NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
November Examination 2018

Course Code:	**MTH301**
Course Title:	**Functional Analysis I**
Credit Unit:	**3**
Time Allowed:	**3 HOURS**
Total:	**70 Marks**
Instruction:	**ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) Let c $f: X \to Y$ be a function between topological spaces X and Y. Show that f is continuous if and only if $f\left(CI(A) \subseteq CI(f(A)\right)$ for all $A \subseteq X$ (7 marks)

   (b) Let X be a topological space and $U \subseteq A \subseteq X$. Suppose U is open in A and A is open in X.

   Show that U is open in X (2marks)

   (c) Define the following terms (i) Relative Topology (ii) Metric Space (6 marks)

   (d) (i) State two types of metric spaces with one example each (ii) Define closed sphere

   (7marks)

2. (a) Define an open-neighbourhood of a point in a topological space (3marks)

   (b) When is a set with a metric defined on it said to be topological equivalent? (3marks)

   (c) Let X and Y be two metric spaces and $f: X \to Y$ be a function. Show that f is continuous. *iff*

   $f(\overline{A}) \subseteq \overline{f(A)}$ for all $A \subseteq X$ (6marks)

3. (a) Define continuity between topological spaces. (3marks)

   (b) Let $f: X \to Y$ be a continuous map and let (x_n) be a sequence in X converging to $x \in X$.

   Show that $(f(x_n))$ in Y converges to $f(x) \in Y$ (6marks)

   (c) Define Homeomorphism between topological spaces (3marks)

4. (a) Let X be a topological space. Show that the identity map on X is a homeomorphism. (4marks)

(b) Let $f: X \to Y$ and $g: Y \to Z$ be homeomorphisms. Show that

$g \circ f: X \to Z$ is a homeomorphism (4marks)

(c) Let X be a topological space and $A \subseteq X$. Show that

$Cl(A) = \{x \in X :$ every neighbourhood of x meet A$\}$ (4marks)

5. (a) Let X be a topological space. What is a cover of X? (3marks)

(b) When is a topological space compact? (2marks)

(c) Show that every cover of topological space X has a finite subcover. *iff* X is compact. (7marks)

6. (a) Let $f: X \to Y$ be a continuous map of topological spaces with X compact.

Show that f(X) is compact. (6marks)

(b) What are separable spaces? (3marks)

(c) When is a topological space said to be dense? (3marks)