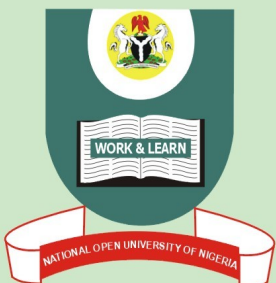


# **PHY 001**

## **ACCESS PHYSICS**



**NATIONAL OPEN UNIVERSITY OF NIGERIA**





**PHY 001**  
**ACCESS PHYSICS**

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Published by  
National Open University of Nigeria

Printed 2008

ISBN: 978-058-593-1

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## **Introduction**

PHY 001: Access Physics is a two-semester, 6 credit-hour foundation (remedial) course in Physics. It is a remedial course offered to students admitted to the School of Science and Technology who are deficient in Physics at the Senior Secondary School level and for whom Physics is compulsory to their programme of study. The course will be done in the first semester simultaneously with other listed courses in the programme on offer. Before registration into second semester is allowed, the student must have remedied the Access Physics course. The examination grade obtained in the remedial course does not count in the GPA of the student at the end of the semester.

As a foundation course meant for students who are deficient in Physics the communicative approach is used. The material has been developed to stimulate and sustain interest in the study of Physics in Nigeria. This is done in recognition that most students dread Physics; resulting in poor performance and an increasing low enrolment in Physics in tertiary institutions in Nigeria despite the desire for technological development.

No compulsory prerequisite is required for this course. It is, however, essential that you revise the Physics topics of the JSS Integrated Science Course to acquaint yourself with what to expect in the related concepts and principles from the course.

The course guide tells you briefly what the course is all about, what course materials you will be using and how you can work your way through these materials. It gives you some guidance on your Tutor-Marked Assignments. You will have detailed information on your Tutor-Marked Assignments in the separate Assignment file. There is/are self-assessment exercise(s) within the body of a unit and/or at the end of



each unit. The exercise(s) is/are an overview of the unit to help you assess yourself at the end of every unit.

## **What You will Learn in this Course**

This course contains several modules. These modules cover the scope and depth of the Physics content level stipulated for the Senior Secondary School programme in conformity with the JCC decision. Mathematics is used to clarify the Physics concepts or principles under consideration as much as possible.

Physics phenomena, facts, laws, definitions, concepts, theories and skills appropriate to Physics permeate the entire modules. The modules dealt with scientific vocabularies, terminologies and conventions including symbols, quantities and units. They described broadly the use of scientific apparatus, including techniques of operation and aspects of safety. They further make you familiar with scientific and technological applications with their social, economic and environmental applications. In addition, they expose you to information handling, experimental and problem -solving techniques.

You will at the end of this course not only have remedied your deficiency in Physics at the Senior Secondary School level but also have laid a solid foundation for further studies in Physics and Physics related courses. I believe you will find the course interesting and motivating.

## **The Course PHY 001**

### **Course Aims**

The aim of this course is to enable you remedy your deficiency in Physics at the Senior Secondary School level. It will help you to lay a solid foundation in Physics on which you will build in your programme of study in the School of Science and Technology. Therefore, the aims of the foundation course in Physics are to enable you:

- (i) acquired proper understanding of the basic principles and applications of Physics;
- (ii) develop scientific skills and attitudes as pre-requisites for further scientific activities;
- (iii) recognize the usefulness and limitations of scientific method and to appreciate its applicability in other disciplines and in everyday life;
- (iv) develop abilities, attitudes and skills that encourage efficient and safe practices;



- (v) develop attitudes relevant to science such as concern for accuracy and
- (vi) precision, objectivity, integrity, initiative and inventiveness.

## Course Objectives

In addition to the aims, the course sets overall objectives which must be achieved. In addition to the course objectives, each of the units has its own specific objectives. You are advised to read properly the specific objectives for each unit at the beginning of that unit. This will help you to ensure that you achieved the objectives. As you go through each unit, you should from time to time go back to these objectives to ascertain the level at which you have progressed.

By the time you have finished going through this course, you should have satisfied these objectives of the Access Physics Course which are to:

- provide you with basic literacy in Physics for functional living in the society;
- enable you acquire basic concepts and principles of Physics as a presentation for further studies;
- enable you acquire essential skills and attitude as a preparation for the technological application of Physics;
- stimulate you and enhance creativity.

## Working through the Course

In order to get the maximum benefit from this Access Course, you would be required to do all that has been stipulated in the Course. That is, you are to STUDY the course units, read the recommended reference textbooks and do all the unit(s) self-assessment exercise(s). At some points in the course you are required to submit assignments for assessment purposes. There is a final examination at the end of the course. This is a 6-credit unit course. As such, it will take you about 40 weeks to complete. One unit requires a total of eight hours (8 hours) study in a week. You will find listed below all the components of the course.

## Course Materials

You will be provided with the following materials:

- (i) Course Guide
- (ii) Study Units in four Modules
- (iii) Assignment File
- (iv) Presentation Schedule



In addition, the course comes with a list of recommended textbooks which though are not compulsory for you to acquire, are necessary for you to use as complements to the study units. It is advised that you acquire some of these textbooks and read them to broaden your scope of understanding and as means of gradual build up of your own library.

## Study Units

The following are the units contained in this course:

### Module 1      Motion

- Unit 1      Units and Dimensions, Scalars and Vectors.
- Unit 2      Molecular Theory of Matter
- Unit 3      Motion
- Unit 4      Motion under Gravity
- Unit 5      Simple Harmonic Motion (S. H. M)

### Module 2      Forces

- Unit 1      Forces, Centre of Gravity and Equilibrium
- Unit 2      Friction in Solids and Liquids
- Unit 3      Linear Momentum
- Unit 4      Simple Machines

### Module 3      Mechanical and Heat Energies

- Unit 1      Mechanical Energy
- Unit 2      Elastic Properties of Solids
- Unit 3      Effects of Heat
- Unit 4      Measurement of Temperature
- Unit 5      Transfer of Heat and Heat Capacities
- Unit 6      Latent Heat and Evaporation
- Unit 7      Expansion of Gases

### Module 4      Light Energy

- Unit 1      Reflection of Light



Unit 2	Reflection at Curved (Spherical) Mirrors
Unit 3	Refraction of Light
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## Assessment

There are three components of assessment for this course. These are:

- The self-assessment questions (SAQs) or Exercises
- The Tutor-Marked Assignments (TMAs)
- The End of Course Examination.

The exercises within each unit of the modules are meant to probe your understanding of the concepts in the unit. It is non-grading and as such does not add up to your final grade in the course.

## Tutor-Marked Assignment

There are some tutor-marked assignments. You must compulsorily answer **ALL OF THESE** and submit them for grading. These assignments are in set of five batteries. Each battery is scored over 20 marks. Your best four batteries of assignments will be selected and used for the final grading. The TMAs constitute 40% of your final grade in this course.

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## Summary

PHY 001 intends to help you remedy your deficiency in Physics at the Senior Secondary School level and to build a solid foundation in Physics. It is meant to be a scaffolding upon which you would build the relevant related courses in the course of your programme of study in the School of Science and Technology. Upon the completion of this course, you are supposed to have laid that solid foundation that is expected of you. Remember that your continuing in your programme in this School depends on your successful performance in this course. That is you must



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## **COURSE GUIDE**

### **PHY 001 ACCESS PHYSICS**

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Unit 1	Reflection of Light
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Printed 2008

ISBN: 978-058-593-1

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## MODULE 1

Unit 1	Motion Units and Dimensions, Fundamental and Derived Quantities and Units, Scalars and Vectors
Unit 2	Molecular Theory of Matter
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Unit 5	Simple Harmonic Motion (SHM)

### UNIT 1 MOTION UNITS AND DIMENSIONS, FUNDAMENTAL AND DERIVED QUANTITIES AND UNITS, SCALARS AND VECTORS

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4.0	Conclusion
5.0	Summary
6.0	Tutor-Marked Assignment
7.0	References/Further Readings



## 1.0 INTRODUCTION

This section is a foundation component of Physics and will introduce you to the concepts of Units, Dimensions, Fundamental and Derived Quantities and their Units as well as Scalars and Vectors. It permeates the entire course and you will come across and in fact use the concepts in subsequent sections.

## 2.0 OBJECTIVES

By the end of this unit, you should be able to:

- discuss the significance of units in physical measurements
- distinguish between fundamental and derived quantities
- determine the dimension of at least 5 physical quantities
- explain the concepts of scalars and vectors using specific examples
- resolve a vector in a given direction
- resolve any number of vectors into two components at right angles to each other
- solve simple problems involving vectors using various methods.

## 3.0 MAIN CONTENT

### 3.1 Units and Dimensions

#### 3.1.1 Units

A unit has to be defined before any kind of measurement can be made. You are aware that different systems of units have been used in the past. However, the system which has gained universal acceptance is the System International d' Unites usually called S. I. Units (adopted in 1960). The S.I Unit is based on the metre as the unit of length; the kilogram (kg) as the Unit of Mass; the second (s) as the Unit of Time, the ampere (A) the unit of current and the Kelvin (K) the unit of temperature. Other Units are given in Tables 1.1 and 1.2.

#### 3.1.2 Dimensions

We often use the term dimension in Physics to describe the relationship between a physical quantity and the fundamental quantities expressed in terms of the symbols M, L, T of the fundamental quantities mass, length and time respectively. You should note that physical quantities can either be dimensional or dimensionless.



- **Dimensionless Quantities** do not depend on the system of unit in which they are measured. That simply means that they have no units. Examples are angles and their trigonometric ratios (ratio of two length); relative density (or specific gravity), (ratio of two densities); efficiency of a machine (ratio of two quantities of work).
- **Dimensional Quantities** on the other hand depend on the magnitude of the fundamental units in which they are measured and are different in different unit systems. If you now consider that the fundamental quantities are denoted by the symbols M, L, and T, you will be able to determine (derive) the dimensions of Physical quantities by your careful study of these illustrations.
  - i) Area (length x breadth) has dimension of  $L \times L = L^2$
  - ii) Volume (length x breadth x depth) has dimensions of  $L \times L \times L = L^3$
  - iii) Velocity (distance  $\div$  time) has dimension of  $L \div T = LT^{-1}$
  - iv) Density (mass  $\div$  volume) has dimensions of  $M \div L^3 = ML^{-3}$
  - v) Acceleration (velocity  $\div$  time) has dimensions of  $L \div T^2 = LT^{-2}$
  - vi) Momentum (mass x velocity) has dimensions of  $(M \times L) \div T = MLT^{-1}$
  - vii) Pressure (force  $\div$  area) = (mass x acceleration)  $\div$  area has dimension of  $MLT^{-2} \div L^2 = MT^{-2} L^{-1}$

### SELF ASSESSMENT EXERCISE 1

Determine the dimensions of force, work, surface tension, (force per unit length) and power (rate of doing work).

## 3.2 Fundamental and Derived Quantities and their Units

### 3.2.1 Fundamental Quantities and their Units

You may have observed that there are two types of physical quantities. One group of such quantities keeps occurring again and again. They are independent of others and cannot be defined in terms of other quantities.

You may also have noticed that most other quantities depend on them. This group of quantities are called fundamental quantities.

- Fundamental quantities are the basic quantities upon which other quantities depend
- Fundamental units are the basic units upon which other units depend. They are the units of the fundamental quantities.



- You will now study Table 1.1 below showing the fundamental quantities and their units.

Table 1.1: **Examples of Fundamental Quantities and their Units**

Quantity	Unit	Unit abbreviation
Length	Metre	m
Time	Second	s
Mass	Kilogram	kg
Electric Current	Ampere	A
Temperature	Kelvin	K
Luminous Intensity	Candela	cd
Amount of substance	Mole	mol

The unit abbreviation is a capital letter when it refers to the name of a person e.g. Ampere (A) or Kelvin (K).

- Can you think of any other quantity that would belong to the above? Give reasons for your answer.

### 3.2.2 Derived Quantities and their Units

At this point you may have realised that all the quantities with the exception of the seven fundamental quantities belong to this group.

- Derived quantities are those quantities derived by some simple combination of the fundamental quantities.
- Derived units are the units of the derived quantities.
- Example of derived quantities, their derivation and their units are summarised in Table 1.2.

Table 1.2: **Examples of Derived Quantities and their Units**



Derived Quantity	Derivation	Derived Unit
Area (A)	Length x breadth	$\text{m}^2$
Volume (V)	Length x breadth x height	$\text{m}^3$
Density	$\frac{\text{mass}}{\text{volume}}$	$\text{kgm}^{-3}$
Velocity (V)	$\frac{\text{displacement}}{\text{time}}$	$\text{ms}^{-1}$
Acceleration (a)	$\frac{\text{change in velocity}}{\text{time}}$	$\text{MS}^{-2}$
Force (F)	Mass x acceleration	Newton; N
Energy or Work (W)	Force x distance	Joule, J (Nm)
Power (P)	$\frac{\text{work}}{\text{time}}$	Watt, W ( $\text{J S}^{-1}$ )
Momentum	Mass x velocity	$\text{kg. m. s}^{-1}$ , Ns
Pressure (P)	$\frac{\text{force}}{\text{area}}$	$\text{N m}^{-2}$ Pascal, Pa
Frequency (f)	$\frac{\text{number of oscillation}}{\text{time}}$	Per second or $\text{s}^{-1}$ (Hertz, Hz)
Electric charge	$\int \text{idt}$	Coulomb (c)
Electric potential difference	Work/charge	Volt (V)
Electromotive force	Work/charge	Volt (V)
Electric resistance	$\frac{\text{Electric potential difference}}{\text{current}}$	Ohm ( $\Omega$ )
Electric capacitance	Charge/Volt	Farad (F)

- Supplementary Units**

You will notice that the Units of plane angle, the radian and of solid angle, the steradian are not classified as fundamental or derived units. They are sometimes called supplementary units.

- For many measurements, you will realise that the units may be too big or too small and so multiples and submultiples of the basic units are used. These are formed by using the following prefixes shown in Table 1.3.

Table 1.3: **Multiple and Submultiples of Units with their Prefixes and Symbols**



Submultiples	Example	Prefix	Symbol
0.1 or $10^{-1}$	decimetre = $10^{-1}$ m	deci	d
0.01 or $10^{-2}$	centimetre = $10^{-2}$ m	centi	c
0.001 or $10^{-3}$	millimetre = $10^{-3}$ m	milli	m
0.000001 or $10^{-6}$	microfarad = $10^{-6}$ f	micro	m
0.000000001 or $10^{-7}$	nanosecond	nano	n
0.000000000001 or $10^{-12}$	picoampere	pico	p
Multiples	Example	Prefix	Symbol
$10^1$	decametre = $10^1$ m	deca	Da
$10^2$	hectometre = $10^2$ m	hecto	H
1000 or $10^3$	kilometre = $10^3$ m	kilo	K
1,000,000 or $10^6$	megawatts = $10^6$ W	mega	M
1,000,000,000 or $10^9$	gigahertz	giga	G
1,000,000,000,000 or $10^{12}$		tera	T

## SELF ASSESSMENT EXERCISE 2

Distinguish between:

- Fundamental and derived quantities; and
- Fundamental and derived units.

### 3.3 Scalars and Vectors

#### 3.3.1 Concept of Scalars

Scalars quantities are those quantities which have only magnitude or numerical value but no direction, examples are length or distance, mass, volume, density, work, time, speed, temperature and energy.

#### 3.3.2 Concept of Vectors

Though we know that many measurable physical quantities have magnitude or numerical value as well as direction, such quantities are not completely described unless their magnitudes and directions are specified. These groups of quantities are known as vector quantities. Examples are: Weight, displacement, velocity, acceleration, force, momentum, electric field etc.

Vector quantities are those quantities which have both magnitude (and size) and direction.

