



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024 1 EXAMINATION

Course Code: MTH381

Credit Unit: 3

Total: 70 Marks

Course Title: Mathematical Methods III

Time Allowed: 3 Hours

Instruction: Answer Question One (1) and Any Other Three (3) Questions

QUESTION ONE

- (a) (i) If functions u, v, w of three independent variables x, y, z , define the Jacobian of the function. If $u = xyz$, $v = x^2 + y^2 + z^2$, $w = x + y + z$, find $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$
- (ii) Given the functions $u = x^2 + y^2$, $v = 2xy$, determine the Jacobian. **10 Marks**
- (b) Determine the Wronskian of set $\{e^x, e^{-x}\}$. **4 Marks**
- (c) If an entire function $f(z)$ is bounded in absolute value for all z , then $f(z)$ must be a constant. Thus prove the Liouville's theorem. **5 Marks**
- (d) Let $f(z)$ be a function that is analytic inside a simple closed path C and on C , except for finitely many singular point z_1, z_2, \dots, z_k inside C , then
$$\oint_C f(z) dz = 2\pi i \sum_{j=1}^k \text{Res } f(z).$$
Hence, prove the Residue theorem. **6 Marks**

QUESTION TWO

- (a) Given $f(x) = 9 \cos 2x$ and $g(x) = 2 \cos^2 x - 2 \sin^2 x$, show that the function is linearly dependent. **3 Marks**
- (b) Compute the integrals $\int_{-\pi/2}^{\pi/2} \int_0^{2 \cos \theta} r^2 dr d\theta$. **6 Marks**
- (c) Evaluate the integrals $\int_0^1 \int_0^1 (x^2 + y^2) dx dy$. **6 Marks**

QUESTION THREE

- (a) If $A = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int_C A \cdot dr$ from $(0,0,0)$ to $(1,1,1)$ along the following paths C . (i) $x = t, y = t^2, z = t^3$. **4 Marks**
- (b) (i) the straight lines from $(0,0,0)$ to $(1,0,0)$, then to $(1,1,0)$, and the to $(1,1,1)$
(ii) the straight line joining $(0,0,0)$ and $(1,1,1)$. **6 Marks**
- (c) Find the total work done in moving a particle in a force field given by $F = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. **5 Marks**

QUESTION FOUR

- (a) Find the modulus and argument of the complex number $\frac{(1+i)(2+i)}{(3-i)}$. **6 Marks**
- (b) Find the n th root of $\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$ if $n = \frac{3}{5}$. **6 Marks**

(c) If $z = \cos \theta + i \sin \theta$, evaluate $z + \frac{1}{z}$.

3 Marks

QUESTION FIVE

(a) Define the (i) limit of the function $f(z)$ (ii) analytic function of $f(z)$.

4 Marks

(b) Without proof, state the existence of an indefinite integral theorem.

Hence, let $w = f(z) = 2zi + 6z$, find u and v when $z = \frac{1}{2} + 4i$.

6 Marks

(c) By Cauchy's integral formula, evaluate $\int_C \frac{2z^3 - 6}{2z - i} dz$.

5 Marks

NOUNTrade.com