



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2023_2 EXAMINATIONS...

Course Code: MTH341

Course Title: Real Analysis

Credit Unit: 3

Time Allowed: 3 Hours

Instruction: Answer Number One (1) and Any Other Four (4) Questions

1. (a) Let $f(x) = \sin \frac{1}{x}$, $x \neq 0$. Discuss the continuity of $f(x)$ at all x_0 except $x_0 = 0$ (3 marks)

(b) Let $f(x) = \sqrt{x}$, $0 \leq x < \infty$. Discuss the continuity of f on $[0, \infty]$ (4 marks)

(c) When do we say ;

(i) a function f is continuous at X_0 ? (3 marks)

(ii) a function f is continuous from the left at X_0 ? (3 marks)

(iii) a function f is continuous from the right at X_0 ? (3 marks)

(d) When is

(i) a function f said to be non-decreasing on an interval? (3 marks)

(ii) a function f said to be monotonic on I ? (3 marks)

2. Define the following

a. (i) Continuity of a function at a point (3 marks)

(ii). Continuity of a function at a point (3 marks)

b. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as $f(x) = x^2 \cos(\frac{1}{x})$ if $x \neq 0$ and

$f(0) = 0$. Find the derivative of f at $x = 0$, if it exists (3 marks)

c. Let a function $f : [0,5] \rightarrow \mathbb{R}$ be defined as $f(x) = \begin{cases} 2x + 1, 0 \leq x \leq 3 \\ x^2 - 2, 3 \leq x \leq 5 \end{cases}$

is f derivable at $x = 3$. (3 marks)

3. Separate the intervals in which the function f defined on \mathbb{R} by

$$f(x) = 2x^3 - 15x^2 + 36x + 5 \text{ for all } x \in \mathbb{R} \text{ is increasing in every interval} \quad (4 \text{ marks})$$

b. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function defined on \mathbb{R} . Show that

f is differentiable on \mathbb{R} . (4 marks)

d. Show that the function f defined on \mathbb{R} by $f(x) = x^3 - 3x^2 + 3x - 5$ for all $x \in \mathbb{R}$ is increasing in every interval (4 marks)

4. Discuss the Rolle's Theorem for the function defined by

i. $f(x) = x^3 - 6x^2 + 11x - 6$ for all $x \in [1, 3]$ (6 marks)

ii. $f(x) = (x - a)^m (x - b)^n$ for all $x \in [a, b]$ where m and n are positive integers (6 marks)

5. Let f be the function defined on $[-1, 2]$ as $f(x) = |x|$. Find the derivative of f (6 marks)

b. Find whether or not the hypothesis and conclusion of Lagrange's mean value theorem

for the functions defined as $f(x) = \frac{1}{x}$ for all $x \in [1, 4]$ (6 marks)

6. Use Cauchy's mean value theorem to the functions f and g defined as $f(x) = x^2$, $g(x) = x$

for all $x \in [a, b]$. (4 marks)

b. Prove that $\frac{\sin \alpha - \sin \beta}{\cos \beta - \cos \alpha} = \cot \theta$. where $0 < \alpha < \theta < \beta < \frac{\pi}{2}$.

(i) a function f said to be non decreasing on an interval I ? (4 marks)

(ii) a function f said to be monotonic on I ? (4 marks)