#### 3.3.3 Distinction between Scalars and Vectors



You can distinguish between scalars and vectors by noting that:

- whereas scalars have only magnitude or numerical value, vectors have both magnitude and direction;
- a scalar can be described completely by indicating only its magnitude whereas a vector can only be specified completely when both its magnitude and direction are included; and
- Scalars add just like ordinary numbers but the addition of vectors must be carried out with due regard to their direction.

### 3.3.4 Vector Representation

Here we consider the various ways of representing a vector.

- In a diagram, a vector is represented by an arrow. The arrow is drawn in the direction of the vector. The arrow head gives the sense of direction. Using a scale, the length of the arrow is chosen to be operational to the magnitude of the vector.

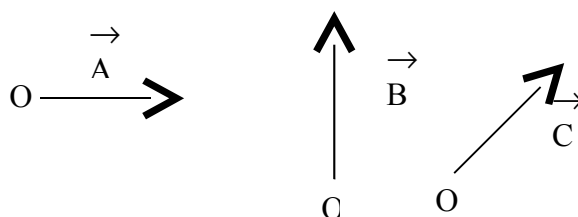


Fig. 1.1: Diagrammatic representation of vectors

- In print, a vector is represented by a bold face symbol such as  $\mathbf{A}$  and  $\mathbf{V}$ . The magnitude of the vector  $\mathbf{A}$  is  $A$  or  $|\mathbf{A}|$
- In writing, it is usual to put an arrow over the symbol to denote a vector quantity e.g.,  $\vec{A}$ ,  $\vec{B}$ ,  $\vec{C}$ , in Fig. 1.1

## 3.4 Addition and Subtraction of Scalars and Vectors

### 3.4.1 Addition and Subtraction of Scalars

You have been informed that the process of adding scalars is straight forward. They add just like ordinary numbers by ordinary algebraic methods.

- Examples:
- $5 \text{ cm}^3 + 7 \text{ cm}^3 = 12 \text{ cm}^3$ ;
  - $5 \text{ kg} - 2 \text{ kg} = 3 \text{ kg}$



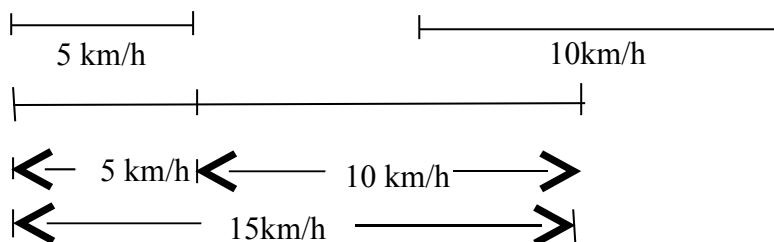


Fig. 1.2: Scalar addition

### 3.4.2 Addition and Subtraction of Vectors

Unlike the scalars, in adding or subtracting vectors, you have to consider both sizes and directions of the vector quantities under consideration.

- Resultant vector

A single vector which has the same effect as two or more vectors acting in the same direction is called the resultant vector. Addition of vectors gives rise to a resultant vector.

#### Examples

Consider the following examples

- Vectors acting in a straight line**
- Vectors acting in the same direction**

Suppose a man walks 2 km due East and his wife walks 3 km due East. What is the resultant vector (velocity)?

#### Solution

Let us represent the magnitude by the scale 1 cm = 1 km; and direction by an arrow head. Therefore, the man covering the distance of 2 km and his wife 3 km both due East would be represented thus:



The resultant vector (R) would take into consideration the magnitude and directions of both journeys as shown below:

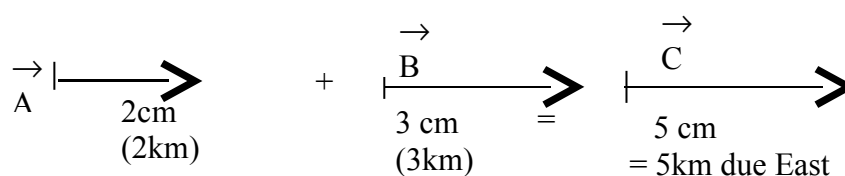




Fig. 1.3: Resultant vector (in one direction)

**b) Vectors acting in opposite directions**

Suppose the man in our example above walks due East while his wife walks due West, the resultant vector would then be as shown in the diagram below:

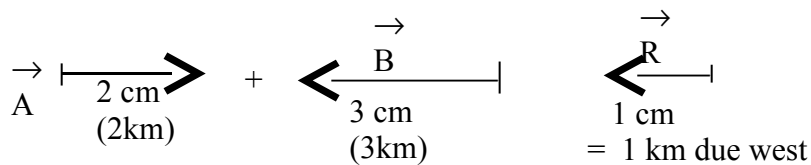


Fig. 1.4: Resultant vector (in opposite direction)

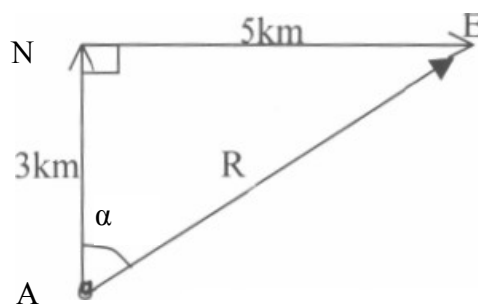
**3.4.3 Vectors Inclined at an Angle**

Let us consider a man starting a journey from a point A and walks 3 km due North, then he turns and walks 5 km due East. What would be his Resultant Vector (displacement)?

- To find the resultant vector, you would either employ the Pythagoras rule (because the vectors are at right angle to each other) or by scale drawing.

**Solution**

(i) Using Pythagoras rule



Scale 1 cm: 1 km

Fig. 1.5: Resultant vector using the Pythagoras rule

- The magnitude of the resultant is given by:

$$\rightarrow \quad \rightarrow \quad \rightarrow$$



$$R^2 = 3^2 + 5^2$$

$$\therefore R = \sqrt{3^2 + 5^2} = \sqrt{9 + 25}$$

$$= \sqrt{34} = 5.83 \text{ km}$$

- To obtain the direction (angle  $\alpha$ ) we use the relation

$$\tan \alpha = \frac{\text{opp}}{\text{Adj}} = \frac{5}{3} = 1.667$$

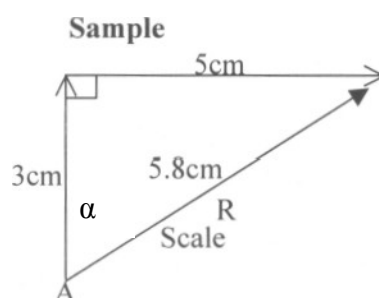
$$\alpha = \tan^{-1} 1.667 = 59.04^\circ$$

- The result of this calculation implies that the resultant vector is 5.83 km, North  $59.04^\circ$  East.

(ii) Using scale drawing

Here you should use a scale of say 1 cm to represent 1 km to find the resultant vector following the directives given below:

- draw 3 cm due North to represent 3 km due North;
- draw 5 cm due East from the head of 3 km due North to represent the 5 km;
- join the tail of 3 kmN to the head of 5 kmE;
- using your ruler, measure the distance AC and convert the result by the scale used; and find the direction by measuring the angle  $\alpha$  using your protractor.



$$R = 5.8 \text{ cm} \times 1 \text{ km} = 5.8 \text{ km} \quad \alpha = 59^\circ \pm 1^\circ$$

Fig. 1.6: Resultant vector using scale drawing

- Note that in general there are three methods of adding or compounding vectors inclined at angles to each other for the purpose of finding the resultant. These are:



- (i) the parallelogram method;
- (ii) the analytical method.
- (iii) the triangle method.

**(i) The Parallelogram method**

In this method you will determine the resultant of two vectors inclined to each other at an angle from the diagonal of a parallelogram drawn with the two vectors as adjacent sides.

**These two vectors must be drawn from a common origin.**

The parallelogram law of vectors states that if two vectors are represented in magnitude and direction by the adjacent sides of a parallelogram, the diagonal of the parallelogram drawn from the point of intersection of the vectors represents the resultant vector in magnitude and direction.

**Worked example**

Find the resultant of two vectors 2N and 5N acting at a point O at an angle of  $60^\circ$  to each other.

**Solution**

- By Scale drawing
  - using a scale (1 cm = 1 N or any other convenient scale) draw a horizontal line;
  - using your ruler, measure (mark) out 5 cm to represent the 5N;
  - using protractor, measure (mark) out angle  $60^\circ$  to the horizontal;
  - draw a line to correspond with the  $60^\circ$  mark;
  - use the ruler to measure (mark) out 2 cm on the line  $OP \rightarrow$  to represent the vector 2N;
  - complete the parallelogram by drawing  $PQ \rightarrow$  parallel and equal to  $OQ$ ;
  - $QC \rightarrow$  parallel and equal to  $AC$ ;
  - join OC ;
  - use the ruler to measure the magnitude OC and the protractor to measure the direction (angle  $\alpha$ ).



- The length of  $\vec{OC}$  gives the magnitude while angle  $\alpha$  gives the direction of the resultant vector.
- Your diagram should look like this.

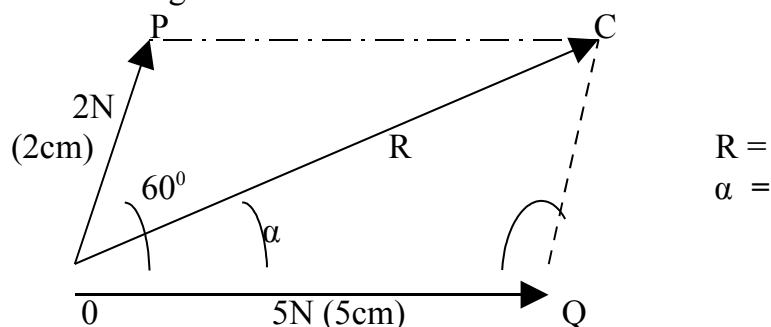


Fig. 1.7: Parallelogram method of determining R

- **The Analytical method**

You can also obtain the resultant vector by Cosine rule. In this case, you only have to draw a sketch of the parallelogram and use the Cosine rule and Sine rule to find the direction.

- Cosine Rule:  $R^2 = OP^2 + OQ^2 + 2 \cdot OP \cdot OQ \cdot \cos 60^\circ$
- Sine Rule:  $\frac{\sin \alpha}{QC} = \frac{\sin B}{OC}$
- To obtain the magnitude using Cosine Rule

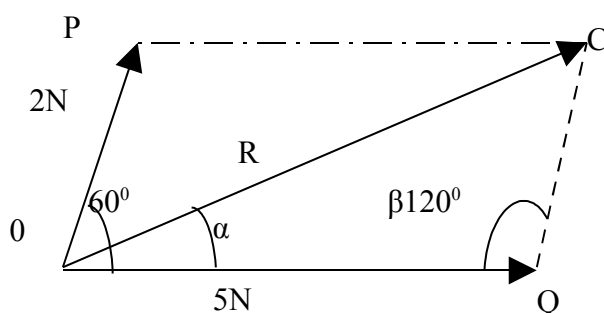


Fig. 1.8: The Cosine rule of obtaining the resultant

$$\begin{aligned}
 \vec{OC}^2 &= \vec{OP}^2 + \vec{OQ}^2 + 2 \vec{OP} \times \vec{OQ} \times \cos 60^\circ \\
 OC &= \sqrt{\vec{OP}^2 + \vec{OQ}^2 + 2 \vec{OP} \times \vec{OQ} (\cos 60^\circ)} \\
 &= \sqrt{2^2 + 5^2 + 2 \times 2 \times 5 \cos 60^\circ}
 \end{aligned}$$



$$= \sqrt{4 + 25 + 20 \times 0.0.5}$$

$$= \sqrt{29 + 10}$$

$$R = OC = \sqrt{39} = 6.25\text{N}$$

- To obtain the direction using Sine rule

$$\frac{\sin \alpha}{OC} = \frac{\sin 120^\circ}{OC} = \frac{0.8660}{OC}$$

$$\sin 120^\circ = \sin (180 - 120^\circ) = \sin 60^\circ$$

$$\sin \alpha = \frac{OC \times 0.8660}{OC}$$

$$= \frac{2 \times 0.8660}{6.25} = \frac{1.732}{6.25} = 0.2656$$

$$\alpha = \sin^{-1} 0.2656$$

$$= 15.4^\circ$$

## (ii) The Triangle method

You can find the resultant of two vectors  $\vec{A}$  and  $\vec{B}$  inclined at an angle  $\theta$  to each other by using the triangle method. To do this, you have to follow these steps:

- Starting from a point O, draw  $\vec{OP}$  (to scale) to represent the vector  $\vec{A}$
- Next, draw the second vector  $\vec{B}$  i.e.  $\vec{PQ}$  (to scale) by placing its tail at the head of the first vector  $\vec{A}$ , ensuring that it is inclined to  $\vec{A}$  by the angle  $\theta$ , and
- Finally, the head of  $\vec{B}$  is joined to the tail of  $\vec{A}$  to make up the third side of the triangle OPQ. OQ represents the resultant of the two vectors  $\vec{A}$  and  $\vec{B}$ .
- The arrows on  $\vec{OP}$ ,  $\vec{PQ}$  and  $\vec{QO}$  follow one another round the triangle. The angle POQ gives the direction of the resultant with respect to the vector  $\vec{A}$  (the above is known as the triangle law of addition of vectors)

## Worked example



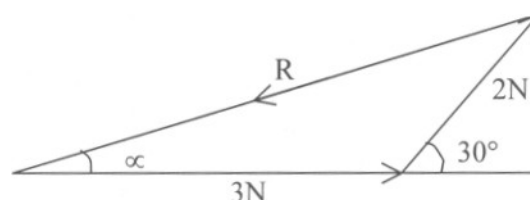
- Two forces 3N and 2N are inclined to each other at  $30^\circ$ . Find the resultant force by the triangle method.

### Solution

Let us use a scale of  $1\text{ cm} = 1\text{ N}$

- first, we draw a horizontal line  $\vec{OA} = 3\text{ cm}$  to represent the 3N force;
- from the tip (head) of  $\vec{OA}$ , we measure out angle  $30^\circ$ ;
- next, we draw a line 2 cm to represent the 2N along the  $30^\circ$  mark (line of inclination to OA extended at  $30^\circ$  mark); and
- finally, we join point O to point B.

The expected result is as shown



Scale:  $2\text{ cm} = 1\text{ N}$

Fig. 1.9: The triangle rule

Measure R and  $\alpha$  to obtain the magnitude and direction of the resultant respectively.

### SELF ASSESSMENT EXERCISE 3

- Outline the differences between Scalars and vectors
- Draw vector diagrams to represent the following vector quantities
  - A vector of 5 km due South
  - A vector of 10 km on a bearing of  $135^\circ$ .
- Find the resultant of the following vectors
  - 
  - 
  -

Fig. 1.10: Vectors in the same and opposite directions



4. A captain directs a ship due North at  $100 \text{ km h}^{-1}$ , while the river is flowing due East at  $30 \text{ km h}^{-1}$ . What is the resultant velocity of the ship?
5. Find the resultant of two vectors  $\vec{A}$  and  $\vec{B}$  of  $3\text{N}$  and  $4\text{N}$  acting at a point  $O$  at an angle of  $45^\circ$  to each other. Use:
  - (i) Parallelogram method
  - (ii) Triangle method.

### 3.5 Resolution of Vectors

Here we consider more than two non-collinear vectors acting at a point. You may obtain the resultant by first reducing the system to two perpendicular vectors with the aid of resolution of vectors and then compounding the two vectors.

