



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**PLOT 91, CADASTRAL ZONE, NNAMDI AZIKIWE EXPRESSWAY, JABI - ABUJA**  
**FACULTY OF SCIENCES**

**DEPARTMENT OF PURE AND APPLIED SCIENCE**

**2020\_2 EXAMINATIONS...**

**COURSE CODE:** PHY 313  
**COURSE TITLE:** MATHEMATICAL METHODS FOR PHYSICS I  
**CREDIT UNIT:** 3  
**TIME ALLOWED:** (2½ HRS)

**INSTRUCTION:** *Answer question 1 and any other four questions*

**QUESTION 1**

Verify that:

- a.  $a(\sqrt{2} - i) - i(1 - \sqrt{2}i) = -2i$  (2mks)
- b.  $(2 - 3i)(-2 + i) = -1 + 8i$  (3mks)
- c. Reduce the quantity  $\frac{5i}{(1-i)(2-i)(3-i)}$  to a real number (3mks)
- d. Show that
  - $\text{Re}(iz) = -\text{Im}z$  (2mks)
  - $\text{Im}(iz) = \text{Re}z$  (2mks)
- e. Write the following functions  $f(z)$  in the forms  $f(z) = u(x, y) + iv(x, y)$  under Cartesian coordinates with  $u(x, y) = \text{Re}(f(z))$  and  $v(x, y) = \text{Im}f(z)$ ,  $f(z) = z^3 + 2 + 1$  (4mks)
- f. Show that if  $f(z)$  is continuous at  $Z_0$ , so is  $|f(z)|$  (5mks)
- g. What is a complex exponential function for  $Z = x + iy$  (1mk)

**QUESTION 2**

Evaluate the following integrals:

- a.  $\int_1^2 (t^2 + i)2 dt$  3mks
- b.  $\int_0^{\pi/4} e - 2it dt$  3mks
- c. Express this equation in the form of  $x + iy$  with  $x, y \in \mathbb{R}$   
$$\frac{i}{1-i} + \frac{1-i}{i}$$
 4mks
- d. What are the two methods used for criterion for convergence? 2mks

### QUESTION 3

- a. If  $C$  is the boundary of a triangle with vertices at the points  $0$ ,  $3i$  and  $-4$  oriented counter clockwise compute the contour integral  $\int_C (e^z - z) dz$  5mks
- b. What is the associative law for multiplication of complex numbers 2mks
- c. Show that  $(z_1 z_2) z_3 = z_1 (z_2 z_3)$  for all  $Z_1, Z_2, Z_3 \in \mathbb{C}$  5mks

### QUESTION 4

Use binomial theorem to expand:

- a.  $(1 + \sqrt{3} i)^{2011}$  6mks
- b.  $(1 + \sqrt{3} i)^{-2011}$  6mks

### QUESTION 5

- a. Show that  $|\cos(z)|^2 = (\cos x)^2 + (\sin y)^2$  for all  $z \in \mathbb{C}$  where  $x = \operatorname{Re}(z)$  and  $y = \operatorname{Im}(z)$  6mks
- b.  $|\sin z|^2 = (\sin x)^2 + (\sin y)^2$  for all complex numbers  $z = x + yi$  6mks

### QUESTION 6

- a. Compute  $\cos\left(\frac{\pi}{3} + i\right)$  6mks
- b. Let  $f(z) = \begin{cases} \frac{z^3}{z^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$  6mks

Show that  $f(z)$  is actually nowhere differentiable i.e. the complex derivative  $f'(z)$

does not exist for any  $Z \in \mathbb{C}$