



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_1 Examinations

Course Code: MTH401
Course Title: General Topology
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks

Instruction: Answer Question One (1) and Any Other 4 Questions

- 1a) State without prove the Holder's inequality. (4 marks)
b) If $f: \mathbb{R}^n \rightarrow \mathbb{R}$, when is f said to be continuous at $\mathbf{a} = (a_1 \cdots a_n)$. (4 marks)
c) Show that if (E, d) is a metric space, $\mathbf{a} \in E$ (a fixed element) and $f: E \rightarrow \mathbb{R}$ such that $f(\mathbf{x}) = d(\mathbf{x}, \mathbf{a})$ for all $\mathbf{x} \in E$. Then f is uniformly continuous on E . (6 marks)

d) Given that $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ such that

$$f(x,y) = \begin{cases} \frac{xy}{x^2 + y^2}, & \text{for } x^2 + y^2 \neq 0 \\ 0, & \text{for } x = y = 0 \end{cases}$$

Verify the continuity of f at $(0,0)$.

(8 marks)

- 2a) Define a metric on a nonempty set E . (3 marks)
b) Verify that the usual metric on \mathbb{R} is a metric (5 marks)
c) State without prove the Cauchy schwartz's inequality. (4 marks)

3a) Define the following terms:

- (i) an open set in a metric space (1 marks)
(ii) an interior point in a metric space (1 marks)
(iii) a closed set in a metric space. (1 marks)

b) Let $E = \mathbb{R}^2$ be endowed with the Euclidean metric $d_2(x,y) = (\sum_{k=1}^n |x_k - y_k|^2)^{\frac{1}{2}}$ for all $\mathbf{x} = (x_1, x_2), \mathbf{y} = (y_1, y_2) \in \mathbb{R}^2$. Describe the sets:

- i) $B((0,0); 1)$ (2 marks)
ii). $\bar{B}((0,0); 1)$ (2 marks)
iii) $S((0,0); 1)$ where $(0,0) \in \mathbb{R}^2$ (2 marks)
iv). $B_r(x_0, y_0)$ for arbitrary where $x_0, y_0 \in \mathbb{R}^2$ and $r > 0$ (3 marks)

4a) Define the following terms

- i) Cauchy Sequence' (2 marks)
ii) Bounded sequence (2 marks)

iii) Cluster point

(2 marks)

b) Let (E, d) , be an arbitrary metric space and let $\{x_n\}$ be a Cauchy sequence in E .

Then is bounded

(6 marks)

5a) State without prove the Pasting Lemma on union of closed sets

(4 marks)

b) Given that $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ such that

$$f(x,y) = \begin{cases} x^2 \sin \frac{1}{y} + y^2 \sin \frac{1}{x}, & \text{for } x \neq 0, y \neq 0 \\ 0, & \text{for } x = 0, y = 0 \end{cases}$$

Verify the continuity of f at $(0,0)$

(4 marks)

bii) Given that $f: \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f(x) = \begin{cases} x^2 + 1, & \text{for } x \leq 0 \\ \frac{1}{2}(x + 2), & \text{for } x \geq 0 \end{cases}$$

Verify the continuity of f on \mathbb{R} .

(4 marks)

6ai) what is a connected metric space?

(2 marks)

ii) What is a connected subspace of a metric space

(2 marks)

bi) Show that $K = (0,2) \cup [3,4]$ is a subspace of a metric space (E, d) is disconnected.

(4 marks)

bii) Show that \mathbb{R}^2 is connected.

(4 marks)