- This is a reverse process of finding two vectors which have the same effect as a resultant vector. The resolution of a vector  $\vec{F}$  means that we can express this vector  $\vec{F}$  as the sum of two other vectors called the components of the original vector  $\vec{F}$ .
- Component of a vector in a given direction is its effective value in that direction. Usually, a vector is resolved into components along mutually perpendicular directions. That is, the horizontal or  $x$ -direction and vertical or  $y$ -direction.
- Suppose a vector  $\vec{F}$  is inclined at an angle  $\theta$  to a horizontal component of the vector  $\vec{F}$  as

$F_x = F \cos \theta$ ; and the vertical component of the vector  $\vec{F}$  as

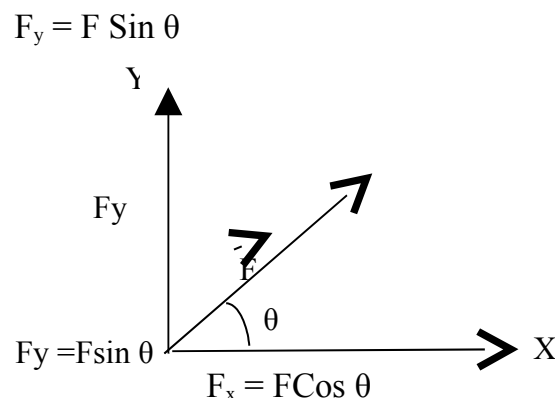


Fig.1.11: Resolution of a vector  $\vec{F}$



### Worked example

A plane flies with a velocity of 500 km /h in a direction N30°E. Find its effective velocity in the easterly and northerly directions.

- You will realise that the question, simply put means to find the two perpendicular components of the velocity.

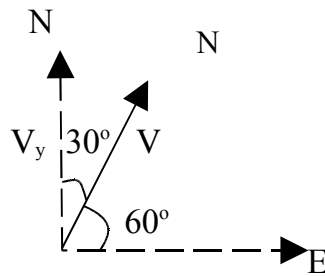


Fig. 1.12: Components of V

- To obtain the velocity in the easterly direction, you will find the horizontal component of the force using the formula  $V_x = V \cos \theta$

$$\begin{aligned} \text{i.e. } V_x &= 500 \cos 60^\circ \\ &= 500 \times 0.5 \\ &= 250 \text{ kmh}^{-1} \end{aligned}$$

- Likewise, to obtain the velocity in the northerly direction, you will find the vertical component of the force using the formula

$$\begin{aligned} V_y &= V \sin \theta \\ \text{i.e. } V_y &= 500 \sin 60^\circ \\ &= 500 \times 0.8660 \\ &= 433 \text{ km h}^{-1} \end{aligned}$$

### 3.6 The Resultant of more than two Vectors

Whenever you are required to find the resultant of more than two vectors, you have to resolve each of the vectors in two perpendicular directions as illustrated in 3.5.

- The right hand or easterly direction is taken as positive while the left hand or westerly direction is taken as negative. Also the southerly or downward direction is taken as negative while the northerly or upward direction is taken as positive.



- After resolving all the forces you then add all the horizontal components  $x$  and the vertical components  $y$ . You then obtain the resultant of  $x$  and  $y$  by the use of the Pythagoras rule.

### Worked example

Determine the resultant of three concurrent forces one of 100N acting horizontally to the right, another of 140N acting upward at an angle of  $60^\circ$  to the horizontal force and the third of 240N acting vertically upward.

### Solution

The following diagram shows the direction of the three forces

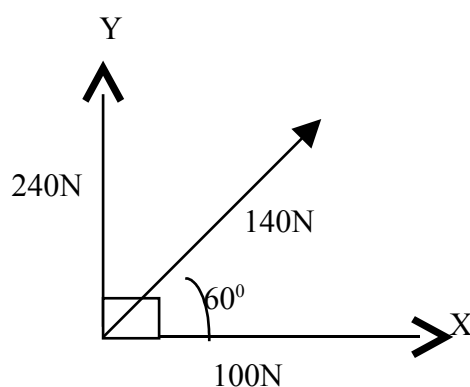


Fig. 1.13: Resolution of three concurrent vectors

Using the resolution of vector method, we shall now resolve the forces into the horizontal and the vertical components as shown in the Table 1.4.

Table 1.4: Resolution of a vector

Force	Inclination to the horizontal $\theta$	Horizontal Component	Vertical Component
240N	$90^\circ$	$240 \cos 90^\circ = 0$	$240 \sin 90^\circ = 240\text{N}$
140N	$60^\circ$	$140 \cos 60^\circ = 70\text{ N}$	$140 \sin 60^\circ = 121.24\text{N}$
100N	0	$100 \cos 0 = 100\text{N}$	$100 \sin 0^\circ = 0$
		$X=170\text{N}$	$Y=361.24\text{N}$



Using Pythagoras rule

$$\begin{aligned}
 \text{The resultant } R &= \sqrt{X^2 + Y^2} \\
 &= \sqrt{170^2 + 361.24^2} \\
 &= \sqrt{28900 + 130494.34} \\
 &= \underline{\underline{399.24\text{N}}}
 \end{aligned}$$

The direction of R is the angle which the resultant R makes with the horizontal and you will obtain this from

$$\tan \theta = \frac{R_y}{R_x} = \frac{361.24}{170} = 2.125$$

$$\begin{aligned}
 \theta &= \tan^{-1} 2.125 \\
 &= 64.8^\circ
 \end{aligned}$$

#### SELF ASSESSMENT EXERCISE 4

Three forces of magnitude 10N; 5N and 4N act on an object in the directions North, West and East respectively. Find the magnitude and direction of their resultant.

#### 4.0 CONCLUSION

In this unit, you have learned about units of measurements, dimensions, fundamental and derived quantities and their units, scalars and vectors. You have also learned to solve numerical questions relating to the unit. Furthermore, you have learned some definitions of the key concepts in the unit using the language of Physics.

#### 5.0 SUMMARY

- Fundamental units are the basic units upon which other units depend. They are the units of the fundamental quantities
- Derived units are the units of the derived quantities
- Fundamental quantities are the basic quantities that are independent of others and cannot be defined in terms of other quantities or derived from them.
- Derived quantities are those obtained by some simple combination of the fundamental quantities.
- Vector quantities, are those quantities which have both magnitude and direction.



- Scalar quantities are those which have only magnitude or numerical value but no direction. Resultant vector is that single vector which would have the same effect in magnitude and direction as the original vectors acting together.
- Components (Resolved parts) of a vector. The component of a vector in a given direction is its effective value in that direction.
- The parallelogram law of vectors states that if two vectors are represented in magnitude and direction by the adjacent sides of a parallelogram, the diagonal of the parallelogram drawn from the point of intersection of the vectors represents the resultant vector in magnitude and direction.

## 6.0 TUTOR-MARKED ASSIGNMENT

1. Distinguish between fundamental and derived quantities. Give five examples of fundamental quantities and their units
2. Which of these quantities: Pressure, volume, velocity, density and momentum are:
  - i) Scalars; and
  - ii) Vectors.
  - iii) Determine their dimensions
3.
  - (a) State the parallelogram law of forces
  - (b) What is meant by the component of a vector in a given direction?
4. A body is under the action of two forces 7N and 10N. Find the resultant of the two forces if:
  - (a) the forces are parallel and act in the same direction.
  - (b) The forces are parallel and act in opposite directions.
5. Four forces of magnitude 10N, 5N, 4N and 6N act on an object in the directions North, West, East and South respectively. Find the magnitude and direction of their resultant (use vector resolution method).



## **7.0 REFERENCES/FURTHER READINGS**

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## UNIT 2      MOLECULAR THEORY OF MATTER

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- 3.0 Main Content
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  - 3.2 Structure of Matter
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    - 3.2.2 Experimental Evidence of the Particle Nature of Matter
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- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor- Marked Assignment
- 7.0 References/Further Readings



## 1.0 INTRODUCTION

In this unit, the Kinetic molecular theory of matter is introduced to you. The theory suggests that matter is made up of small particles called molecules which are in constant motion. You will also learn about the particle nature of matter, molecular and atomic structure and the nature and size of a molecule. The three states of matter, assumptions of the Kinetic molecular theory and use of the Kinetic theory to explain the three states of matter and some common physical phenomena are also considered. The unit ends with a brief discussion of the structure of simple crystals.

## 2.0 OBJECTIVES

At the end of the unit, you should be able to:

- state the molecular theory and explain its justification
- explain how molecules of a substance move relative to other molecules of the same substance
- decide the atomic and molecular structures of matter.
- state the constituents of the atom
- estimate the size of a molecule
- use the molecular theory to explain the three states of matter and some physical phenomena such as change of state, pressure exerted by a gas, osmosis diffusion and surface tension
- state the fundamental assumptions of the kinetic molecular theory of matter
- distinguish between crystalline and non-crystalline or amorphous substances
- give at least two examples of a crystal and describe the structure of simple crystals.

## 3.0 MAIN CONTENT

### 3.1 Concept of Matter

Matter is anything that has weight and occupies space. Every object or substance is made up of matter. Many of the properties and behaviour of substances can best be explained by assuming that all substances are composed of small particles called molecules. The assumption that matter is made up of tiny particles (molecules) which are in constant motion is known as the "molecular theory of matter."



## 3.2 Structure of Matter

### 3.2.1 Evidence of the Particle Nature of Matter

- (i) Many substances in solid form can easily be crushed to powder form e.g. piece of chalk, lump of clay, charcoal and piece of stone.
- (ii) A dry stick or dry wood is easily broken into smaller bits
- (iii) Solubility - if you drop a cube of sugar into a cup of water and turn the water, the sugar "disappears". That is, it dissolves in water.
- (iv) If you scrape the surface of a piece of chalk, you will see thousands of very tiny particles flake off and float through the air (Note that what you see are not the smallest particles possible i.e. molecules. There is a limit to the size of particles the eyes can see)
- (v) If a beam of light (e.g. sunlight) is entering a dusty room through a window, you will observe a chaotic motion of the dust particles in the air.

### 3.2.2 Experimental Evidence of the Particle Nature of Matter

Experimental evidence of the atomic or molecular nature of matter is the Brownian motion named after the botanist, Robert Brown, who discovered the phenomenon in 1827. While he was observing tiny pollen grains suspended in water under a microscope, Brown noticed that the tiny pollen grains moved about in zigzag paths even though the water appeared to be perfectly still. This is due to the molecules of water pollen grains were being knocked about which were vigorously moving about.

Brownian motion can be demonstrated by the smoke-cell experiment. The apparatus is shown in Fig. 2.1.



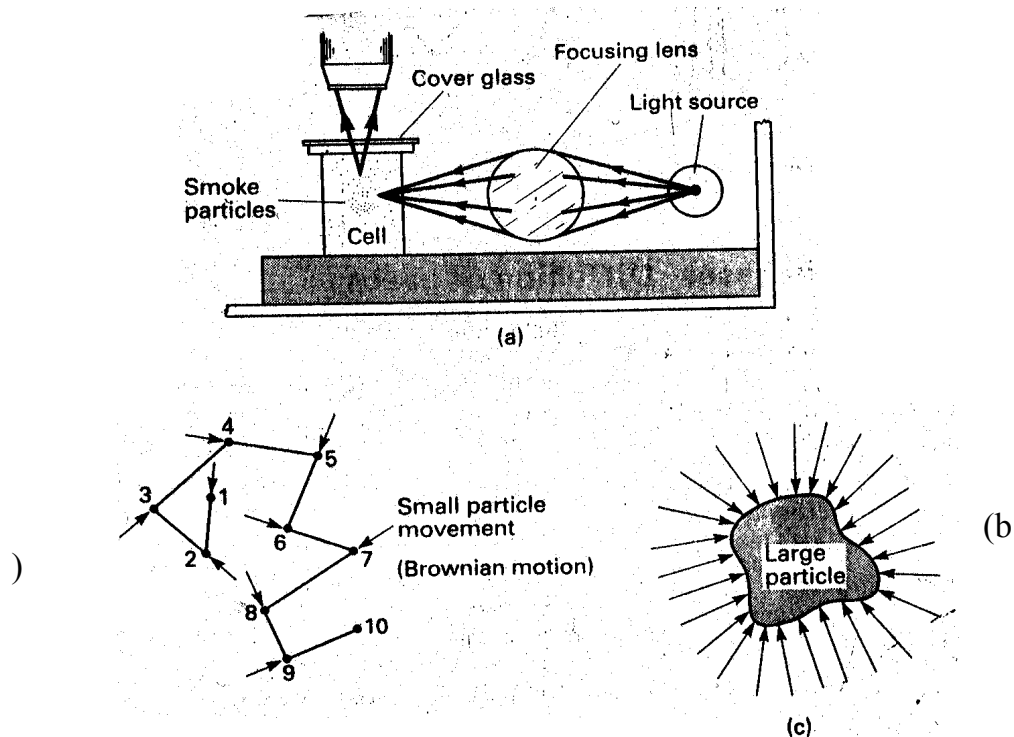


Fig. 2.1: Molecular motion in gas: Brownian (random) motion

Collect some smoke from a smouldering piece of cloth or wood by means of a syringe and introduce it into the cell. Replace the cover quickly and adjust the focus of the microscope until the fine particles come into view clearly. You will observe the smoke particles as black dots which move about irregularly like a (drunkard) drunken man staggering about. The particles dart from one place to another very suddenly, some going out of focus, others coming into focus, but always in motion.

### Explanation of Brownian motion

The irregular movement of the smoke particles is due to the motion of the invisible air molecules which bombard each particle from all sides. The particle is very small and the number of molecules of air hitting one side is not balanced by the number of molecules hitting the opposite side at the same instant. Therefore, the particle moves in the direction of the resultant force and when it moves to another place, the same thing happens. Why can't the motion of the particles be due to convection? In that case the particles would move upwards continually and not zigzag from side to side.

Brownian motion in liquids can be demonstrated similarly as follows:



Place on a clean microscope slide a few drops of diluted aquadag (fine graphite particles suspended in water) or photopake (a similar suspension used for blacking negatives) and cover the liquid with a cover-slip. Project an image of the slide on a screen using a micro projector so that the particles can be seen. The graphite particles are then seen to be moving about in an irregular manner, thus showing Brownian motion in a liquid. In this case, the irregular motion of the graphite particles is due to their bombardment by the surrounding water molecules which are constantly moving about in different directions. Brownian motion is important for two reasons.

- (i) It provides evidence for the existence of the tiny particles of matter called molecules
- (ii) It gives evidence that molecules are in a constant state of random motion.

### 3.2.3 Simple Atomic Structure

#### Definition

An atom is the smallest indivisible particle of an element which can take part in a chemical change.

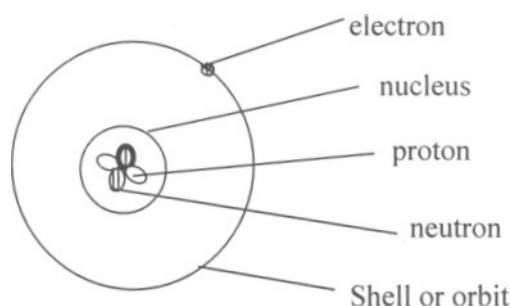


Fig. 2.2: Structure of an atom

Fig.2.2 shows the simple structure of an atom. It consists of two parts: the nucleus and the electrons. The nucleus is the heavy portion of the atom and is made up of two types of particles called protons and neutrons. The protons carry a positive charge while neutrons carry no charge. The electrons carry a negative charge and circle in orbits around the heavy nucleus. The numbers of orbits depend on the substance, for example hydrogen has only one orbit while oxygen has two. An electron is very light (about  $\frac{1}{1840}$  of the mass of the proton). The negative charge of an electron is equal to the positive charge of a proton and the number of electrons in an atom is equal to the number of protons. The atom is therefore electrically neutral.



**SELF ASSESSMENT EXERCISE 1**

1. Define matter and state the molecular theory of matter
2. Mention five simple evidences which you would use to justify the molecular theory of matter
3. Describe an experiment to demonstrate Brownian motion. Why is Brownian motion important?
4. With the aid of a labelled diagram describe the simple structure of an atom
5. Name the constituents of an atom and briefly state the properties of each one.

**3.3 Molecules**

**3.3.1 Definition:** A molecule is the smallest particle of a substance which can have a separate existence and still retain the properties of that substance.

