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**NATIONAL OPEN UNIVERSITY OF NIGERIA**

**14-16 AHMADU BELLO WAY, VICTORIA ISLAND LAGOS**

**SEPTEMBER/OCTOBER 2015 EXAMINATION**

**SCHOOL OF SCIENCE AND TECHNOLOGY**

**COURSE CODE: MTH 411**

**COURSE TITLE: MEASURE THEORY& INTEGRATION**

**TIME: 3 HOURS TOTAL: 70%**

**CREDIT UNIT: 3**

**INSTRUCTION:ANSWER ALL QUESTIONS IN SECTION A AND ANY TWO (2) FROM SECTION B.**

 MTH 411 (section A)

A1. Let E be a subset of and int(E), the set of all interior points of E, then int(E) = if and only if (E) = 0 (Here denotes the outer measure)

(i). Is the above statement true? 1mark

(ii). Justify your answer in a. 2½marks

A2. There is a subset A of which is not measurable, but such that B={xA: x is irrational} is measurable.

(i) Is the above statement true or false? 1mark (ii) Justify your answer. 2½marks

A3. (i) Does there exist a non-measurable subset of whose complement in has outer

measure zero? 1mark

 (ii) Support your answer. 2½marks

A4. (i) Do there exist two non-measurable sets, whose union is measurable? 1mark

 (ii) Justify your answer. 2½marks

A5. If the boundary of Ωhas outer measure zero then Ω is measurable.

 (i) Is this true or false? 1mark

 (ii) Justify your answer in (i) above 2½marks

A6. Let A⊂ [0, 1] be a non-measurable set. Let B ={ (x,0), x.

 (i) Is B a Lebesgue measurable subset of ? 1mark

 (ii) If the closure of E has Lebesgue measure zero, then E has Lebesgue

measure zero. 2½marks

A7. If f: [0,1] R is continuous a.e, then f is measurable.

 (i) Is the above statement true or false? 1mark

 (ii) Justify your answer in (i) above . 2½marks

A8. (i) Does there exist a non-measurable function such that is measurable? 1mark

 (ii) Justify your answer. 2½marks

A9 Letfn be a sequence of continuous functions Lebesgueintegrable on [0, ) which

converges uniformly to a function fLebesgueintegrable on [0, ).

 (i) Is it true that dx = 0 1mark

 (ii) Justify your answer 2½marks

A10. Let { k }be kʃ a sequence of non-negative measurable function on **R** such that kʃa.e in **R**. Then K exists and

(i) Is the above statement true or false? 1mark

(ii) Justify your answer. 2½marks

**SECTION B *ANSWER TWO QUESTIONS ONLY***

1(a) Explain carefully what is meant by the Lebesque Outer Measure μ\*(E) of a subset E of the real line **R** – 5marks

1(b) Prove that for any two subsets A and B of **R**, if A B, then μ\*(A) ≤ μ\* (B)

 – 8marks

1(c) Let E1, E2, E3 . . . , En be disjoint measurable subset of E with μ(E)<, then

Every linear combination S = with real coefficient a1, a2, a3, . . . am is

measurable simple function and IE(s) = . -7marks

2(a) Find the length of the set  -7marks

2(b) Prove that if E is any countable set of real numbers, the = 0 -6marks

2(c) Prove that for every countable familyof subsets of **R,** we have μ\*  ≤  -7marks

3(a) Show that a subset E of **R’** is Lebesque measurable if and only if every subset X of **R’** we have  =  +  -8marks

3(b) Prove that if a subset E of **R’** is measurable, then so is its complement -5marks.

3(c) Prove that if two subsets A and B of the real line are measurable, then so is

*A* -7marks