NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
2021_1 Examinations

Course Code:	**MTH315**
Course Title:	**Analytical Dynamics**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One (1) and Any Other 4 Questions**

1. (a) Determine the number of degrees of freedom in each of the following cases:
   (i) 12 particles moving freely in a plane (ii) 23 particles moving freely in space. (4 marks)

   (b) A system of particles consists of a 2-gram mass located at (1, 0, -3), a 7-gram mass at (-1,1,3) and 2-gram mass at (1, -1, 1). Find the center of mass. (6 marks)

   (c) A uniform beam is 100m long and has a mass 100kg and masses of 60kg and 80 kg are suspended from its ends; at what point must the beam be supported so that it may rest horizontally? (8 marks)

   d) A quadrilateral ABCD has masses 1,5, 2 and 3 units located at its vertices, A(3,-2,2), B(2,-2,3), C(1,-2,4) and D(4,1,3). Find the coordinates of the center of mass (4 marks)

2. Three particles of masses 3,2,4 respectively have position vectors
   $r_1 = (t+2)i - 2t^2 j + k$,
   $r_2 = -2ti + 3t^2 j + k$,
   $r_3 = \frac{1}{2}t^2 i - tj + 2tk$, where t is time.
   Find (a) the velocity of the center of mass at time $t = 0$ (b) the acceleration at t=1.(12 marks)
3. A particle of mass 10 units moves along a space curve whose position vector is given as a function of time t by
   $$r = (2t^3 - 3t + 2)i + (6t^2 - t)j + t^3 k$$
   At time $t = 1$, find the (a) momentum (b) force field. (12 marks)
4. A particle of mass 2 moves in a force field depending on time t given by $F = 6t^2 i - 2tj + 4tk$. Assuming that at $t = 0$ the particle is located at $r_0 = 2i - 3j + k$ and has velocity $v_0 = i + 3j - k$, find (a) the velocity (b) the position at any time t. (12 marks)

5. A particle moves along the x axis in a force field having potential

$V = ax^2 - bx^3 + \frac{2}{3}$,

where a and b are positive constants. Determine the point(s) of equilibrium.

(12 marks)

6. (a) State without proof the Liouville's theorem in Hamiltonian theory. (4 marks)

(b) Minimise the integral

$I = \int_0^{\frac{\pi}{2}} \left[3\left(\frac{dy}{dt}\right)^2 - 3y^2 + 6ty\right] dt$, $y(0) = \frac{\pi}{2}$ and $y\left(\frac{\pi}{2}\right) = 0$ (8 marks)