

NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS

Course Code: MTH421
Course Title: Ordinary Differential equations
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question Number One and any Other Four Questions

1 (a) (i) Classify the DE $\frac{y''-2}{y'+3} = x$ (3 marks)

(ii) Is the following DE Linear or nonlinear ? $\frac{y''-2}{y'+3} = xy$ (1 mark)

(iii) find the particular solution of the IVP

$$y' = 6x^2y, y(3) = 1,$$

And give its interval of existence (3 marks)

(b) (i) Show that the function $f(x) = \frac{1}{1+x^2}$ is a solution of $(1+x^2)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$ on every interval $a < x < b$ of the x -axis. (3 marks)

(b) (ii) given that every solution of

$$\frac{dy}{dx} + y = 2xe^{-x}$$

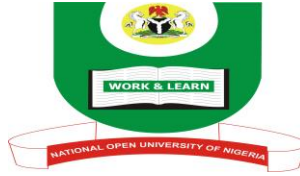
Maybe be written in the form $y = (x^2 + c)e^{-x}$, for some choice of the arbitrary constant c , solve the given initial value problem

$$\frac{dy}{dx} + y = 2xe^{-x} \quad y(0) = 2 \quad (3 \text{ marks})$$

(c) Use the operator method described in this section to find the general solution of the given linear system $5x' + y' - 5x - y = 0$ (5 marks)

(d) Determine the stability property of the critical point at the origin for the following system

$$y_1' = y_1^3 - y_1^3 \quad (4 \text{ marks})$$



$$y_2' = 2y_1y_1^2 + 2y_1y_1^2 + 2y_2^3$$

2. Use the operator method to find the general solution of the linear system

$$x' + y' - 2x - 4y = e' \quad \text{(12 marks)}$$

3. consider the linear system

$$x' = 5x + 3y, \quad y' = 4x + y$$

(a) Show that

$$x = 3e^{7t}, \quad y = 2e^{7t} \quad \text{and} \quad x = e^{-t} \quad y = -2e^{-t}$$

Are solution of the system. (5 marks)

(b) Show that the two solutions of part (a) are linearly independent on every interval $a \leq t \leq b$, and write the general solution of the system. (2 marks)

(c) Find the solution

$$y = f(t), \quad y = g(t)$$

of the system which is such that $f(0) = 0$ and $g(0) = 8$ (5 marks)

4. (a) show that the solutions of the following system of differential equations remain bounded at $t \rightarrow \infty$

$$u' = v - u$$

$$v' = -u \quad \text{(3 marks)}$$

4 (b) let A be the matrix given by: $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$. Find

(i) The eigenvalues (3 marks)

(ii) The generalized eigenspaces (3 marks)

(iii) A fundamental matrix for the system $y'(t) = Ay$. (3 marks)

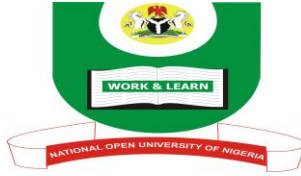
5. Let $V(x, y) = x^2(x - 1)^2 + y^2$. Consider the dynamical system

$$\frac{dx}{dt} = -\frac{\partial V}{\partial x},$$

$$\frac{dy}{dt} = -\frac{\partial V}{\partial y},$$

(a) Find the critical points of this system and determine their linear stability. (5 marks)

(b) Show that V decreases along any solution of the system. (2 marks)



(c) Use (b) to prove that if $z_0 = (x_0, y_0)$ is an isolated minimum of V then z_0 is an asymptotically stable equilibrium. **(5 marks)**

6. (a) Consider the boundary value problem

$$x \frac{d^2 w}{dx^2} + (a - x) \frac{dw}{dx} = -\lambda w$$

$$w(L) = w(R) = 0$$

Where $a, L (> 0)$ and $R (> L)$ are real constants.

By casting the problem in self-adjoint form shows that the eigenfunctions, w_1 and w_2 , corresponding to different eigen values, λ_1 and λ_2 are orthogonal in the sense that

$$\int_L^R e^{-x} x^{a-1} w_1 w_2 dx = \int_L^R e^{-x} x^a \frac{dw_1}{dx} \frac{dw_2}{dx} dx = 0$$

Show also that

$$\lambda_i \frac{\int_L^R e^{-x} x^a \left(\frac{dw_i}{dx}\right)^2 dx}{\int_L^R e^{-x} x^{a-1} w_i^2 dx}$$

And hence that all eigenvalues are positive. **(6 marks)**

6 (b) Determine the eigenvalues and eigenfunctions of the Sturm-Liouville problem

$$y'' + \lambda y = 0, \quad 0 \leq x \leq L$$

$$y(0) = 0, \quad y(L) = 0 \quad \textbf{(6 marks)}$$