



NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES DEPARTMENT OF MATHEMATICS

Course Code: MTH 308

Course Title: Introduction to Mathematical Modelling

Credit Unit: 3

Time Allowed: 3 Hours

Total: 70 Marks

Instruction: Answer Question Number One and Any Other Four Questions

1. (a) By the use of mathematical expressions, write down the dimensional formulas of the following:

- i. Velocity (2 marks)
- ii. Acceleration (2 marks)
- iii. Force (2 marks)
- iv. Work-done (2 marks)

(b) i. Given that $\frac{dQ}{dt} = kQ$

with solution, $Q = Q_0 e^{-kt}$ where Q_0 is the amount present at time $t = 0$. k is a proportionality constant. What is the dimension of k ? (2 marks)

ii. Interpret the solution obtained for a formulation of the model of a simple pendulum given as:

$$T_0 = 2\pi \sqrt{\frac{l}{g}} \quad (3 \text{ marks})$$

(c) Explain the following steps involved in problem formulation:

- i. Stating the question (3 marks)
- ii. Identifying the relevant factors (3 marks)
- iii. Mathematical description (3 marks)

2. Explain the following concepts as they relate to Mathematical Modeling with relevant examples

- (a) Empirical Models (6 marks)
- (b) Theoretical Model (6 marks)

3. (a) A formulation of the simple pendulum model involves the two differential equations

$$m \frac{d^2x}{dt^2} = -T \sin \theta$$

$$m \frac{d^2y}{dt^2} = T \cos \theta - mg$$

Where T and θ are unknown. Obtain a non-linear differential equation in terms of T and θ **(8 marks)**

(b) With the usual notation of the simple pendulum model, find T_0 if $\theta_0 = 20^\circ$ given that $l = 20 \text{ cm}$ and $g = 980 \text{ cm/sec}^2$ **(4 marks)**

4. (a) Briefly explain the type of modelling will you use for the launching of a rocket/satellite for meteorological purposes. **(4marks)**

(b) Briefly explain the following processes in Mathematical Modelling.

(i) Validation of a model. **(4 marks)**

(ii) Interpreting the solution of a model **(4 marks)**

5. (a) (i) A raindrop, beginning at rest, falls from a cloud 705.6 m above the ground. By adopting a well-known model, determine how long it takes for the raindrop to reach the ground?

(4 marks)

(ii) State two things that could contradict the adopted model in (i) above **(4 marks)**

(b) Explain the concept of evaluation of a mathematical model. **(4 marks)**

6. (a) Suppose that D_t , S_t , and p_t represent the respective demand, supply and price at time t respectively, formulate a difference equation for the dynamic stability of market equilibrium with D_t and S_t given as:

$$D_t = ap_t + b$$

$$S_t = Ap_t + B$$

Where a, b, A and B are all consonants

(8 marks)

(b) List four main classification of Mathematical Models

(4 marks)