



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2023_2 EXAMINATIONS

Course Code: MTH401

Course Title: General Topology I

Credit Unit: 3

Time Allowed: 3 Hours

Instruction: Attempt Number One (1) and Any Other Four (4) Questions

- (a) Define the following terms:

 - condensing point **(3 marks)**
 - accumulation point of F , where F is a subset of E of the metric space (E, d) . **(3 marks)**

(a) Let (E, d_E) be a metric space and let (Y, d_Y) be a subspace of X . Let A be subset of Y . Show that A is closed in Y if and only if there exists a set F which is closed in E such that $A = Y \cap F$. **(8 marks)**

(b) Verify that d is a metric on the real line: $(\mathbb{R}, |\cdot|)$, where \mathbb{R} denotes the set of real numbers and $d : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ be defined by $d(x, y) := |x - y|$ for all $x, y \in \mathbb{R}$. **(8 marks)**
- Show that every compact subset of a metric space is closed and bounded. **(12 marks)**
- Show that a point x of a metric space (E, d) is a cluster point of a sequence $\{x_n\}$ in E if and only if there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \rightarrow x$ as $k \rightarrow \infty$. **(12 marks)**
- Prove that limits of a sequence are unique (i.e. if $\{x_n\}$ converges to both x and x' then $x = x'$). **(12 marks)**
- Prove that $\{x_n\}$ converges to x in E , if and only if $\{d(x_n, x)\}$ converges to 0 in \mathbb{R} . **(12 marks)**
- Prove that every convergent sequence in any metric space is Cauchy. **(12 marks)**