



NATIONAL OPEN UNIVERSITY OF NIGERIA
PLOT 91, CADASTRAL ZONE, NNAMDI AZIKIWE EXPRESSWAY, JABI - ABUJA
FACULTY OF SCIENCES
DEPARTMENT OF PURE AND APPLIED SCIENCES
SEPTEMBER, 2020_1 EXAMINATION

COURSE CODE: PHY313
COURSE TITLE: MATHEMATICAL METHOD FOR PHYSICS II
CREDIT UNIT: 3
TIME ALLOWED: (2½ HRS)

INSTRUCTION: *Answer question 1 and any other four questions*

1. a. Obtain the $\frac{df}{dz}$ of the function $f(z) = 4x + y + i(-x + 4y)$ [3 marks]

b. Evaluate the continuity of the followings: (i) $f(z) = \begin{cases} \frac{z^3 - iz + i}{z - i}, & z \neq i \\ 0 & z = 0 \end{cases}$ at $z = i$ [3 marks]

c. Obtain the limit of the following function $\lim_{z \rightarrow 1+i} \frac{z^2 - z + 1 - i}{z^2 - 2z + 2}$ [3 marks]

d. Evaluate the following integral using Residue theorem $\int_c \frac{1+z}{z(2-z)} dz$ [3 marks]

e. Obtain the residue of $\frac{1}{(z^2 + 1)^3}$ at $z = i$ [3 marks]

f. State three rules in obtaining Singularity. [3 marks]

g. Obtain the singularities of the following functions: (i) $f(z) = \sin \frac{1}{z}$ (ii) $g(z) = \frac{e^{1/z}}{z^2}$ [4 marks]

2. a. Show that $f(z) = e^z$ is analytic and that $\frac{de^z}{dz} = e^z$ [4 marks]

b. Using the complex Integral, show that $\int_0^1 (t-1)^3 dt = -\frac{5}{4}$ [4 marks]

c. Show that $\int_0^{\frac{\pi}{2}} e^{t+it} dt = \frac{1}{2}(e^{\pi/2} - 1) + \frac{i}{2}(e^{\pi/2} + 1)$ [4 marks]

3. a. Determine the pole of the function $f(z) = \frac{1}{z^4 + 1}$ [4 marks]

b. Define an Analytic function in a domain, hence write an expression for a function $f(z)$ to be analytic. [4 marks]

c. from equation 1e above, show that $f(z)$ satisfies Laplace's equation. [4 marks]

4. a. State Cauchy Residue Theorem

[4 marks]

b. Show that the function $f(z) = |z|^2$ is continuous everywhere but not differentiable except at the origin. [4 marks]

c. Using Residue theorem, evaluate $\frac{1}{2\pi i} \int_C \frac{e^z dz}{z^2(z^2 + 2z + 2)}$ where C is the circle $|z| = 3$. [4 marks]

5. a. If a function $f(z)$ is analytic in a region R, then obtain its derivative at any point $z = a$ of R [4 marks]

b. Prove that $\int_C \frac{dz}{z-a} = 2\pi i$, where C is the circle $|z - a| = r$ [4 marks]

c. Use Cauchy's integral formula to evaluate $\int_C \frac{z}{(z^2 - 3z + 2)} dz$ where c is the circle $|z - 2| = \frac{1}{2}$ [4 marks]

6. a. Obtain the residue of $f(z) = \frac{ze^z}{(z-a)^3}$ at its pole [4 marks]

b. Use the complex variable technique to obtain the value of the integral $\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta}$ [4 marks]

c. Evaluate $\int_0^{\infty} \frac{\cos mx}{(x^2 + 1)} dx$ [4 marks]