



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
November Examination 2018

Course Code: MTH421
Course Title: Ordinary Differential Equation
Credit Unit: 3
Time Allowed: 3 HOURS
Total: 70 Marks
Instruction: ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS

1 (a) i) State the Lyapunov theorem of a system (ii) Define positive definite and positive semi definite of a system. (6marks)

(b). Show that the solution e^x , e^{-x} and e^{2x} , of $y''' - 2y'' - y' + 2y = 0$ are linearly independent. (6marks)

(c) Consider the system
$$\begin{aligned}\dot{X}_1 &= x_1^3 - 2x_1x_2^2, \\ \dot{X}_2 &= x_1^2x_2 - x_2^3\end{aligned}$$

with the function $V(x_1, x_2)$ defined by $V(x_1, x_2) = x_1^2 + x_1^2x_2^2 + x_2^4$ prove that the system is asymptotically stable in the sense of Lyapunov (10marks)

2. (a). Define and describe stability at a critical point. (3marks)

(b) Show that a necessary and sufficient condition that f be solution of the nth order ODE

$L_n y = 0$ is the solution of the $(n-1)$ first order linear differential equation $P[y, g(x)] = c$, where $P(u, v)$ is the bilinear concomitant associate with L_n , the function g is a nontrivial solution of the adjoint equation $\overline{L}_n y = 0$, and c is arbitrary constant. (5marks)

(c) Consider the non-linear system
$$\begin{aligned}\dot{x} &= x + 4y - x^2 \\ \dot{y} &= 6x - y + 2xy\end{aligned}$$

Determine the type and stability at the critical point $(0,0)$ of the nonlinear autonomous system. (4marks)

3. (a) Define the terms Lipschitz continuity and Lipschitz constant of a function in a region.

(4marks)

(b). Find Picards first three approximations y_1, y_2, y_3 for the initial value problem

$y' = x^2 + y, \quad y(0) = 1, \quad D: |x| \leq 1, \quad |y-1| \leq 1$. Determine the largest interval on which the theorem guaranteed the existence and uniqueness of a solution. (4marks)

(c) Find nontrivial solutions of the Sturm-Liouville problem

$$\frac{d^2 y}{dx^2} + y = 0, \quad y(0) = 0, \quad y'(0) = 0. \quad (4marks)$$

4.(a) Solve the differential equations

$$(i) (1+x)ydx + (1-y)xdy = 0 \quad (ii) \frac{dy}{dx} = \frac{1+y}{1+x} \quad (4marks)$$

(b) Convert the differential equation $x^2 y'' - xy' + y = \log x$ into a differential equation with constant coefficients. (4marks)

(c) Let I_n be the adjoint operator on L_n . Show that L_n is the adjoint operator of $\overline{L_n}$. (4marks)

5. (a) Discuss solution of the homogeneous linear system. (4marks)

(b) Solve the system $\dot{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ \sin t \end{pmatrix}, \quad x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (4marks)

(c) Solve the differential equation $\ddot{X} - 2\ddot{X} - \dot{X} + 2X = 0,$
subject to $x(0) = 0, \dot{x}(0) = 0, \ddot{x}(0) = 1$ (4marks)

6. (a) Describe the method of undetermined coefficients. (3marks)

(b) Find the general solution of $y^{iv} - 5y''' + 6y'' + 4y' - 8y = 0$ (4marks)

(c) Using the method of undetermined coefficients solve the differential equation

$$i) y'' - 2y' - 3y = 2e^x - 10\sin x$$
$$ii) y'' - 2y' - 3y = 2e^x - 10\sin x, \quad y(0) = 2, \quad y'(0) = 4.72 \quad (5marks)$$