

**National Open University Of Nigeria**

**Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi - Abuja**

**Faculty of Sciences**

**October/November Examination 2016**

Course Code: MTH 412

Course Title: Functional Analysis II

Credit Unit: 3

Time Allowed: 3Hours

Total Marks: 70%

**INSTRUCTION: ANSWER ANY FOUR (4) QUESTIONS**

1(a) Let X = **R**2. For each vector = ( x1, x2) ε X. Define ****2 : X →R by

****2 = ()½. Show that **** is a norm on X. (9marks)

1(b) The surface of a unit sphere centered around the origin of a linear space with the $ \ell^p$-norm is the locus of points $ \{ (x_1,x_2,\cdots\}$. Show that

$\displaystyle \left(\sum^\infty_{k=1}\vert x_k\vert^p\right)^{\frac{1}{p}}=1~.
$ (8½marks)

2(a) Let X be a normed space. For all xε X \{o}, show that there exists x\* such that = ‖nd ‖ =1 (8½marks)

2(b) Let X and Y be normed spaces. Show that a linear transformation T : X→Y is open if and only if T(x1) contains Yε for some ε˃0 (9marks)

3(a) What is a Convex set? If xεRnand if r >0; show that the ball

B(s\*, r) ={ tεRn: ‖s\* - t‖ < r} centred at s\* of radius r is a convex set (8marks)

3(b) Let u, and v be vectors in **R**n, show that the line L through u in the direction of v given by L = { u+αv : αε**R}** is a convex set**.** (9½marks)

4 For p [ 1, ] and x ε . Let ‖x‖p = ()1/p .Show that  = { x ε : ‖x‖p } is a subspace of  and (, ‖.‖ ) is a Banach, the p-norm direct sum of the Banach spaces Ea  (17½marks)

5(a) Let ( X, ρ ) be a complete metric space, and let EX. Show that ( E, ρE)is complete if and only if it is closed. (Where ρEis the subspace metric induced

by ρ). (8½marks)

5(b) If X is a topological space and E is a Banach space. Show that Cb(X, E) is a Banach space with respect to the supremum norm ‖.‖ (9marks)

6 Let P be a sublinear functional on the real vector space X and suppose that M is a subspace of X. If f ε M’ is a linear functional that satisfies f(m) ≤ p(m) for all m ε M ( P is a majorant for f ), then show that there exists a linear functional F X’ such that F \ M = f and F(x) ≤ p(x) for all x ε X (17½marks)