



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2024 1 EXAMINATION

Course Code: MTH 301

Course Title: Functional Analysis

Credit Unit: 3

Time Allowed: 3 Hours

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other 3 Questions

Q1 (a) Define each of the following:

- (i) a complete metric space (X, d) . **(3marks)**
- (ii) a complete subset. **(3marks)**

(b) Given the space R with the usual metric. Establish that it is complete. **(12 marks)**

(c) Show that if $\{A_i\}_{i \in I}$ is a family of connected subsets of X such that $\bigcap_{i \in I} A_i \neq \emptyset$, then $A = \bigcup_{i \in I} A_i$ is connected. **(7 marks)**

Q2 (a) (i) What is a topological space (X, τ) ? **(4 marks)**

ii) Using the axiom of addition. Show that the system $(R, +, \cdot)$ is an ordered field. **(5marks)**

(b) Differentiate between open set and closed set. **(6marks)**

Q3 (a) Define each of the following:

- (i) a separation **(3marks)**
- (ii) a connected subset Y of X . **(3marks)**

(b) Show that if $\{A_i\}_{i \in I}$ is a family of connected subsets of X such that $\bigcap_{i \in I} A_i \neq \emptyset$, then $A = \bigcup_{i \in I} A_i$ is connected. **(5 marks)**

(c) Differentiate between connected and disconnected topological spaces. **(4marks)**

Q4 (a) Define each of the following:

- i) a continuous function f at the point x_0 . **(4 marks)**
- ii) a continuous function f . **(3marks)**

(b) Show that if $f: X \rightarrow Y$ be a function from a metric space (X, d) to another metric (Y, ρ) and let $x_0 \in X$. Then f is continuous at x_0 if for every sequence $\{x_n\}$ such that $x_n \rightarrow x, f(x_n) \rightarrow f(x_0)$. **(8 marks)**

Q5 (a) Define neighborhood of x .

(4marks)

(b) Established that a subset A of X is open if and only if its complement A^c is closed in X .

(11 marks)

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