

**NATIONAL OPEN UNIVERSITY OF NIGERIA**

**Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.**

**FACULTY OF SCIENCES**

**DEPARTMENT OF MATHEMATICS**

**JULY EXAMINATION 2017\_1**

**Course Code: MTH312**

**Course Title: GROUPS AND RINGS**

**Credit Unit: 3**

**Time Allowed: 3 HOURS**

**Total Marks: 70**

**Instruction: ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) (i) Define a normal subgroup. (2 Marks)

 (ii) Show that every subgroup H of an abelian group G is a normal subgroup. (4 Marks)

(b) Given H as the subgroup of S3 consisting of elements (1) and (12) and W consisting of

permutations (1), (123), and (132), show that H is not a normal subgroup but W is a normal subgroup. (8 Marks)

(c) Given that N is a subgroup of G. Show that the following statement are equivalent

 i). The subgroup N is normal in G (2 Marks)

 ii). For all  ,  (3 Marks)

 iii). For all  ,  (3 Marks)

1. (a) let  be a group homomorphism, show that
2. Ker f is normal subgroup of G1  (4 Marks)
3. Im f is a subgroup of G2 (4 Marks)

(b) If  is an onto group homomorphism and S is a subset that generates G1, show

 that f(S) generates G2  (4 Marks)

1. (a) If H and K are subgroups of a group G, with K normal in G, show that 

(4 Marks)

(b) If a group G be the internal direct product of its subgroups H and K, show that:

1. Each can be uniquely expressed as x=hk, where  (4 Marks)
2. hk=kh  (4 Marks)

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 4 (a) Let R be a Boolean ring (i.e. , show that also show that R must be commutative (3 Marks)

(b) Let R be a ring, if for a,b,c elements of R, show that

1. a0=0=0a (3 Marks)
2. a(-b)=(-a)b=-(ab) (3 Marks)
3. (-a)(-b)=ab (3 Marks)

5 (a) (i) Find the principal ideals of Z10 generated by and  (3 Marks)

(ii) find the nil radicals of Z8 and p(X) (3 Marks)

(b) Let R be a ring with identity 1, if 1 is an ideal of R and , show that 1=R (6 Marks)

1. a) State and prove the Cayley’s theorem. (6 Marks)

b) Use the Cayley’s table show that  is a group (6 Marks)