**3.3.2 Structure, Nature and Size of Molecules**

- (i) Most substances cannot exist by themselves as individual atoms, rather they combine their atoms with themselves or with other atoms to form molecules. Thus a molecule may be made up of similar atoms of the same element or different atoms of two or more elements. For example a molecule of hydrogen is made up of two atoms of hydrogen but a molecule of water consists of two atoms of hydrogen and one atom of oxygen.
- (ii) The molecules of any pure substance are identical they have the same structure, the same mass and the same mechanical properties.
- (iii) Molecules are formed by atoms combining in simple proportions
- (iv) The simplest model of a molecule is that of a rigid sphere (like a small billiard ball) which can move and collide with other molecules or with a wall and exert attractive or repulsive forces on neighbouring molecules. The molecular forces decrease as the distance separating the molecules increase.
- (v) Molecules are in constant motion. The motion is random (haphazard or zigzag) in liquids and gases but oscillatory or vibrational in solids.
- (vi) The size of a molecule is extremely small - of the order of  $10^{-9}$  -  $10^{-10}$  m. as a result of the small size, molecules cannot be seen



with the naked eyes or even with the aid of a microscope: Again because of the small size, one gramme of an element contains several millions of molecules. For example one gramme of hydrogen contains about  $10^{23}$  molecules.

### 3.3.3 Some Ideas about Molecular Size

It is difficult to imagine how tiny the diameter of a molecule actually is. Even the numerical value estimated to be of the order of  $10^{-10}$  m ( $10^{-9}$  cm) is also not easily and accurately conjectured. The following may enable you to have a better idea of the size of a molecule:

- (i) If a fine hair is magnified until its thickness is that of a wide street, a molecule in the hair would then look like a speck of dust in the street
- (ii) The tip of a pin contains millions of molecules.
- (iii) Two grains of hydrogen contain  $6 \times 10^{23}$  molecules.

If the whole population of the world were to count such a huge number of molecules individually at the rate of five molecules per second, it would take 100 years to count all of them.

### 3.3.4 Estimating the Size of a Molecule

You can estimate the size or diameter of a molecule by performing the oil film experiment. The principle of the oil film experiment was discovered by Lord Raleigh in 1890. It was known that certain oils when dropped on the surface of water, would spread to form a circular film with the molecules standing up - right. Lord Raleigh argued that if a drop of oil is placed on top of a water surface, the oil will spread out on top of the water surface until the thickness of the oil film is one molecule thick. He, therefore, used this reasoning to obtain the first estimate of the diameter of a molecule. You can repeat Lord Raleigh's experiment as follows:

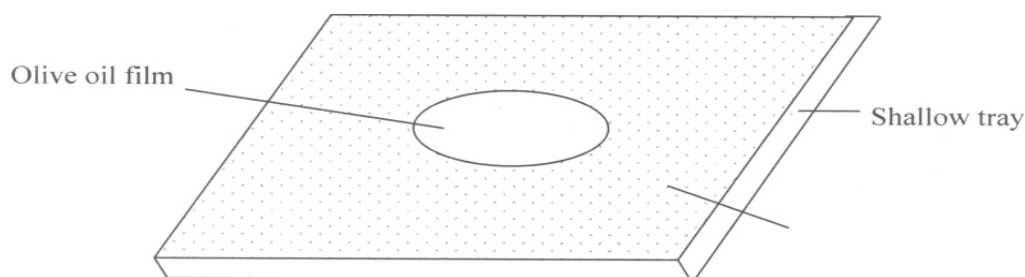


Fig. 2.3: Oil film experiment



- (i) Fill a shallow tray with water and allow it to stand until the water is at rest.
- (ii) Sprinkle some lycopodium powder lightly on the surface of the water
- (iii) Using a graduated pipette with a fine bore, take up a small volume of olive oil and note the reading on the pipette scale
- (iv) Drop a very small quantity of the oil on the water surface. Note again the reading on the pipette and obtain the volume of the oil dropped by subtracting the second from the first pipette reading.
- (v) Allow the oil to spread, pushing the lycopodium powder outwards and forming a clear thin circular film of oil on the water surface.
- (vi) Measure the diameter of the oil film to the nearest centimeter using a half millimetre scale. Calculate the thickness of the oil film as follows:

Let diameter of oil film =  $d$  cm

Let volume of oil drop =  $V$  cm<sup>3</sup>

$$\text{Area of oil film} = \pi \left( \frac{d}{2} \right)^2 \text{ cm}^2$$

$$\therefore \text{Thickness of oil film} = \frac{\text{Volume}}{\text{Area}}$$

$$= \frac{V}{\pi \left( \frac{d}{2} \right)^2}$$

$$= \frac{4V}{\pi d^2} \text{ cm}$$

If you have performed the experiment accurately you should get a value of about  $2 \times 10^{-7}$  cm as the thickness of the oil film. Hence the size of an oil molecule is taken as about  $2 \times 10^{-7}$  cm.



## SELF ASSESSMENT EXERCISE 2

1. Define molecules and distinguish between a molecule and an atom of a substance.
2. Outline briefly the structure and nature of a molecule.
3. The actual size of a molecule is almost impossible to imagine. Discuss.
4. Describe an experiment which you would use to estimate the thickness of an oil molecule. State the principle employed in the experiment you have described.

### 3.4 States of Matter

Matter exists in three main states, namely solid, liquid and gas. Solids have fixed shape and volume. They cannot be poured. Liquids have fixed or constant volumes but they assume the shape of the container. They can be poured. Gases have no fixed shape, but always occupy the shape of the container. They can be poured.

### 3.5 The Kinetic Molecular Theory of Matter

The theory states that matter is made up of tiny particles called molecules which are in constant motion.

#### 3.5.1 Fundamental Assumptions of the Kinetic Molecular Theory

- (i) Matter exists either in solid, liquid or gaseous state.
- ii) All substances consist of molecules, the smallest particle which can exist independently.
- (iii) In solids the molecules vibrate about a mean or fixed position. The forces between the molecules are strong and may be attractive or repulsive. All true solids have a crystalline structure in which the atoms are arranged in regular patterns called lattices.
- (i v) In liquids the molecules move freely in all directions. In addition to vibrational energy, they have translational energy. The Kinetic energy of the liquid molecules is greater than in solids.
- (v) In gases the molecules are in constant motion and are further apart than in solids and liquids. They move at high speeds and have translational, vibrational and in addition rotational energy if



the molecules are made of two or more atoms. The attractive or cohesive force is negligible, so, gases are perfectly free to expand and completely fill the vessels containing them. Gas molecules have the greatest Kinetic energy. Because the intermolecular forces are small, the motion of molecules in the gaseous state is linear until collision takes place either with other molecules or with the walls of the container.

### 3.5.2 Basic Assumptions of the Kinetic Theory of Gases

The Kinetic theory of matter has been more completely developed for gases than for solids and liquids. This is because the problems involved are much simpler in the case of gases. The simplest substance to which the theory has been applied is the ideal gas. The fundamental assumptions of the theory are as follows:

- (i) Gases consist of many very small particles called molecules, which are like perfectly elastic spheres and are usually in constant random motion.
- (ii) Molecules exert no forces on one another except when they collide. Therefore, between collisions with other molecules or with the walls of the container, they move in straight lines.
- (iii) Collisions of molecules with one another or with the walls of the container are perfectly elastic. This means that the total Kinetic energy of two molecules before collision is the same as that after collision, and that when a molecule collides with the wall its Kinetic energy is unchanged.
- (iv) The duration of a collision is negligible compared with the time between collisions
- (v) Molecules are separated by distances which are very large compared with the size of the molecules (or the volume of the molecules is negligible when compared with the volume of the container); they are, however, distributed uniformly throughout the container.
- (vi) Any finite volume of the gas contains a very large number of molecules. This assumption is supported by experimental evidence because under standard temperature and pressure (s.t.p), there are about  $3 \times 10^{19}$  molecules per  $\text{cm}^3$  of any real gas.



### SELF ASSESSMENT EXERCISE 3

1. What do you understand by the Kinetic molecular theory of matter?
2. State (i) The assumptions of the Kinetic theory of matter, (ii) The assumptions of the Kinetic theory of gases.
3. Distinguish between the Kinetic theory of matter and the Kinetic theory of gases

### 3.6 Explanation of some Physical Properties and Processes Using the Kinetic Molecular Theory

#### 3.6.1 The Three Basic States of Matter

- (a) In solids, molecules vibrate about a mean position. The forces between the neighbouring molecules are strong and the attractive and repulsive forces between them balance. Solids therefore have characteristic crystalline structures in which the atoms are arranged in regular patterns called lattices.
- (b) In liquids, the forces between the molecules are less strong than in solids, but they are both cohesive forces. So, the molecules move freely through the liquid, exchanging partners or neighbours as they go. The liquid can thus take the shape of the container and can be poured because the molecules are very free and are in continuous random motion.
- (c) In gases, the molecules are much further apart than those of solids and liquids. They move freely and at high speeds in the space in which they exist. As in liquids they are in constant random motion. The attractive or cohesive force is negligible, and hence a gas can be poured and is perfectly free to expand and completely fill the vessel containing it.

#### 3.6.2 Pressure Exerted by a Gas

A gas consists of molecules in random motion. As the molecules collide with one another and with the walls of the container, they change direction of motion. Therefore, there is always change in momentum, and rate of change of momentum exists at any given time. According to Newton's second law of motion, rate of change of momentum is force. So, a force is exerted on the container as the molecules change momentum. The force per unit area thus exerted is the pressure exerted by the gas.



### 3.6.3 Diffusion

If hydrogen sulphide gas is released at one end of the room, the rotten-egg smell soon fills the whole room.

If a perfume bottle is opened at one corner of a room the perfume smell soon spreads to every corner of the room.

In both cases the molecules of the gas, because of their incessant random motion, have quickly travelled throughout the room and filled the whole space with their smell. The process is known as diffusion.

#### Definition

Diffusion is the tendency of molecules to migrate and fill an-empty space as a result of their random thermal motions.

Diffusion takes place in liquids and gases but more slowly in solids. Its rate depends on the density and temperature of the gas. Light gases at high temperatures diffuse faster than heavy gases at low temperatures.

### 3.6.4 Change of State (Melting, Vaporisation)

Melting - Heat lessens the cohesion which holds molecules together in the crystal lattice of a solid. At the melting point heat breaks the lattice completely and the molecules are free to move about randomly, though still held together by the force of cohesion. During melting the temperature is constant while heat is absorbed to break the lattice.

### 3.6.5 Boiling

At boiling point the temperature of the boiling liquid is again constant. Heat absorbed is used to overcome the force of cohesion by breaking the attractive forces holding the liquid molecules together. Thus the molecules break away and become molecules of gas. The same amount of heat absorbed for melting is given out for fusion (just like in the case of vaporisation and condensation).

### 3.6.6 Evaporation

In a liquid the molecules move randomly with different speeds and they have an average Kinetic energy (k.e). Some of the molecules near the surface which are moving with speed greater than the average speed succeed in escaping from the attraction of their neighbours and jump out of the liquid. This is evaporation. The molecules would have left the liquid to form the vapour. As they escape from the liquid with this extra



energy the temperature of the liquid reduces. Thus we have the cooling effect of evaporation.

### **SELF ASSESSMENT EXERCISE 4**

Use the Kinetic molecular theory to explain the following:

- a) The three states of matter
- b) Pressure exerted by a gas
- c) Diffusion
- d) Change of state (melting, solidification, vaporisation, condensation)
- e) Boiling
- f) Evaporation.

## **3.7 Properties of Fluids at Rest**

### **3.7.1 Adhesion and Cohesion**

Adhesion is the force of attraction between molecules of different substances, e.g. molecules of water and glass.

Cohesion is the force of attraction between molecules of the same substance, e.g. water molecules.

The different behaviours of water and mercury when spilled on a clean glass surface is explained by adhesion and cohesion: The adhesion of water to glass is greater than the cohesion between water molecules, so water spreads out on clean glass and wets it. In the case of mercury, the cohesion of mercury molecules is stronger than the adhesion of mercury to glass. As a result mercury does not wet glass but rather forms into globlets or spherical beads on the glass surface.

### **3.7.2 Surface Tension**

The surface of a liquid behaves as if it were a stretched elastic skin with the result that it can hold an object denser than the liquid gently placed on it. This phenomenon is known as surface tension.

#### **Definition**

Surface tension is the force acting along the surface of a liquid, causing the liquid surface to behave like a stretched elastic scene.



### Examples of the effect of surface tension

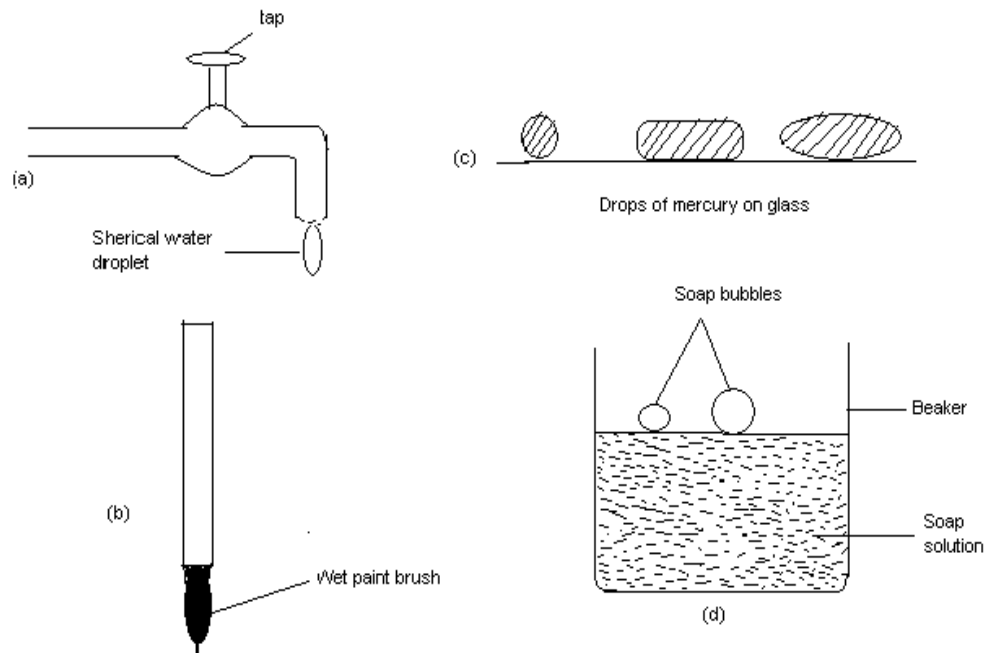


Fig. 2.4: Effects of surface tension

- (i) If water is allowed to drip very slowly from a water tap, the shape of each small drop is spherical (Fig. 2.4a)
- (ii) If a dry paint brush is dipped in water and withdrawn, the hairs of the brush cling to one another (Fig. 2.4b)
- (iii) Droplets of mercury on a clean hard surface such as glass are spherical in shape. Larger drops, because of their weight, are flattened at the top with curved sides (Fig. 2.4c)
- (iv) Soap bubbles are spherical in shape (Fig. 2.4d).  
(You should be able to give some other examples of the effect of surface tension)

### Molecular explanation of surface tension

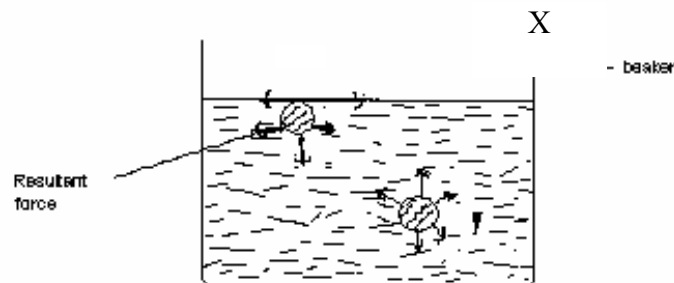


Fig. 2.5: Molecular forces in a liquid

Consider two molecules X and Y in a liquid as in Fig. 2.5. X is on the surface while Y is well below the surface. Y will be attracted uniformly in all directions and is not pulled in any one particular direction, i.e.



there is no resultant force on it as a result of the attraction of the molecules surrounding it. On the other hand X is only attracted by the molecules within the body of the liquid. It therefore experiences a resultant inward force (pull) at right angles to the liquid surface.

