NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
October Examination 2019

Course Code: **MTH 382**
Course Title: **Mathematical Methods IV**
Credit Unit: **3**
Time allowed: **3 HOURS**
Instruction: **Answer Question Number One and Any Other Four Questions**

1. (a) Define each of the following:
   (i) an ordinary differential equation. **(2 marks)**
   (ii) a partial differential equation **(1 mark)**
   (iii) an order of a differential equation **(2 marks)**
   (iv) a gamma function $\Gamma(\alpha)$ **(2 marks)**

   (b) Show that:
   (i) $\Gamma(1)=1$ **(3 marks)**
   (ii) $\Gamma(n)=(n-1)\Gamma(n-1)$ **(4 marks)**

   (c) An elastic string with non-zero initial displacement is displaced from its equilibrium position and then released with zero velocity at time $t=0$ to vibrate freely. if $u(x,t)$ satisfy the wave equation $\alpha^2 u_{xx}=u_{tt}, 0<x<1, t>0$ and $\lambda<0$. Find $u(x,t)$ by the method of separation of variables. **(8 marks)**

2. (a) Use the method of successive approximation to obtain the solution of the differential equation $\frac{dy}{dx}=1+xy$ up to the third approximation when $x_0=0$ and $y_0=0$ **(8 marks)**

   (b) Show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$. **(4 marks)**

3. (a) Evaluate $\int_0^\infty t^{\frac{1}{4}} e^{-\sqrt{t}}\,dt$. **(6 marks)**

(b) Given that $\Gamma(n+1)=n!$. Evaluate $2\beta(10,6)$. **(6 marks)**

4. (a) Define each of the following:

(i)Hyper-geometric functions **(2 marks)**

(ii) Bessel's equation of index *v*. **(2 marks)**

(b) Show that $2F(\alpha,\beta,\beta,x)=(1-x)^{-\alpha}$. **(4 marks)**

(c) Prove that $P_2(x)=\frac{1}{2}(3x^2-1)$ by Rodrigue's formula.. **(4 marks)**

5. (a) Show that the two definitions of gamma function are equivalent. **(9 marks)**
(b) State the relationship between gamma and beta functions.
If $R(p)>0$ and $R(q)>0$ **(3 marks)**

6. Find the solution of the Bessel's equation
$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-v^2)y=0$. **(12 marks)**