



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.**

**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
**October Examination 2019**

**Course Code: MTH315**  
**Course Title: Analytical Dynamics**  
**Credit Unit: 3**  
**Time Allowed: 3 Hours**  
**Total: 70 Marks**  
**Instructions: Answer Question Number One and Any Other Four Questions**

1. (a) When is the angular momentum  $L_i$  of the  $i^{th}$  particle said to be conserved **(3 marks)**  
(b) Let  $r_1, r_2, \dots, r_N$  be the position vectors of a system of  $N$  particles of masses  $m_1, m_2, \dots, m_N$  respectively. Define  
(i) The kinetic energy of the system **(2 marks)**  
(ii) The total external torque acting on the system **(2 marks)**  
(c) Define  
(i) Degree of Freedom **(2 marks)**  
(ii) Centre of Mass (centroid) **(2 marks)**  
(b) What is a virtual displacement of a system of  $N$  particles? **(3 marks)**  
(d) A particle of mass  $m$  is connected to a mass  $M$  by means of an inextensible string of length  $L$ . Find the equation of motion leading to small oscillations of the system **(8 marks)**
2. (a) A particle of constant mass  $m$  moves in space under the influence of a force field  $F$ . Assuming that at times  $t_1$  and  $t_2$ , the velocity is  $v_1$  and  $v_2$  respectively, prove that the work done is the change in kinetic energy i.e.  $\int_{t_1}^{t_2} F \cdot dr = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$  **(4 marks)**  
(b) Prove that if  $F$  is the force acting on a particle and  $v$  is the velocity of the particle, then the power applied to the particle is given by  $P = F \cdot v$  **(4 marks)**  
(c) Define the following terms  
i. Holonomic constraint **(2 marks)**  
ii. Non-holonomic constraint **(2 marks)**

3. (a) When is a particle said to move with simple harmonic motion? **(3 marks)**
- (b) Define the following terms
- (i) Amplitude of the motion **(2 marks)**
- (ii) Period of the oscillation **(2 marks)**
- (iii) Rheonomous constraint **(2 marks)**
- (c) Give three examples of simple harmonic motion **(3 marks)**
4. (a) Under what condition can a particle suspended by an elastic string experience simple harmonic motion? **(3 marks)**
- (b) One end of an elastic string of length 24cm is fixed and the other suspended end, a mass of 5kg is attached, which when in equilibrium stretches the string 4cm. The mass is pulled down at a distance of 3cm below its equilibrium position then released. Find the period of oscillation and the maximum kinetic energy of the mass **(9 marks)**
5. (a) State the law for the Impact of spheres **(3 marks)**
- (b) A mass of 10kg rests on a rough horizontal table with coefficient of friction  $\frac{1}{2}$ . It is attached to one end of a light inextensible string which passes through a smooth hole in a mass of 4kg at its free end. If the mass 4g describes a horizontal circle with a velocity of 8m/sec and the mass on the table is on the point of slipping. Find the radius of the circle and the length of string below the table **(9 marks)**
6. (a) State the Newton's Laws of Motion **(3 marks)**
- (b) Three forces of magnitude  $15Q$ ,  $10Q$ ,  $5Q$  act on a particle in directions which make  $120^\circ$  with one another. Find their resultant **(9 marks)**