All molecules on or near the surface experience this resultant force downwards with the result that the liquid surface will always tend to shrink or contract and make the surface area as small as possible. The surface is therefore in a state of strain or tension which is called surface tension.

### 3.7.3 Capillarity

#### Definition

Capillarity (or capillary action) is the tendency of a liquid to rise or fall in a narrow tube

Illustration of capillarity

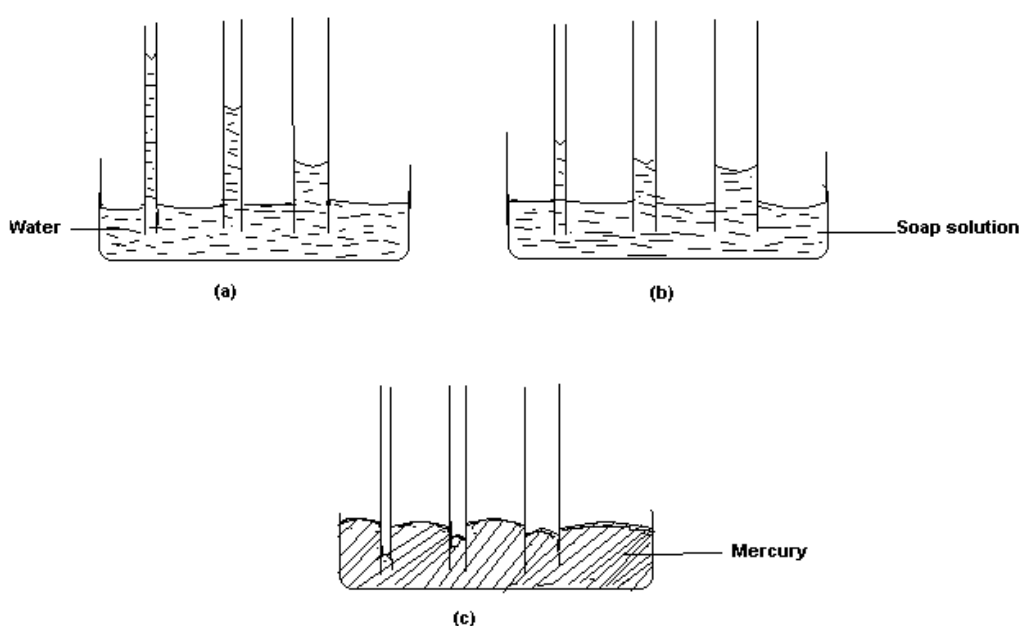


Fig. 2.6: Demonstration of capillary action

- (i) Dip three capillary tubes with fine bores but with different diameters into clean water as in Fig. 2.6 (a) above. You will observe that the water rises in the tubes but the narrower the bore the greater the height to which the water rises (Fig. 2.6 a).



- (ii) Repeat the experiment with soap solution. You will observe a similar situation except that the heights of soap solution are lower than in the case of water (Fig. 2.6b)
- (iii) Repeat the experiment again with mercury. You will observe that the level of mercury falls in the three tubes. The mercury level is depressed below the level of mercury outside the tubes (i.e. in the container). Again the narrower the tube the lower the level of mercury (Fig 2.6c).

### **Explanation of Capillarity**

- (i) The meniscus of water or soap solution is curved upwards (concave) because the adhesion of water and soap solution to glass is greater than the cohesion of water or soap. Therefore water or soap solution wets the glass tube and so spreads a thin film of water/soap solution on the inner surface of the tube. The adhesive forces thus force the water (soap solution) to creep up the inside of the tube. The water/soap solution is held up as it creeps by surface tension forces acting around the circumference of the meniscus. The water/soap solution thus keeps rising in the tube until the weight of the column of water/soap solution balances the surface tension acting at the top of the column.
- (ii) In the case of mercury the cohesion of mercury molecules which is greater than the adhesion of mercury to glass causes the mercury level to be depressed in the tube. The surface tension forces acting around the circumference of the tube holds down the mercury column as it is depressed by cohesive forces of the mercury molecules. The depression continues until the weight of the mercury column in the tube is equal to the surface tension.

You should try and identify other examples of capillarity e.g. absorption of liquid by blotting paper, the rise of oil in wicks of lamps, etc.

### **3.7.4 Osmosis**

#### **Definition**

Osmosis is the tendency of a solvent to pass from a dilute solution, through a semi permeable membrane into a concentrated solution.

Semi-permeable membrane is a substance such as cellophane, parchment or vegetable material which would allow some molecules of liquid to diffuse through them but not others. Such a membrane may



allow the molecules of a solvent to pass through it but not those of a solute.

### Demonstration of osmosis

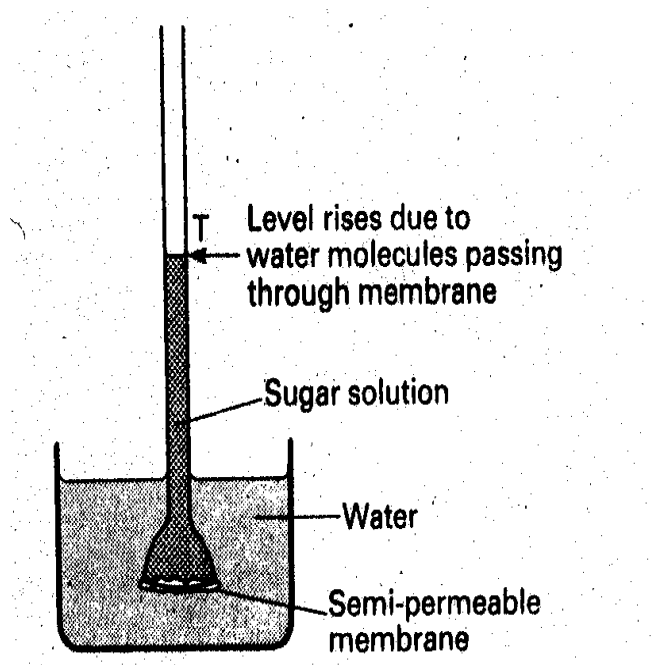


Fig.  
Demonstration of osmosis

2.7:

- (i) Tie a semi-permeable membrane across the mouth of a thistle funnel and pour some concentrated sugar solution into the funnel.
- (ii) Immerse the funnel in a beaker of water as shown in Fig. 2.7 and note the level of sugar solution in the funnel.
- (iii) Allow the set up to stand for a day, and check the level of sugar solution in the funnel again. Test the water in the beaker for sugar. You will observe that the level of the sugar solution in the inverted thistle funnel has risen and that the water in the beaker is free from sugar.

### Explanation

The molecules of sugar (the solute) cannot pass through the semi-permeable membrane, but molecules of water (the solvent) can do so. Sugar and water molecules are bombarding the membrane on one side and only water molecules on the other side. Thus, more water molecules move up into the funnel than down out of it and so the level of liquid in the thistle funnel rises.



### 3.7.5 Viscosity

Brief mention should be made here of the force of viscosity which occurs in fluids at rest or in motion.

**Viscosity** is "friction" in fluids (liquids and gases). It may be defined as "**the internal friction between the layers of a liquid or gas in motion**". Liquids that pour slowly are more viscous than those that pour faster. Thus very cold honey is more viscous than very cold water.

The movement of one layer of fluid over a neighbouring layer is opposed by forces of viscosity. Again, when a stone or a ball-bearing falls through a viscous (thick) liquid, the downward motion of the object is opposed by the viscosity of the liquid. The greater the viscosity of the liquid, the greater the opposition to the movement of the object and hence the slower its motion.

A more detailed discussion of viscosity is done in Module 2, Unit 2.

### SELF ASSESSMENT EXERCISE 5

1. Distinguish between adhesion and cohesion.
2. Define surface tension and mention any three effects caused by surface tension
3. Explain the meaning of  
(i) Capillarity, (ii) Viscosity and describe an experiment to illustrate each of them.
4. How does the Kinetic molecular theory explain surface tension, capillarity and osmosis?
5. Why is it easier to pour out water from a container than to pour out engine oil?

### 3.8 Crystalline and Amorphous Substances

Solids are usually classified into two groups:

- (i) Crystals or crystalline solids;
- (ii) Noncrystal or non-crystalline solids;

The difference between crystals and non-crystals is the arrangement of atoms or molecules in the solid.



### 3.8.1 Crystals

#### Definition

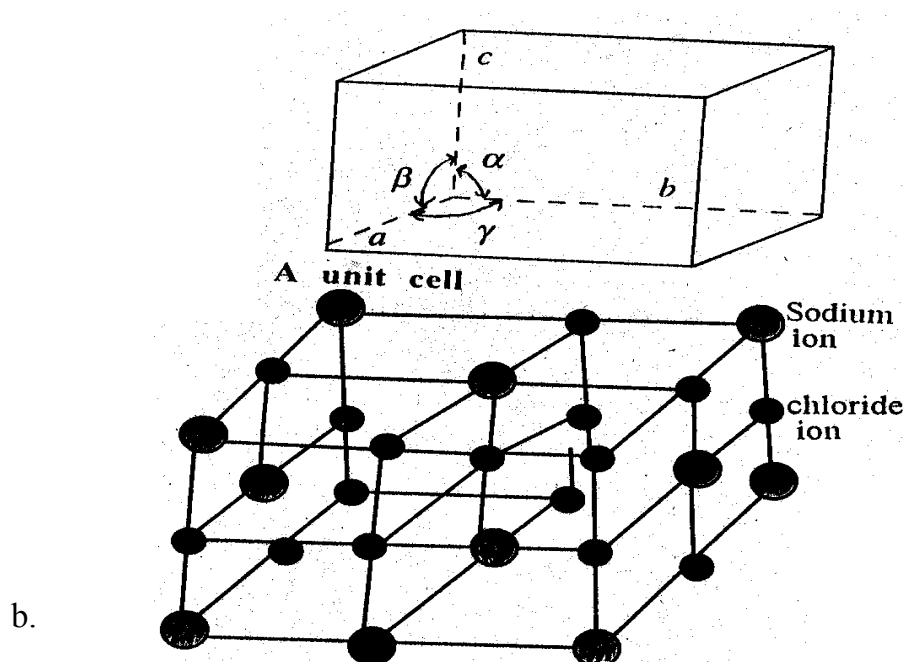
A crystal is a piece of solid matter in which the atoms, molecules or ions are arranged in a highly regular repeating pattern called lattice.

#### Crystal Lattice

The particles in a crystal are arranged in regular 3-dimensional framework or pattern called crystal lattice which repeats over and over again in all directions. The high degree of regularity and order in the arrangement of the molecules is the principal feature distinguishing solids from liquids. Particles in a liquid are jumbled and highly disorganised as they move about. They are even more disorganised in a gas. Examples of common crystals are: sodium chloride, zinc sulphide, chromium, iron and platinum salts.

### 3.8.2 Structure of Simple Crystals

A simple crystal is made up of a huge number of simple basic units or building blocks called unit cells (Fig. 2.8a). If you stack these units up and, down, side by side and in all directions, you can build the whole lattice. Unit cells are of three types, giving rise to 3 types of lattice and hence 3 types of crystals, simple cubic lattice, face-centred cubic lattice and body-centred cubic lattice.



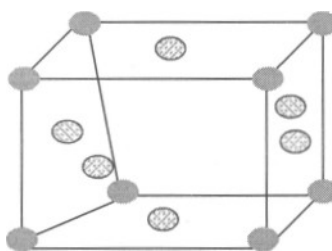
Sodium chloride, NaCl

Fig. 2.8: Simple cubic crystal



### (a) Simple Cubic Crystal

Here the atoms or molecules or ions are placed at the corners of imaginary cubes stacked side by side, up and down like building blocks (Fig.2.8b). An example is the Sodium chloride (NaCl) crystal. In the lattice, the atoms of Na and Cl alternate positions in the cube in each of the 3 directions. Each atom within solid, thus has six immediate neighbours.



Zinc sulphide, ZnS

Fig. 2.9: Face-centred cubic crystal

### (b) Face-Centred Cubic Crystal

The unit cell has identical particles at each of the corners plus another particle in the centre of each face as shown in Fig. 2.9. A typical face-centred cubic.

Crystal is Zinc sulphide (ZnS). Other crystals in this group include crystals of common metals like copper, silver, aluminium, lead, etc.

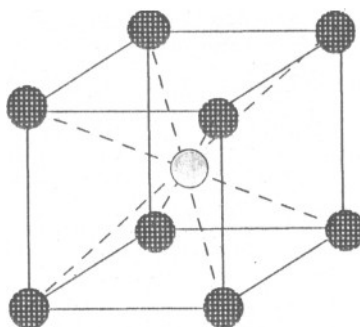


Fig. 2.10: Body-centred cubic crystal

### (c) Body-centred Cubic Crystal

The unit cell has identical particles at each corner of the cube plus one in the centre of the cell as illustrated in [Fig. 2.10. Examples](#) include; chromium, iron and platinum salts.



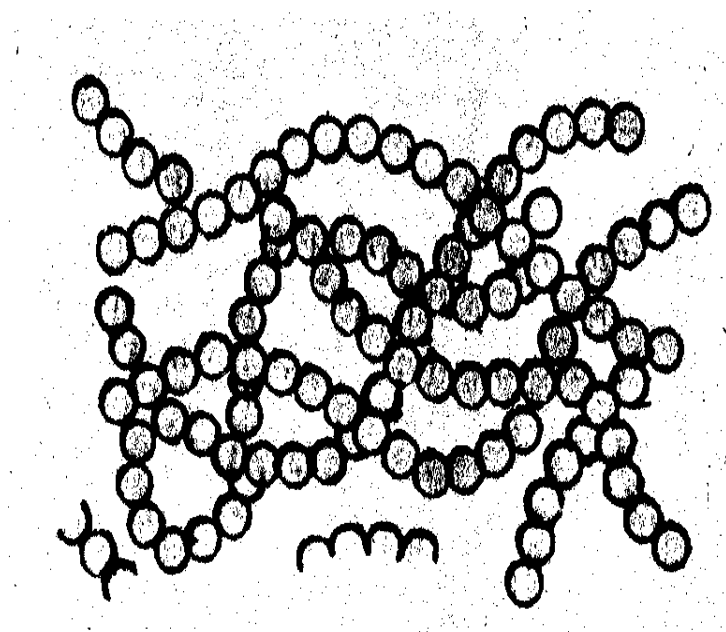


Fig.2.11: Chain-like molecules of amorphous substance

### 3.8.3 Non-Crystalline and Amorphous Solids

The atoms of non-crystals are not regularly arranged as in the case of crystals. They are said to be "amorphous" that is having no definite shape or form; not organised. In a number of ways amorphous substances resemble liquids more than solids. Examples of amorphous solids are glass and plastics. Amorphous solids never form crystals. They are usually made up of long, chain-like molecules that are intertwined in the liquid state just like strands of earthworms (Fig. 2.11)

### 3.8.4 Heating and Cooling of Crystals and Non-Crystals

When heated, crystalline solids, with time, melt at constant temperature. When the melting process is completed, temperature rises until at another constant temperature, the liquid begins to change to gas. Fig. 2.12 shows the heating curve for a crystalline solid.



