



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024 1 EXAMINATION

Course Code: MTH305

Credit Unit: 3

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other Four (4) Questions

Course Title: Complex Analysis II

Time Allowed: 3 Hours

- 1(a) Find the residue of $\frac{e^{iaz}}{z^4}$ at $z = 0$ [5 Marks]
- (b) Evaluate $\lim_{z \rightarrow 1+i} (2z^2 - z + 5)$ [4 Marks]
- (c) Prove that $\sum_{n=0}^{\infty} \frac{(100+75i)^n}{n!}$ Converges [5 Marks]
- (d) Find the modulus of the complex number $\frac{1+i}{1-i}$. [4 Marks]
- (e) Determine the type of singularity of $f(z) = \frac{\sin z}{z^2}$ at $z = 0$ [4 Marks]
2. (a) Find the harmonic conjugate of $U(x, y) = x^4 - 6x^2y^2 - y^4$. [7 Marks]
- (b) Evaluate $\lim_{z \rightarrow 0} \frac{1 - \cos z}{\sin z^2}$ [5 Marks]
3. (a) Compute the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^{2n+1}}{n!}$ [8 Marks]
- (b) Evaluate $\int_{1+i}^{2+3i} (12z^2 - 4iz) dz$ [4 Marks]
4. (a) If $z_1 = 1 + i$, $z_2 = -2 - 2i$ and $z_3 = 1 - i$
- (i) Evaluate $|z_1 \cdot z_2 \cdot z_3|$. [3 Marks]
- (ii) Find the dot product of $\overline{z_1 + z_2 + z_3}$. [3 Marks]
- (b) Compute $\int_{\gamma} \bar{z} dz$ along the straight line from 0 to $1 + i$. [6 Marks]
5. (a) Find all the complex roots of $z^5 = 32$. [7 Marks]
- (b) Show that $|z + w| \leq |z| + |w|$ for $w, z \in \mathbb{C}$. [5 Marks]
6. (a) Using Cauchy's Residue theorem, evaluate $\int_C \frac{z \cos z}{(z - \frac{\pi}{2})^2} dz$; along $|z - 1| = 1$ [8 Marks]
- (b) Show, by definition, that $\lim_{z \rightarrow 2} 4z - 3 = 5$ [4 Marks]