**NATIONAL OPEN UNIVERSITY OF NIGERIA**
**University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi, Abuja**

**FACULTY OF SCIENCES**
**DEPARTMENT OF MATHEMATICS**
**2022_1 EXAMINATION**

**Course Code: MTH 305**
**Course Title:  Complex Analysis II**
**Credit Unit: 3**
**Time Allowed: 3 Hours**
**Total: 70 Marks**
**Instruction: Answer Question One (1) and Any Other 3 Questions**

Q1 (a) Define each of the following:

(i)   Limit of a complex function $f(z)$.                    **(5 marks)**
(ii)   Essential singularity.                                **(3 marks)**

(b) Establish that $sin^2 z + cos^2 z = 1$                   **(7 marks)**

(c) Determine the poles and the residues at the poles of $f(z) = \frac{3z+1}{(z^2-z-2)}$ **(6 marks)**

(d) State the Residue theorem.                               **(4 marks)**

Q2 (a) State the Cauchy integral formula                     **(5 marks)**

(b) If c is a curve $y = x^3 - 3x^2 + 4x - 1$ joining the points (1,1) and (2,3), show that $\int_c (12z^2 - 4iz)dz$ is independent of the path joining (1,1) and (2,3).     **(10 marks)**

Q3 (a) Differentiate between a single valued and a multiple valued complex function $w(z)$.

                                                             **(5 marks)**

(b) Prove that $cosh^2 z - sinh^2 z = 1$                     **(10 marks)**

Q4 (a) Define each of the following:

  (i)  A continuous complex function $f$ at a point.  **(3 marks)**
  (ii) bounded complex function.  **(4 marks)**

  (b) Find the Laurent series expansion of $f(z) = \frac{1}{z-3}$ valid for $|z| < 3$.  **(8 marks)**

Q5 (a) Define a harmonic function.  **(7 marks)**

  (b)The derivative of the function $f(z) = z^2$ exists everywhere, show that the Cauchy-Riemann equations are satisfied everywhere.  **(8 marks)**

Q6 (a) Define an isolated singular point.  **(5 marks)**

  (b) Determine the poles and the residues at the poles of $f(z) = \left(\frac{z+1}{z-1}\right)^2$.  **(6 marks)**

  (c) State the Moreira's theorem  **(4 marks)**