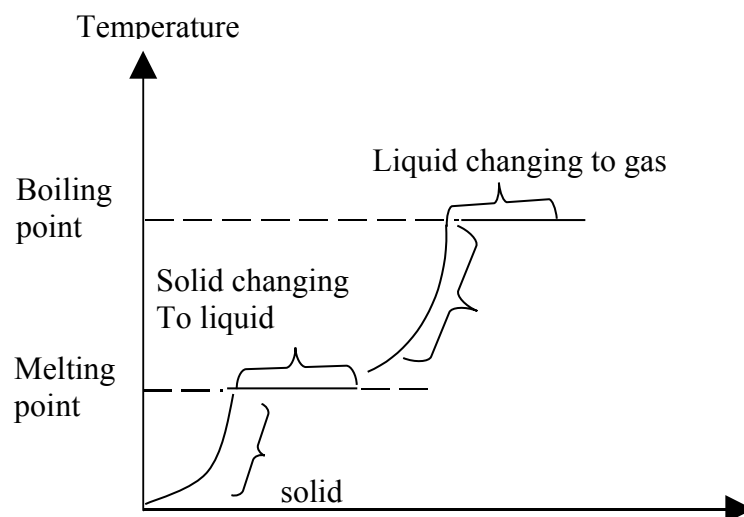


Fig. 2.12: Heating curve for crystalline substances

When crystalline substances are cooled, they begin to solidify at some constant temperature, forming solid [crystal](#). [Fig.2.13](#) shows a typical cooling curve for a crystal cooling from the liquid phase. As soon as the last bit of liquid has solidified, the temperature of the substance drops sharply.

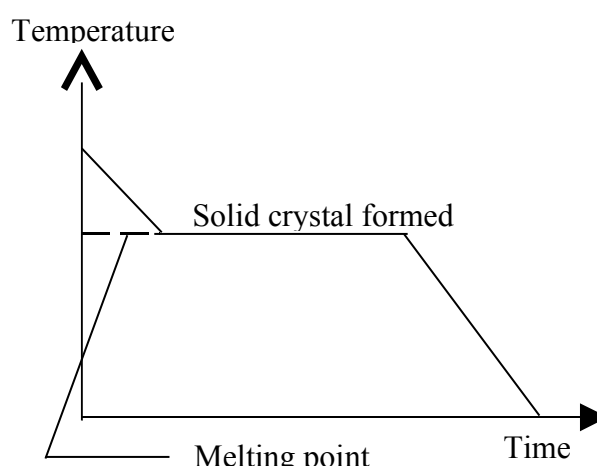
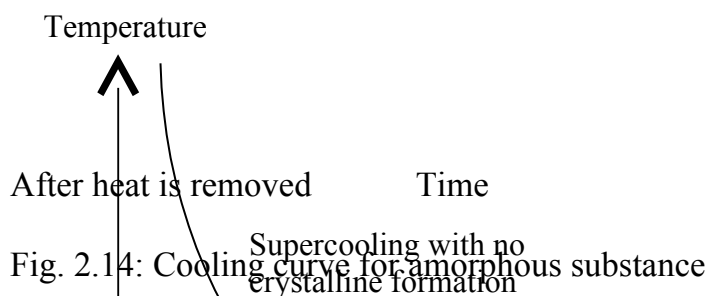


Fig. 2.13: Cooling curve for crystalline substance

Amorphous substances soften gradually when heated, unlike crystalline substances which will melt suddenly. For this reason, glass or plastic can be heated to soften it and then bend it. When cooled, amorphous substances.





behave rather oddly. They lose heat steadily and very fast and solidify without forming crystals. The cooling of the "liquid" continues until the solid hardens. Crystals are never formed (and cannot be formed) because the molecules of the amorphous substance cannot be tangled before they are frozen. Fig. 2.14 shows the cooling curve for a typical amorphous substance. The process is called "supercooling". The amorphous substance (glass, rubber or plastic) is "supercooled liquid" and not a "solid" in the true sense of the word.

### SELF ASSESSMENT EXERCISE 6

1. Define and give three examples of a crystal
2. Distinguish between crystalline and amorphous substances. Give two examples of each
3. Describe the structure of
  - (i) Sodium chloride crystal,
  - (ii) Zinc sulphide crystal, and
  - (ii) Iron crystal
4. Name three types of crystal structure and give one example of each.
5. Explain the following terms
  - (i) Amorphous substance
  - (ii) Crystal lattice
  - (iii) Supercooled liquid



## 4.0 CONCLUSION

In this unit you learned about the molecular theory of matter; specifically, you learned about the the concept, structure and nature of matter. In addition, you learned about the molecule, states of matter, the kinetic molecular theory and the assumptions of the theory. Furthermore, you learned boiling, evaporation, surface tension, capillarity, osmosis and viscosity. Finally, you learned crystal and amorphous solids.

## 5.0 SUMMARY

- Simple evidences of the molecular nature of matter exist in everyday life, e.g. a number of substances in solid form is easily broken into pieces or crushed into powder.
- A molecule is the smallest particle of a substance which can have a separate existence and still retain the properties of that substance.
- The size of a molecule is about  $10^{-9}$  -  $10^{-10}$  m. The molecular size can be empirically estimated using the oil film experiment.
- the molecular Kinetic theory of matter is based on a number of fundamental assumptions which are used to explain the three states of matter (solid, liquid and gas) as well as physical properties such as pressure exerted by a gas, change of state, boiling, evaporation, surface tension, cohesion, adhesion, capillarity and osmosis.
- Solids can be divided into two groups - crystalline solids and non-crystalline or amorphous solids
- A crystalline solid is one in which the atoms or molecules or ions which make up the solid are arranged in a regularly repeating pattern called lattice (crystal lattice). Crystals are very small particles of a crystalline solid. They grow out from the solution of the solid when the solution is cooled to freezing point. Crystalline solids are usually soluble in water.
- Non-crystalline solids do not form crystals. They are said to be amorphous, i.e. without shape or form. When they lose heat, they are supercooled and have no definite melting (or freezing) point. They soften gradually when heated, e.g. glass, plastic, rubber. They usually do not dissolve in water.
- The structure of a crystalline solid can be any one of the following:
  - (i) Simple cubic,
  - (ii) face-centred cubic and
  - (iii) body-centred cubic.



## 6.0 TUTOR-MARKED ASSIGNMENT

1. Mention one experimental evidence and three evidences from daily life experience which suggest that matter is made up of small particles.
2. Using a labelled diagram only, illustrates the simple structure of an atom. State two properties of each of any three constituents of an atom which you have labelled.
3. Distinguish between the Kinetic molecular theory of matter and the Kinetic theory of gases. State any three basic assumptions of each theory.
4. Explain each of the following observations:
  - (a) The smell of a dead rat located at a hidden corner of a room, soon fills the entire room.
  - (b) A thermometer suspended in boiling water does not indicate any rise in temperature although heat is steadily supplied to the vessel containing the water
  - (c) Drops of water, which issue slowly from a tap, are spherical in shape
  - (d) A drop of water spreads out on a clean glass sheet and wets it but a drop of mercury forms a spherical globlet on the glass surface without wetting it.
5.
  - (a) Distinguish between crystalline and amorphous substances and explain what is meant by "a true solid".
  - (b) Draw sketch diagrams to illustrate the structure of each of the following crystals: Chromium, Zinc sulphide and Sodium chloride crystals.

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## UNIT 3 MOTION IN A STRAIGHT LINE

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### 1.0 INTRODUCTION

The term motion is usually associated with movement of objects when forces act on them. You can use the term to describe any object that is not at rest such as an aeroplane in flight or a man walking along the road.

Motion can take place in one, two or three dimensions. When a body moves along a straight line, we speak of one dimensional motion. Two dimensional motion is motion in a plane. For example, the motion of footballers on a field. Three dimensional motions take place in space. Example is the flight of mosquito in a room.



## 2.0 OBJECTIVES

At the end of this unit, you should be able to:

- list the types of motion in a given environment
- classify a list of familiar motion into random, rotational, oscillatory and translational motion;
- identify for a given motion the two bodies between which there is a relative separation
- distinguish between distance and displacement in a translational motion
- distinguish between speed and velocity
- plot a distance-time graph and deduce the speed of motion from the gradient or slope of the graph
- determine speed and velocity when simple problems are set involving distance or displacement and time
- explain the concept of uniform motion, instantaneous speed/velocity and uniform acceleration
- determine acceleration from a velocity - time graph
- deduce the three equations of motion from a ( $v - t$ ) graph with initial velocity and constant acceleration
- explain the terms used in the equations of motion.

## 3.0 MAIN CONTENT

### 3.1 Types of Motion

There are four main types of motion:

- (i) random motion
- (ii) translational motion
- (iii) rotational motion and
- (iv) oscillatory motion.

#### 3.1.1 Random motion

An object is said to undergo random motion when it moves in an irregular manner with no preferred direction or orientation.

### Examples



- Movement of players in a football field;
- Brownian motion - an irregular motion of particles suspended in water or of smoke particles suspended in air.

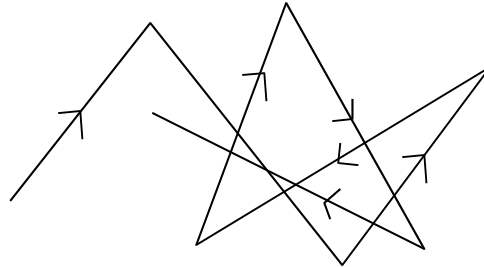


Fig. 3.1: Random motion of gas molecule

### 3.1.2 Linear/Translational Motion

When rigid objects move away from one point in space to another without rotating, the motion is said to be translational. In translational motion, you will observe that every point in the body remains relatively fixed to one another. Each part of an object undergoing pure translational motion follows the same path.

#### Examples

- When you move from your home to the study centre;
- A car travelling from Enugu to Abuja, along the road;
- An insect crawling from one point to another.

### 3.1.3 Rotational Motion

An object undergoes rotational motion when its movement requires its complete turning. We mean that all points in the body move in concentric circles such as the point P in a rotating wheel. The centre of these circles all lie on a line called the axis of rotation.

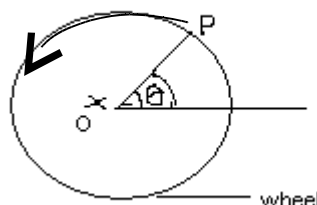


Fig. 3.2:  
wheel

Rotational motion of a

Axis of rotation passes into the page

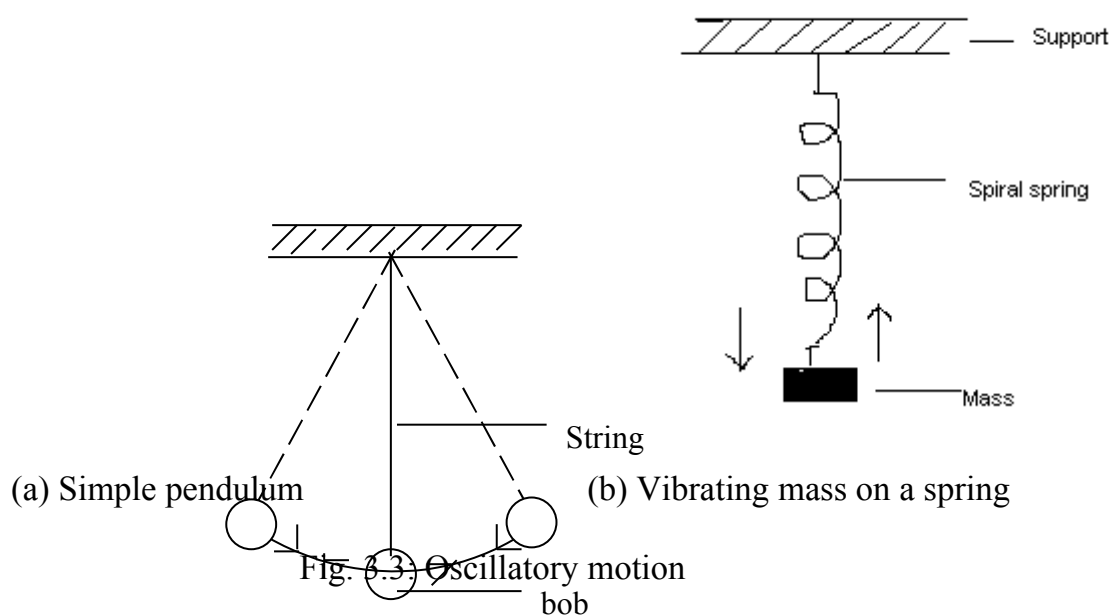
#### Examples



- Rotational motion of a record disc
- The movement of the blades of an electric fan;
- The earth spinning about its axis.

### 3.1.4 Oscillatory (or Vibrational ) Motion

You describe an object as undergoing oscillatory motion when its movement is periodic. That is, the object undergoes a 'to and fro' movement about an axis, reversing the direction of its motion and returning regularly to its original position.



#### Example

- The motion of a pendulum as it swings back and forth, Fig. 3.3. (a)
- The vertical motion of a disturbed mass hung from a spiral spring or an elastic band Fig .3.3. (b)

You are aware that the actual motion of most objects is quite complicated. The motion of an object is usually a combination of these different types of motion.

You know that:



- The wheel of a car in motion combines rotational motion and translational motion;
- The earth moves round the sun (translational) as well as spins about its north south axis (rotational);
- A complex gas molecule with many atoms may undergo random, translational, rotational and oscillatory motions.

### 3.2 Relative Motion

If you are sitting in a train travelling pass a railway platform on which your friend is standing, there is said to be relative motion between you and your friend.

A person sitting still at the back seat of a moving bus has the same speed as that of the bus relative to the earth. But if he now walks towards the driver of the bus, he has a speed relative to the earth which is more than that of the bus.

Now, imagine the bus is moving at  $30\text{m/s}$  and the person walks at  $2\text{m/s}$  towards the driver, his forward speed relative to the earth is  $32\text{m/s}$  ( $30\text{m/s} + 2\text{m/s}$ ). But when he walks back from the driver to his back seat with the speed of  $2\text{m/s}$ , his speed relative to the earth is now  $18\text{m/s}$  ( $30\text{m/s} - 2\text{m/s}$ ).

Relative to the earth, the speeds of the person are different in the two occasions. In this illustration, the points of reference are the earth and the bus.

When you travel in a bus look down on the floor of the bus. The bus will appear to you to be stationary.

This is because you and the bus are moving with the same speed and hence there is zero or no relative motion between two of you. But if you look out of the window the trees outside the bus will appear to be moving in opposite direction as the bus. Trees do not move. The illusion of moving trees is due to the relative motion between you in the moving bus and the stationary trees.

An object undergoes relative motion when there are two bodies between which there exists a relative separation. Such a motion involves the change of position of one object relative to another which may or may not be fixed.

#### SELF ASSESSMENT EXERCISE 1



1. What is motion?
2. List and discuss the four main types of motion.
3. List five examples each of these main types of motion that you come across in your environment (do not use examples already given in the unit).
4. Give five examples of motions which involve more than one type of motion
5. What do you understand by relative motion?

### 3.3 Parameters Used in Describing Motion

The parameters used in describing motion are time, displacement (distance in a specified direction), speed, velocity and acceleration.

Example: If a boy walks 30 m eastward and then 40 m northward the distance he has walked is 70 m ( $30 + 40\text{m}$ ). His displacement is 50 m  $\text{N}38^\circ\text{E}$ . ( $\sqrt{30^2 + 40^2}$ )-at an angle  $38^\circ$  East of North)

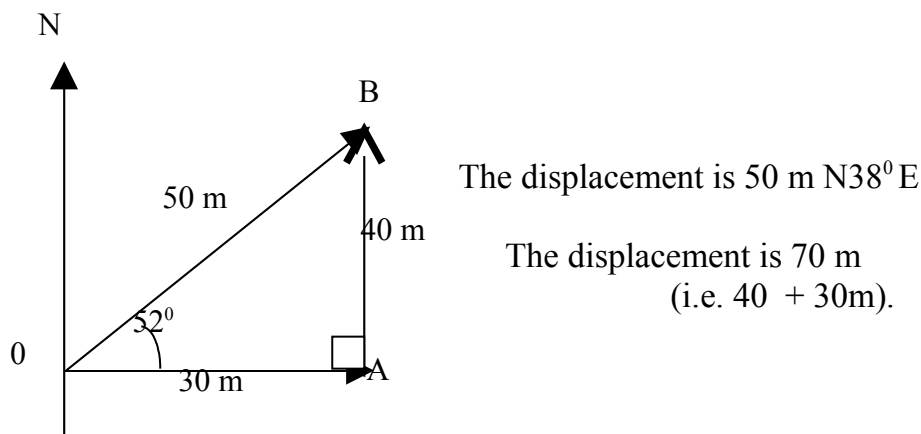


Fig. 3.4: Displacement and distance

Think of and give other good examples to show distance and displacement.

