



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2023_2 EXAMINATIONS

Course Code: MTH 307

Course Title: Numerical Analysis II

Credit Unit: 3

Time Allowed: 3 Hours

Instruction: Answer Question Number One and Any Other Four Questions

- 1a. Define Polynomial and Polynomial equation. **(3 marks)**
- b. What is the degree of the polynomial involved in these equations:
i. $(5x^2 + 4)(x - 9)$ ii. $(3x^5 + 7x)(1/x + x)$ **(4 marks)**
- c. Find the Cubic approximation to e^x by using Chebyshev polynomial. **(8 marks)**
- d. Calculate a linear least square approximation to $f(x)$, if $f(x) = e^x$ on interval $[-1, 1]$.

(7 marks)

2a. Define $P_n(x)$ by RODRIGUE'S formula and state the three properties of $P_n(x)$. **(4 marks)**

b. Prove the orthogonality of Chebyshev polynomial with respect to weight function

$$w(x) = (1 - x^2)^{-\frac{1}{2}} \text{ where}$$

$$\int_{x_0=-1}^{x_1=1} T_n(x)T_m(x)w(x)dx = \begin{cases} 0, & m \neq n \\ \frac{\pi}{2}, & m = n \neq 0 \\ \pi, & m = n = 0 \end{cases} \quad \mathbf{(8 \text{ marks})}$$

3a. If $f(x)$ be a continuous function for $a \leq x \leq b$, then $C[a, b]$ the set of all continuous real valued function in the interval $[a, b]$, State the infinite or Chebyshev norm and its properties.

(5 marks)

b. Construct the cubic Spline Interpolant to $f(x) = x$ with knots $-1 \ 0 \ 1$ subject to clamped boundary conditions. **(7 marks)**

- 4a. Evaluate $\int_0^{\frac{\pi}{3}} \sin x dx$ with $h = \pi/12$, correct to 5 decimal places using Trapezoidal rule. **(5marks)**
- b. Generate the Chebyshev polynomial up to degree 6 in power of x **(7marks)**
- 5a. Find the fourth degree least square polynomial of $/x/$ over $[-1, 1]$ by means of Legendre Polynomial. **(6marks)**
- b. Compute the min-max polynomial $q_1^*(x)$ to e^x on interval $[-1, 1]$ **(6marks)**
- 6a. Use conditions to classify Partial Differential Equation (PDE) and state the examples for each. **(6marks)**
- b. Solve Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the boundary conditions:
 $u(x, 0) = 1, u(0, y) = 0, u(1, y) = 0, u(x, 1) = 1; 0 \leq x \leq 1, 0 \leq y \leq 1$ **(6marks)**