



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.**

**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
**September Examination 2020\_1**

**Course Code: MTH 421**

**Course Title: Ordinary Differential Equations**

**Credit Unit: 3**

**Time Allowed: 3 Hours**

**Total: 70 Marks**

**Instruction: Answer Question Number One (1) and Any Other Four (4) Questions**

1) a. Solve the differential equations

i)  $y' = 1 + y^2$                       ii)  $y' = ky$                        $y(0) = y_0$                       **(6 marks)**

b. Apply the method of variation of parameters to solve

i)  $y'' + y = \tan x$                       **(6 marks)**

ii)  $y'' + 7y' - 8y = e^{2x}$

c. Solve the differential equations

(i)  $(1+x)ydx + (1-y)xdy = 0$                       (ii)  $\frac{dy}{dx} = \frac{1+y}{1+x}$                       **(6 marks)**

d. Describe the homogeneous linear equations with constant coefficients. **(4 marks)**

2) a. Describe the method of underdetermined coefficients. **(3 marks)**

b. Find the general solution of  $y^{iv} - 5y''' + 6y'' + 4y' - 8y = 0$  **(4 marks)**

c. Using the method of undetermined coefficients solve the differential equation

i)  $y'' - 2y' - 3y = 2e^x - 10\sin x$                       **(5 marks)**

ii)  $y'' - 2y' - 3y = 2e^x - 10\sin x$  ,                       $y(0) = 2$ ,  $y'(0) = 4.72$

3) a. Define the following

(i) Uniform convergence (ii) Pointwise convergent (iii) Convergent sequence  
**(5 marks)**

b. Solve  $x^2 \frac{d^2 y}{dx^2} + 7x \frac{dy}{dx} + 8y = 0$ , determine a solution of its adjoint equation by inspection. **(3 marks)**

c. Solve the differential equations

i)  $\frac{dy}{dx} + 2xy = 3x$     (ii)  $(D^2 + 4D + 3)y = \ell^{3x}$  **(4 marks)**

4) a. Use matrices to solve the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} - 12x = 0, \quad x(0) = 0, \quad x'(0) = 8$$
 **(4 marks)**

b. Solve the following systems of differential equation

$$\frac{dx}{dy} = y + 1; \quad \frac{dy}{dx} = x + 1$$
 **(4 marks)**

c. If  $\phi$  is a fundamental matrix for  $\dot{X} = A(t)x$ . Show that  $\psi$  is a fundamental matrix for its adjoint  $\dot{X} = A^T(t)x$  iff  $\psi^T \phi = c$  for some constant nonsingular C matrix C.

**(4 marks)**

5) a. Define the terms Lipschitz continuity and Lipschitz constant of a function in a region.

**(3 marks)**

b. Find Picards first three approximations  $y_1, y_2, y_3$  for the initial value problem  $y' = x^2 + y$ ,  $y(0) = 1$ ,  $D: |x| \leq 1, |y - 1| \leq 1$ . Determine the largest interval on which theorem guaranteed the existence and uniqueness of a solution.

**(5 marks)**

c. Find nontrivial solutions of the Sturm-Liouville problem

$$\frac{d^2y}{dx^2} + y = 0, \quad y(0) = 0, \quad y'(0) = 0.$$
 **(4 marks)**

6) a. i) State the Lyapunov theorem of the system

**(4 marks)**

b. Show that the solution  $\ell^x, \ell^x$  and  $\ell^{2x}$ , of  $y''' - 2y'' - y' + 2y = 0$  are linearly independent.

**(4 marks)**

c. Consider the system

$$\begin{aligned} \dot{X}_1 &= x_1^3 - 2x_1x_2^2, \\ \dot{X}_2 &= x_1^2x_2 - x_2^3 \end{aligned}$$

with the function  $V(x_1, x_2)$  defined by  $V(x_1, x_2) = x_1^2 + x_1^2x_2^2 + x_2^4$  prove that the system is asymptotically stable in the sense of Lyapunov.

**(4 marks)**