#### 3.3.1 Concept of Speed

In describing the motion of a body, we note both the distance and time it takes to cover that distance. The rate at which a body covers a distance is called the speed of the body.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$



$$\text{i.e. Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$\text{Symbolically, speed (v)} = \frac{\text{distance (s)}}{\text{time (t)}}$$

$$\text{i.e. } v = \frac{s}{t} \quad (3.1)$$

You should note that:

- Speed is a scalar quantity;
- We talk of average speed because speed may not be constant all the time. For instance when a driver slows down at road bends, stops for school children to cross the road or may even increase his speed;
- Speed at a particular instance (instantaneous speed) is obtained by timing a short section of the journey around that instance. Speedometer is the instrument fixed in vehicle for this purpose.

### Uniform Speed

When a body covers equal distances in equal time intervals, no matter how small the time interval may be, the speed is said to be a uniform speed or constant speed.

### 3.3.2 Concept of Velocity

You should, as a student of Physics, know that velocity and speed can not be used interchangeable as is used in everyday language. In speed no direction is specified (scalar) while in velocity the direction is specified (vector).

Velocity is the rate of change of displacement with time or the rate of change of distance with time or the rate of change of distance with time in a given direction.

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\text{i.e. } v = \frac{s}{t} \quad (3.2)$$

Note that:



- When a car moves at constant speed in a straight line, its velocity is constant;
- When a car moves round a circular path with constant speed, the velocity will be changing because of the change in direction at different points round the circle;
- Velocity is a vector quantity and its unit is metre per second ( $\text{ms}^{-1}$ ).

### Non- Uniform Velocity

If a body moves round a circular path at constant speed, then it is said to move with non - uniform velocity because its direction of motion is constantly changing.

### Uniform Velocity

When a body moves with equal displacement in equal time intervals no matter how small the time intervals may be, the velocity is said to be uniform or constant.

### 3.3.3 Concept of Acceleration/Retardation

An object may be moving with non-uniform velocity (changing velocity). The change could be increase or decrease in velocity. When the velocity of a moving object is increasing, the object is said to be accelerating. When it is decreasing, the object is said to be retarding.

Acceleration (retardation) is defined as the rate of change of velocity with time.

Specifically,

$$\begin{aligned} \text{Acceleration (a)} &= \frac{\text{increase in velocity (v)}}{\text{time taken (t)}} \\ \text{i.e. Acceleration (a)} &= \frac{\text{velocity change}}{\text{time taken to make the change}} \end{aligned}$$

$$\Rightarrow a = \frac{v}{t} \quad (3.3a)$$

$$\text{Retardation (- a)} = \frac{\text{velocity change}}{\text{time taken to make the change}}$$

$$\Rightarrow -a = \frac{v}{t} \quad (3.3b)$$

S. 1. Unit = metre per second squared ( $\text{ms}^{-2}$ )



Uniform acceleration is defined as the motion of an object whose velocity increases by equal amount in equal time interval however small the time intervals may be.

- **Worked examples**

1. A car has a velocity of 72 km/h. How far does it travel in 1/2 min?

- **Solution**

You must always use the same kind of units in calculations.

$$36 \text{ km/h} = 10 \text{ m/s}$$

$$\therefore 72 \text{ km/h} = 20 \text{ m/s}$$

$$1/2 \text{ min} = 30 \text{ s}$$

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\begin{aligned} \Rightarrow \text{displacement} &= \text{velocity} \times \text{time} \\ &= 20 \text{ m/s} \times 30 \text{ s} \\ &= \underline{\underline{600 \text{ m}}} \end{aligned}$$

2. Suppose a train travels at 54 km/h and accelerates uniformly to 144 km/h in 10s. Find the uniform acceleration.

- **Solution**

$$36 \text{ km/h} = 10 \text{ m/s}$$

$$54 \text{ km/h} = \frac{54}{36} \times 10 \text{ m/s} = 15 \text{ m/s}$$

$$144 \text{ km/h} = 144 \times \frac{10}{36} \text{ m/s} = 40 \text{ m/s}$$

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$\Rightarrow a = \frac{(40-15) \text{ m/s}}{10 \text{ s}} = \frac{25 \text{ m/s}}{10 \text{ s}}$$

$$= 2.5 \text{ m/s}^2$$



## SELF ASSESSMENT EXERCISE 2

1. A car moves round a circular track with a uniform speed of 100 km/h. Show in a diagram, vectors which represent the velocity of four consecutive positions A, B, C, D equally spaced round the track. Is
  - (a) the velocity and
  - (b) the speed the same at A, B, C and D?
2. Differentiate between:
  - (i) average and instantaneous velocities
  - (ii) acceleration and retardation
  - (iii) distance and displacement.
3. With an average velocity of 820 km/h an aeroplane in flight spent 1.25hrs to move from Lagos to Sokoto. How far is Lagos from Sokoto by air?
4. A car travels 120m in 5seconds. What is its average velocity?
5. In an attempt to over take a luxury bus, a car driver increased his velocity from 20 m/s to 60 m/s within a period of 15 seconds. Determine the acceleration of the car.

### 3.4 Distance/Displacement - Time graph

You are familiar with the motion along a straight line in which the actual distance covered is equal to the displacement and the speed is equal to the magnitude of the velocity. Such motions can be represented on distance/displacement - time graph as shown in Fig. 3.5 below.

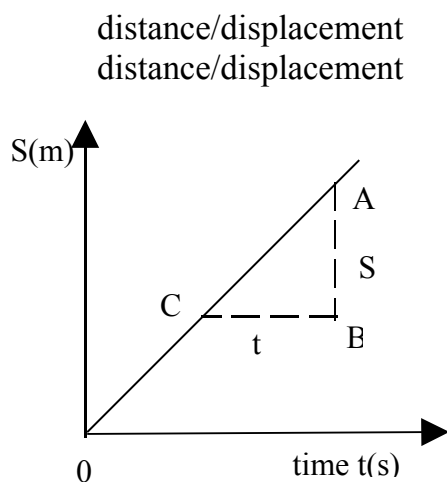


Fig. 3.5(a)

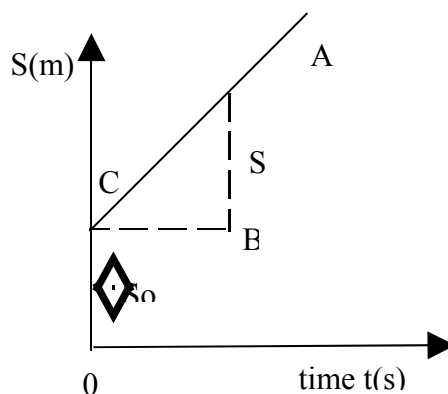


Fig. 3.5(b)

In Fig. 3.5 (a), distance  $s = 0$  when  $t = 0$ .



That is, the timing is started when the body passes a reference point 0 on the path.

In Fig. 3.5b,  $S = S_0$  at  $t = 0$ .

This means that timing was started when the body was at a distance  $S_0$  from the reference point on the path.

The straight line graph shows that the body covers equal distances in equal time (speed is uniform).

The slope of the graph =  $\frac{AB}{BC} = \frac{s}{t}$  represents the speed ( $v$ ) of the body.

The distances covered in the above diagrams are for:

Fig. 3.5a,  $S = vt$  i.e. slope  $\times$  time

Fig. 3.5b,  $S = S_0 + vt$  i.e.  $S_0 +$  slope  $\times$  time

For Fig. 3.6 representing the motion of a body when the speed changes with time, the instantaneous speed at any instance is the slope of the graph at that instance.

For example, the instantaneous speed at P is represented by  $\frac{AB}{BC}$ , the slope of the tangent to the curve at P.

distance/displacement

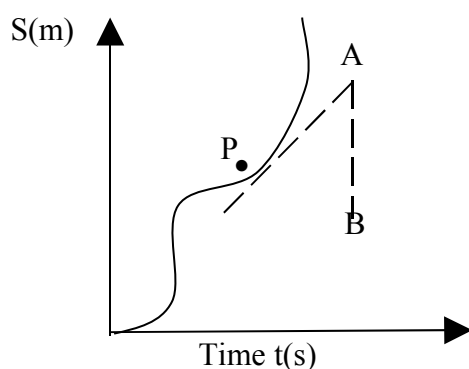


Fig. 3.6: Motion of a body with varying speed



### 3.5 Speed/Velocity - Time graph

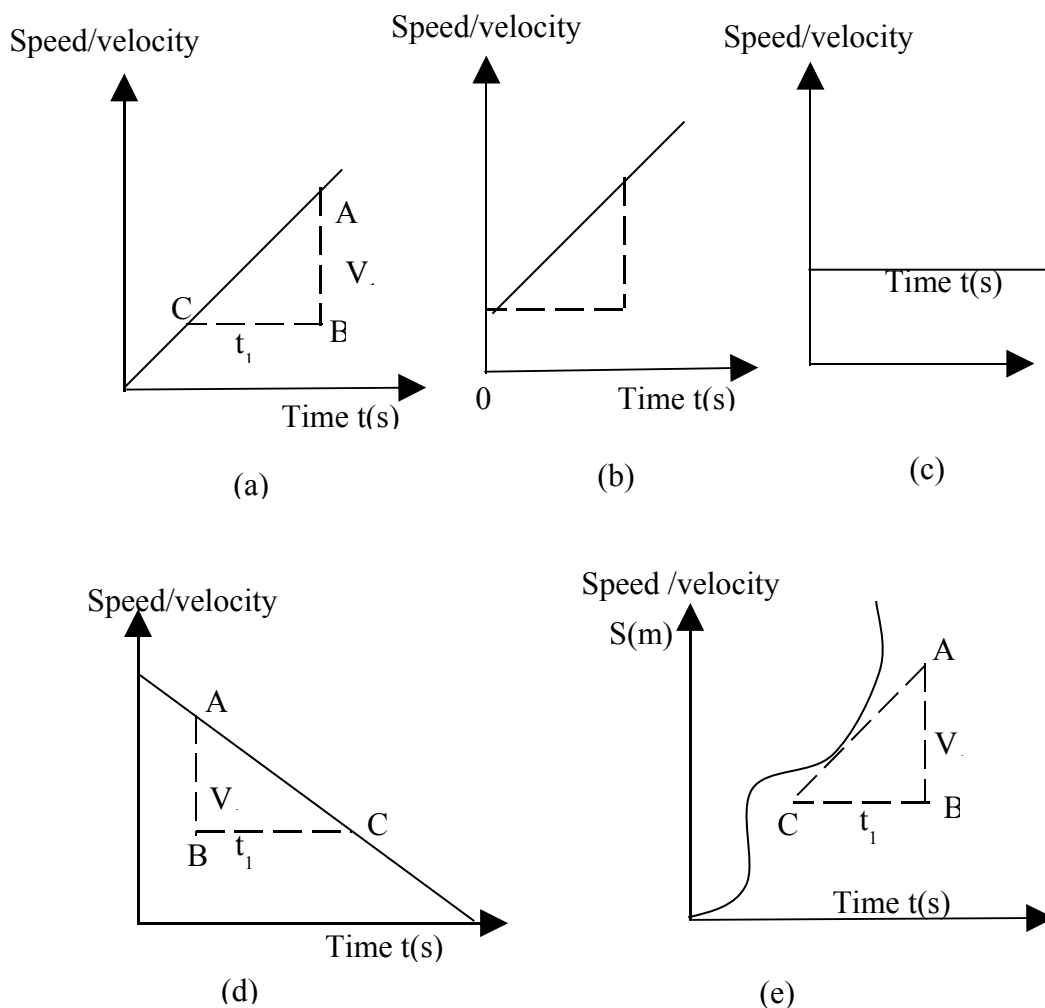


Fig. 3.7: Speed/ velocity time graphs

The speed/velocity time graph of a moving object could be plotted as shown in Fig. 3.7 above. The graph provides information about the moving object. You can deduce the following from the graph.

- the slope of the graph gives the acceleration of the moving object;
- the area under the graph is the distance covered;
- a straight line graph parallel to X - axis shows that the object is travelling with uniform velocity. That is, the object has zero acceleration or retardation,
- a straight line graph inclined to the X - axis shows that the object is undergoing uniform acceleration or retardation;
- a non-linear graph shows that the object is undergoing non uniform acceleration or retardation. The acceleration or retardation at any point is the slope of the tangent to the curve at that point.



**Worked example**

An object travelling along a straight path has the following velocities for the first 12 seconds of the motion.

Velocities (m/s)	0.0	5.0	10.0	15.0	15.0	7.5	0.0
Time (s)	0	2	4	6	8	10	12

Sketch the velocity - time graph for the motion.

Obtain from the graph:

- the uniform acceleration during the first stage of the journey;
- the retardation during the last stage of the journey;
- for how long did the object travel altogether; and
- what was the total distance travelled.

Solution

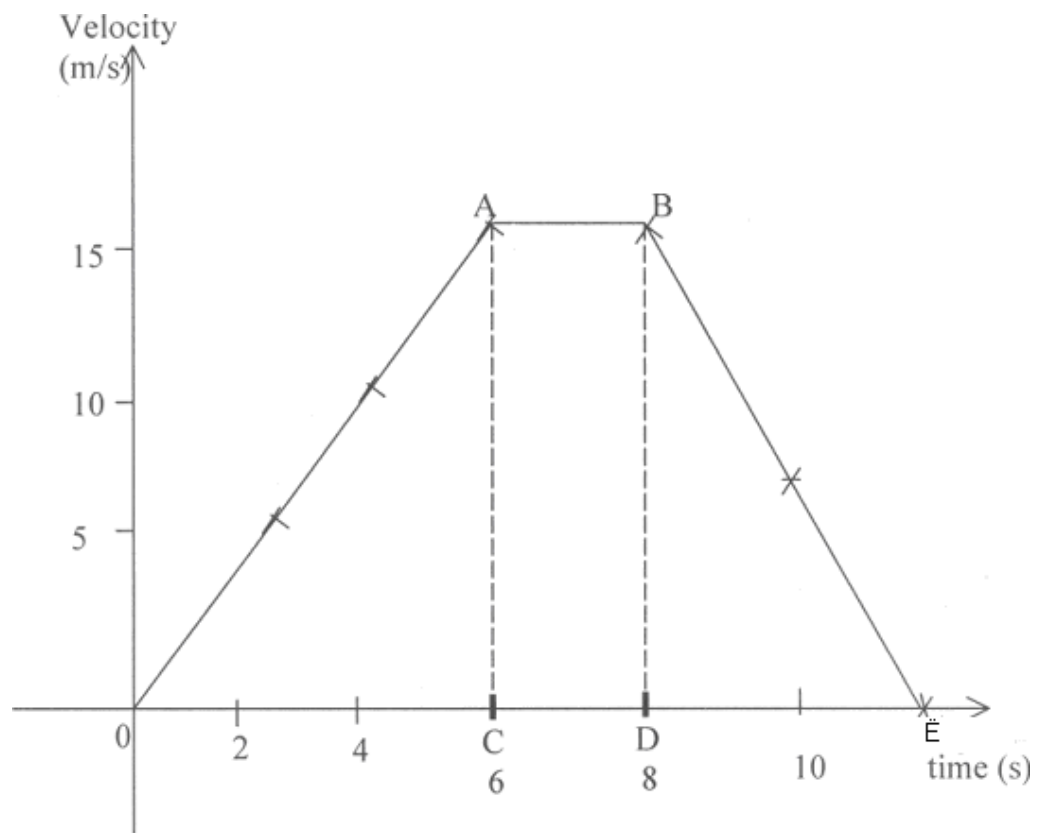


Fig. 3.8: Velocity time graph



From the graph,

- (a) Acceleration OA = gradient of the line OA

$$\frac{AC}{OC} = \frac{5-0}{6-0} = \frac{15\text{m/s}}{6\text{s}} = 2.5\text{m/s}^2$$

- (b) Retardation = gradient of the line BE

$$= \frac{BD}{DE} = \frac{15-0}{12-8} = \frac{15\text{m/s}}{4\text{s}} = 3.75\text{m/s}^2$$

- (c) Total time of travel  
= 6s + 2s + 4s = 12seconds.

