NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS

Course Code:	**MTH315**
Course Title:	**Analytical Dynamics**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One and Any Other Four Questions**

1. (a) Determine the number of degrees of freedom in each of the following cases:

   (i) Seven particles moving freely in a plane **(2 marks)**

   (ii) Ten particles moving freely in space. **(2 marks)**

   (b) A system of particles consists of a 3 gram mass located at (2, 0, -1), a 5 gram mass at (-5,1,3) and 2 gram mass at (3, -1, 1). Find the center of mass. **(6 marks)**

   (c) A uniform beam is 72m long and has a mass 200kg and masses of 120kg and 160 kg are suspended from its ends; at what point must the beam be supported so that it may rest horizontally? **(8 marks)**

   (d) A pentagon QRSXW has masses 7, 1, 5, 2 and 3 units located at its vertices Q(1,2,-1), R(3,-2,2), S(2,-2,3), X(1,-2,4) and W(4,1,3). Find the coordinates of the center of mass. **(4 marks)**

2. Three particles of masses 1,3,6 respectively have position vectors

   $r_1 = (2t+2)i - 11t^2 j + (t^3 + 6t - 10)k,$

   $r_2 = -5ti + 3t^2 j + k,$

   $r_3 = \frac{1}{3}t^3 i - tj + 2tk$, where t is time. Find

   (a) The velocity of the center of mass at time $t = 1$ **(8 marks)**

   (b) The acceleration at t=1. **(4 marks)**

3. A particle of mass 5 units moves along a space curve whose position vector is given as a function of time t by

   $$r = (6t^3 - t + 2)i + (2t^2 - t)j + (3t^3)k$$

   At time $t = 2$, find the

   (a) momentum **(7 marks)**

(b) force field. **(5 marks)**

4. A particle of mass 2 moves in a force field depending on time t given by $F = 42t^2i - 8tj + 4tk$. Assuming that at $t = 0$ the particle is located at $r_0 = i - 2j + k$ and has velocity $v_0 = 2i + 3j - 4k$, find

   (a) the velocity, **(6 marks)**

   (b) the position at any time t. **(6 marks)**

5. A particle moves along the x axis in a force field having potential

   $V = \frac{\alpha}{3}x^3 - \frac{\beta}{3}x^2 + 2\gamma x + 10\delta,$

   where α, β, γ and δ are positive constants.

   Determine the point(s) of equilibrium. **(12marks)**

6. (a) State without proof the Liouville's theorem. **(4marks)**

   (b) Minimise the integral

   $I = \int_0^{\frac{\pi}{2}} \left[\left(\frac{dy}{dt}\right)^2 - y^2 + 2ty\right] dt$, $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$ **(8 marks)**