

**National Open University Of Nigeria**

**Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi - Abuja**

**Faculty of Sciences**

**October/November Examination 2016**

Course Code: MTH301

Course Title: Functional Analysis I

Credit Unit: 3

Time Allowed: 3Hours

Total Marks: 70%

**INSTRUCTION: ANSWER ANY FOUR (4) QUESTIONS**

1(a) Define a metric space? Give one example of a metric space. (4½marks

1(b) What is meant by a topological space? Give an example of a topological space.

 (5marks)

1(c) Define the length or norm of a vector x ε R3 (8marks)

2(a) Let E||.|| be a normed space. Then the addition , scalar multiplication and the norm mapping itself are continuous ( with respect to metric induced by the norm). That is, if Xn $\rightarrow $x, Yn$ \rightarrow $ y and in E and **λ**n$\rightarrow $ λ in **R.**

Show that Xn + Yn $\rightarrow $ x + y, Xn$\rightarrow $ x and **λ**nXn$\rightarrow $ λx and ‖ Xn ‖$ \rightarrow $ ‖ x ‖ (8½marks)

2(b) Let X be a complete metric space and {An} is countable collection of dense open subset of X. Show that An is not empty. (9marks)

3(a) Let M = { A, d} be a metric space. Given any four points x, y, z, t ε A. Prove that

 d(x, z) + d(y, t) ≥ |d(x, y)- d(z, t)| (9marks)

3(b) Let ‖.‖ be a normed space. Then E is complete if and only if every absolutely convergent series in E converges (8½marks)

4(a) Let X be a normed space and let Y be a closed subspace of X. If f is continuous, bounded, real-valued function in Y. Show that we can find a continuous, bounded real-valued function F on X such that F\y = f and ‖F ‖$\infty $ = ‖f ‖$\infty $

 (8½marks)

4(b) Prove that for any s, t ε,

 max(s,v) = ½[t+ v + |t-v|],

min(t,v) = ½[t+v- |t-v|].

 (9marks)

5(a) Let ( X, H) and ( Y, S) be topological spaces. Show that a function f : X$\rightarrow $ Y is continuous is and only if f(Xa) $\rightarrow $f(x) for every net (Xa)aεA such that Xa$\rightarrow $ x

 (10marks)

5(b) Define the legth or norm of a vector x ε R2 (7½marks)

6 Let (S, d) and (T, d) be metric spaces and f a mapping of S into T. Let τ1 and τ2 be the topologies determined by d and d1 respectively. Then f(S, τ)(T, τ) is continuous if and only if Sn→S→f(Sn,τ)→f(s); that is if s1, s2, . . . , sn, . . . , is a sequence of points in (S, d) converging to x, show that the sequence of points f(s1), f(s2), . . . , f(sn), . . . in (T, d) converges to x. (17½marks)