NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS

Course Code: MTH422
Course Title: Partial Differential Equations
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One and Any Other Four Questions

Q1 (a) Define each of the following:

  i)   a nth order partial differential equation **(2 marks)**
  ii)   a linear partial differential equation   **(2 marks)**
  iii) a slip   **(3 marks)**

  (b) Find the general solution of $xz_x + yz_y = z$. **(8 marks)**

  (c) Given that $xp + yq = pq$. Find the initial element, if $x = x_0, y = 0$ and $z = \frac{x_0}{2}, z(x, 0) = \frac{x}{2}$.   **(7 marks)**

Q2 (a) State the conservation law   **(3 marks)**

  (b) Suppose u is the density of the substance and F is the flux, show that $ut + a(u)u_x = 0$.

                                                             (9 marks)

Q3 (a) State the types of solutions a partial differential equation can have   **(3 marks)**

  (b) Consider $z = px + qy + f(p, q)$, find the complete solution   **(9 marks)**

Q4 (a) How can a second order semi-linear partial differential equation be classified? **(3 marks)**

  (b) Given that $z\frac{\partial^2 u}{\partial x \partial y} + 2x\frac{\partial^2 u}{\partial y \partial z} = 0$ is hyperbolic-parabolic in R^3 and

$$A_{ij} = \begin{pmatrix} 0 & z & 0 \\ z & 0 & x \\ 0 & x & 0 \end{pmatrix}. \quad \text{Find } \lambda. \qquad \textbf{(9 marks)}$$

Q5 (a) What is a well posed partial differential equation?   **(3 marks)**

(b) Show that characteristics are invariant under regular transformation. **(9 marks)**

Q6 (a) Define an element of a stripe   **(2 marks)**

(b) Copy and complete the following:

i) $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial y} = cosxy$ is _____ order and _____ homogeneous PDE   **(2 marks)**

ii) $\frac{\partial^2 u}{\partial x \partial y} + \left(\frac{\partial u}{\partial x}\right)^2 = \frac{\partial y}{\partial z} + z^3$ is _____ order and _____linear PDE. **(2 marks)**

iii) $\frac{\partial^2 u}{\partial t} - c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$ is _____ order and _____ homogeneous PDE   **(2 marks)**

iv) $\left(\frac{\partial^2 u}{\partial x^2}\right)^3 + \left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial y} = 0$ is _____ order and ____ linear PDE   **(2 marks)**

(c) Solve $4(1 + z^3) = 9z^4 pq$ **(2 marks)**