 0$, $0 \leq x \leq 1$,
- (iv) $u = 4$ when $y = 1$, $0 \leq x \leq 1$.

[11 Marks]

QUESTION FIVE

- (a) What is a partial Integration? [2 Marks]
- (b) Find the general solution of the differential equation $T'' + 4T = 0$. [4 Marks]
- (c) Solve the differential equation $\frac{\partial^2 u}{\partial x \partial y} = \cos(x + y)$ given that $\frac{\partial u}{\partial x} = 2$ when $y = 0$, and $u = y^2$ when $x = 0$. [9 Marks]

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