NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OFMATHEMATICS

Course Code:	**MTH421**
Course Title:	**Ordinary Differential equations**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question Number One and any Other Four Questions**

1 (a) (i) Classify the DE $\frac{y''-2}{y'+3} = x$ **(3 marks)**

(ii) Is the following DE Linear or nonlinear ? $\frac{y''-2}{y'+3} = xy$ **(1 mark)**

(iii) find the particular solution of the IVP

$$y' = 6x^2y, y(3) = 1,$$

And give its interval of existence **(3 marks)**

(b) (i) Show that the function $f(x) = \frac{1}{1+x^2}$ is a solution of $(1 + x^2)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$ on every interval $a < x < b$ of the $x - axis$. **(3 marks)**

(b) (ii) given that every solution of

$$\frac{dy}{dx} + y = 2xe^{-x}$$

Maybe be written in the form $y = (x^2 + c)e^{-x}$, for some choice of the arbitrary constant c, solve the given initial value problem

$$\frac{dy}{dx} + y = 2xe^{-x} \quad y(0) = 2$$ **(3 marks)**

(c) Use the operator method described in this section to find the general solution of the given linear system $5x' + y' - 5x - y = 0$ **(5 marks)**

(d) Determine the stability property of the critical point at the origin for the following system

$y_1' = {y_1}^3 - {y_1}^3$ **(4 marks)**

$$y_2' = 2y_1{y_1}^2 + 2y_1{y_1}^2 + 2{y_2}^3$$

2. Use the operator method to find the general solution of the linear system

$x' + y' - 2x - 4y = e'$ **(12 marks)**

3. consider the linear system

$$x' = 5x + 3y, \quad y' = 4x + y$$

(a) Show that

$$x = 3e^{7t}, \qquad y = 2e^{7t} \;\; and \;\; x = e^{-t} \;\; y = -2e^{-t}$$

Are solution of the system. **(5 marks)**

(b) Show that the two solutions of part (a) are linearly independent on every interval $a \le t \le b$, and write the general solution of the system. **(2 marks)**

(c) Find the solution

$$y = f(t), \quad y = g(t)$$

of the system which is such that $f(0) = 0$ and $g(0) = 8$ **(5 marks)**

4. (a) show that the solutions of the following system of differential equations remain bounded at $t \to \infty$

$$u' = v - u$$

$$v' = -u$$ **(3 marks)**

4 (b) let A be the matrix given by: $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$. Find

(i) The eigenvalues **(3 marks)**

(ii) The generalized eigenspaces **(3 marks)**

(iii) A fundamental matrix for the system $y'(t) = A_y$. **(3 marks)**

5. Let $V(x, y) = x^2(x-1)^2 + y^2$. Consider the dynamical system

$$\frac{dx}{dt} = -\frac{\partial V}{\partial x},$$

$$\frac{dy}{dt} = -\frac{\partial V}{\partial y},$$

(a) Find the critical points of this system and determine their linear stability. **(5 marks)**

(b) Show that V decreases along any solution of the system. **(2 marks)**

(c) Use (b) to prove that if $z_0 = (x_0, y_0)$ is an isolated minimum of V then z_0 is an asymptotically stable equilibrium. **(5 marks)**

6. (a) Consider the boundary value problem

$$x\frac{d^2w}{dx^2} + (a-x)\frac{dw}{dx} = -\lambda w$$

$$w(L) = w(R) = 0$$

Where $a, L(> 0)$ and $R(> L)$ are real constants.

By casting the problem in self-adjoint form shows that the eigenfunctions, w_1 and w_2, corresponding to different eigen values, λ_1 and λ_2 are orthogonal in the sense that

$$\int_L^R e^{-x}x^{a-1}w_1w_2dx = \int_L^R e^{-x}x^a\frac{dw_1}{dx}\frac{dw_2}{dx}dx = 0$$

Show also that

$$\lambda_i\frac{\int_L^R e^{-x}x^a\left(\frac{dw_i}{dx}\right)^2 dx}{\int_L^R e^{-x}x^{a-1}{w_i}^2dx}$$

And hence that all eigenvalues are positive. **(6 marks)**

6 (b) Determine the eigenvalues and eigenfunctions of the Sturm-Liouville problem

$$y'' + \lambda y = 0, \quad 0 \le x \le L$$

$$y(0) = 0,\ y(L) = 0$$ **(6 marks)**