- (d) The area under the velocity - time graph gives the total distance travelled.

In time OC, distance covered,

$$= \text{area of } \triangle OAC = \frac{1}{2} OC \times AC$$

$$= \frac{1}{2} \times 6 \times 15 = 45\text{m}$$

In time CD, distance covered,

$$= \text{area of rectangle CABD} = CD \times BC$$

$$= 2 \times 15 = 30\text{m}$$

In time DE, distance covered

$$= \text{area of } \triangle BDE = \frac{1}{2} DE \times BD$$

$$= \frac{1}{2} \times 4 \times 15 = 30\text{ m}$$

Total distance travelled = (45 + 30 + 30) m

$$= \underline{105\text{ m}}$$



### SELF ASSESSMENT EXERCISE 3

An electric train moves from rest with a uniform acceleration of  $1.5\text{m/s}^2$  for the first 10s and continues accelerating at  $0.5\text{m/s}^2$  for a further 20s. It continues at constant speed for 90s and finally, takes 30s to decelerate uniformly to rest.

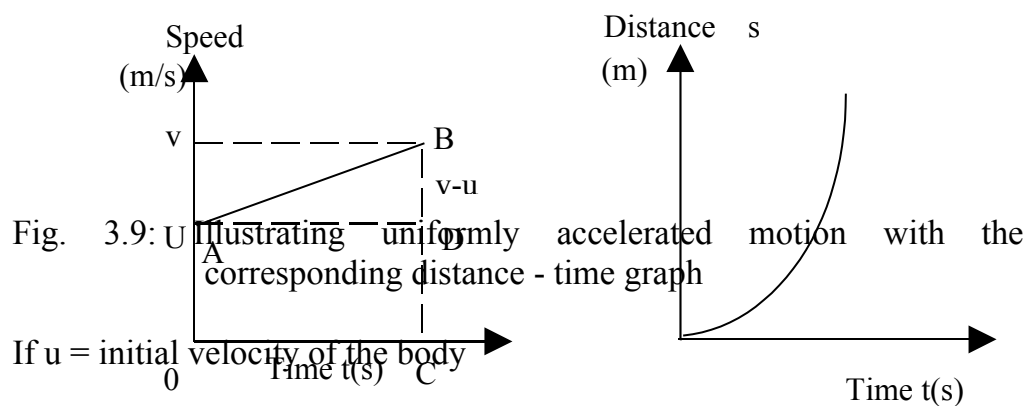
- Draw a graph of speed against time for the journey;
- From your graph or otherwise, deduce the total distance travelled.
- What is the average speed of the train for the whole journey?
- What was the deceleration during the last stage of the motion?
- For how long did the object travel altogether?

### 3.6 Derivation of the Equations of Uniformly Accelerated Linear Motion

You can apply equations of uniformly accelerated motion when a body moves under constant (uniform) acceleration along a straight line.

Here both the velocity and the acceleration are in the same direction which is along the line of motion.

The speed - time graph of such a motion is shown in Fig. 3.9.



$v$  = velocity at time  $t$

$a$  = the uniform acceleration



You recall then that by definition,

$$a = \frac{\text{change in velocity}}{\text{time interval for the change}} = \frac{v - u}{t}$$

$$\Rightarrow at = v - u$$

$$\text{or } v = u + at \text{ --- (3.4)}$$

You already know that the distance  $S$ , travelled by the body during the time interval is represented by the area of the trapezium OABC

$$\text{That is, } S = \frac{u + v}{2} \times t$$

$$= \text{average velocity} \times \text{time interval}$$

From equation 3.4;  $v = u + at$

Therefore, substituting for  $v$  in the equation above we obtain

$$S = \frac{u + u + at}{2} \times t$$

$$\text{i.e. } S = \frac{2u + at}{2} \times t = \frac{2u}{2} + \frac{at}{2} \times t$$

$$\Rightarrow S = u + \frac{1}{2} at + t$$

$$S = u + \frac{1}{2} at^2 \text{ ..... (3.5)}$$

You can still make  $t$  the subject of the first equation  $v = u + at$

$$\text{i.e. } t = \frac{v - u}{a}$$

To obtain the third equation of motion, you will substitute for  $t$  in equation 3.4 using the relation for  $t$  above.

i.e.  $S = ut + \frac{1}{2} at^2$  becomes

$$S = u \left( \frac{v - u}{a} \right) + \frac{1}{2} a \left( \frac{v - u}{a} \right)^2$$

Multiplying through by '2' gives

$$2S = 2u \left( \frac{v - u}{a} \right) + \left( \frac{a}{a} \frac{v - u}{a} \right)^2$$

$$2S = \frac{2uv - 2u^2}{a} + \frac{1}{a} \left( v^2 - 2uv + u^2 \right)$$



Multiplying through by 'a'

$$2aS = 2UV - 2U^2 + V^2 - 2UV + U^2$$

$$2aS = V^2 + U^2 - 2U^2$$

$$2aS = V^2 - U^2$$

If you make  $V^2$  the subject of the relation you obtain

$$V^2 = U^2 + 2aS \text{--- (3.6)}$$

The equations 3.4, 3.5 and 3.6 are generally referred to as the three equations of uniformly accelerated linear motion.

You should note that if the body starts from rest, the initial velocity

$U = 0$  so that the

1<sup>st</sup> equation  $v = u + at$  becomes  $v = at$

2<sup>nd</sup> equation  $S = ut + \frac{1}{2}at^2$  becomes  $S = \frac{1}{2}at^2$

3<sup>rd</sup> equation  $v^2 = u^2 + 2aS$  becomes  $V^2 = 2aS$

Similarly, if a body starts with a velocity  $u$  from a distance  $S_0$ , its distance  $S$ , at time  $t$ , is given by

$$S = S_0 + ut + \frac{1}{2}at^2.$$

### 3.7 Applications of the Three Equations of Motion in Solving Numerical Problems

- **Worked example**

A train, starting from rest, accelerates uniformly so that it attains a speed of  $108 \text{ kmh}^{-1}$  in 2 minutes. It travels at this speed for 10 minutes and is then brought to rest with uniform retardation after 3 minutes. Calculate the distance travelled by the train. Determine also the retardation.

The above problem can be solved by calculation and graphical methods. We shall solve by calculation here while you repeat it by graphical method later.



- **Solution**

First, you will identify the motion of the train which are in three parts:

- (a) under uniform acceleration;
- (b) at constant speed; and
- (c) under uniform retardation.

The sketch diagram is as shown in Fig. 3.9

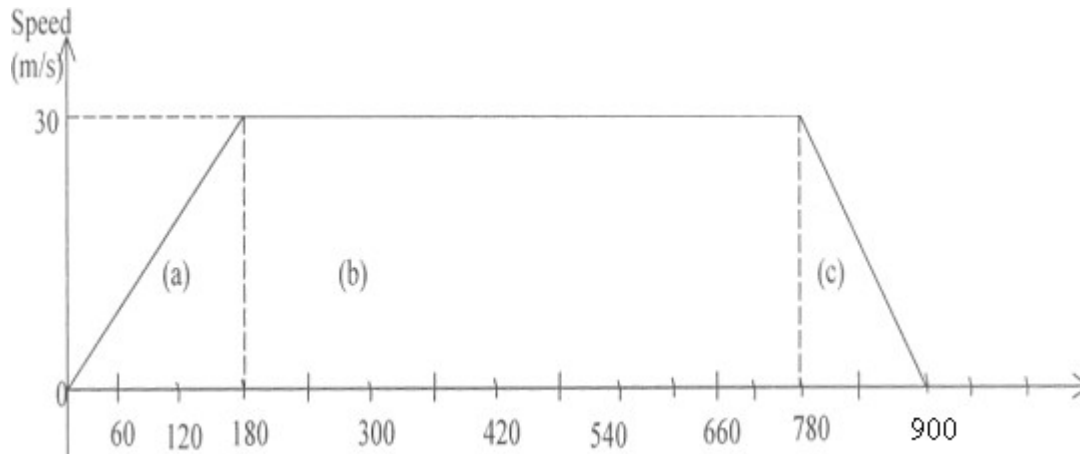


Fig. 3.10: Sketch of the motion in our sample (Worked example)

In part (a), initial velocity,  $u = 0$ ;  
final velocity after 3 mins  $V_1 = 108 \text{ kmh}^{-1}$

$$= \frac{108 \times 1000}{3600} = 30 \text{ m/s}$$

time interval  $t_1 = 180 \text{ s}$

- distance travelled,  $S = \frac{1}{2} V_1 t_1$  ( $u = 0$ )

$$S_1 = \frac{1}{2} \times 30 \text{ m/s} \times 180 \text{ s} = 2700 \text{ m}$$

In part (b), constant velocity,  $u_2 = v_1 = 30 \text{ m/s}$   
Time interval,  $t_2 = 600 \text{ s}$

- distance travelled;  $S_2 = u_2 t_2$  ( $a = 0$ )

$$S_2 = 30 \text{ m/s} \times 600 = 18000 \text{ m}$$

In part (c), initial velocity  $u_3 = v_1 = 30 \text{ m/s}$   
final velocity,  $V_3 = 0$ ;  
time interval,  $t_3 = 120 \text{ s}$



- distance travelled,  $S_3 = \frac{1}{2} U_3 t_3$

$$S_3 = \frac{1}{2} \times 30\text{m/s} \times 120\text{s} = 1800 \text{ m}$$

- Total distance travelled by the train

$$\begin{aligned} S &= S_1 + S_2 + S_3 \\ &= 2700\text{m} + 18000\text{m} + 1800 \text{ m} \\ &= 22,500 \text{ m} \\ &= 22.5 \text{ km} \end{aligned}$$

The retardation took place at the 'C' part of the motion and is given by

$$\begin{aligned} \text{Retardation} &= \frac{U_3 - V_3}{t_3} \\ &= \frac{30-0}{120\text{s}} \\ &= 0.25 \text{ ms}^{-2} \end{aligned}$$

#### SELF ASSESSMENT EXERCISE 4

- A car is traveling with uniform velocity of  $72 \text{ kmh}^{-1}$ . What distance does it cover in 20s?
- Define uniform acceleration and average speed. A car starts from rest with a uniform acceleration of  $1.5\text{m/s}^2$  until it reaches a speed of  $24\text{ms}^{-1}$ . It travels at this speed for 30s and then comes to rest with uniform retardation in 10s. Draw on a graph paper a velocity – time graph of the motion. Using calculation and graphical methods determine:
  - the average speed of the car over the whole journey;
  - the distance travelled by the train; and
  - the retardation in the last 10 second.
- Use calculation method to solve all the problems treated earlier in this unit.

#### 4.0 CONCLUSION

In this unit you learned about the four types of motion namely random, translational, rotational and oscillatory motion. The study highlighted the fact that motion of an object is usually a combination of these different types of motion. Also discussed was the idea of relative motion



showing that motion involves the separation of an object relative to the observer or point of reference. Furthermore, the parameters used in describing motion namely time, position, distance, displacement, speed, velocity and acceleration were examined. Finally, you studied distance/displacement - time graph as well as speed/velocity - time graph and how to solve problems relating to them.

## 5.0 SUMMARY

- You can distinguish between four types of motion: -
  - (i) random motion in which a body moves irregularly with no preferred direction.
  - (ii) translational motion in which every point in a moving body remains fixed in relation to other points,
  - (iii) rotational motion where every point in a body moves in circles, and
  - (iv) oscillatory or vibratory motion in which a body moves to and fro, reversing its direction, and returning regularly to its position.
- Relative motion of a body A with respect to another body B is the motion A as observed from B. B is the frame of reference. In general, all motion involves the separation of an object relative to an observer (or point of reference).
- Displacement is the distance travelled in a specified direction
- Speed is the distance travelled in unit time.
- Velocity is the rate of change of displacement with time
- When a body moves with equal displacement in equal time interval no matter how small the time interval may be, the velocity is said to be a uniform or constant velocity.
- The slope of a displacement/distance - time graph is the velocity/speed.
- Instantaneous velocity is the velocity at any instant in time
- Acceleration is the rate of change of velocity with time.
- The area under a velocity- time graph gives the total displacement.



## 6.0 TUTOR -MARKED ASSIGNMENT

1.
  - (a) List the four main types of motion
  - (b) A bus is traveling along a straight road at  $100\text{kmh}^{-1}$  and the bus conductor walks at  $8\text{kmh}^{-1}$  on the floor of the bus and in the same direction as the bus. Find the speed of the conductor relative to the road in meters per second.
2. Distinguish between
  - (a) distance and displacement
  - (b) speed and velocity
  - (c) How can you find speed on a distance -time graph?
3. A car starting from rest, accelerates uniformly so that it attains a speed of  $72\text{ km/hr}$  in 2 minutes. It travels at this speed for 5 minutes and is then brought to rest with uniform retardation after 3 minutes, Calculate the distance travelled by the train. Determine also the retardation.
4. The speedometer of a racing car reads the following values of velocity  $v$  in the time  $t$  shown, starting from rest.

Time (s)	0	2	4	6	8	10	12	18	24	30	32	34	36	38	40	42
Velocity (m/s)	0	12	24	36	48	60	60	60	60	60	50	40	30	20	10	0

Sketch the velocity - time graph for the motion and obtain from the graph:

- (a) the uniform acceleration during the first stage of the journey;
- (b) the retardation during the last stage of the journey; and
- (c) the total distance travelled.



## **7.0 REFERENCES/FURTHER READINGS**

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## UNIT 4 MOTION UNDER GRAVITY

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### 1.0 INTRODUCTION

In unit 3 you studied motion in a straight line. Here you will learn how fundamental laws of motion apply to motions along curved paths such as the motion of projectiles along parabolic curves. A projectile motion is a special case of motion in a plane. It has no motive power of its own and will travel freely under the action of gravity and air resistance only when it is launched into space.

### 2.0 OBJECTIVES

At the end of this unit, you will be able to:

- identify a projectile motion
- derive the time of flight, maximum height and range of a projectile
- solve simple problems involving the range, height and time of flight of a projectile
- note the applications of projectile motion in warfare and sport.



### 3.0 MAIN CONTENT

#### 3.1 Concept of Projectile

When you release a piece of stone from your catapult against a bird perching on a tree branch, the stone travels in a parabolic path towards the bird.

Also, if you throw a tennis ball against a wall, the path of the ball towards the wall is a parabola. You will notice the same type of curve when you project a ball horizontally from the top of a building. The stone or ball you projected is known as projectile, while the motion of the stone or ball projected is known as projectile motion. The path followed by a projectile is called its trajectory.

Examples of projectile motion are the motions of

- kicked or thrown balls;
- jumping animals;
- objects dropped from windows;
- bombs released from aircraft;
- bullets fired from a rifle or gun
- a stone released from a catapult;
- an athlete doing the high jump.

#### 3.2 Motion of Projectile

We will first consider the simple case of a stone thrown horizontally with an initial velocity  $u$  from the top of a high wall or a cliff of height  $h$ .

The stone is subject to two independent motions - a horizontal and a vertical motion. Once the stone is free and before it hits the ground, no horizontal force acts upon it. Its horizontal velocity will remain unchanged until it hits the ground as shown in Fig. 4.1.



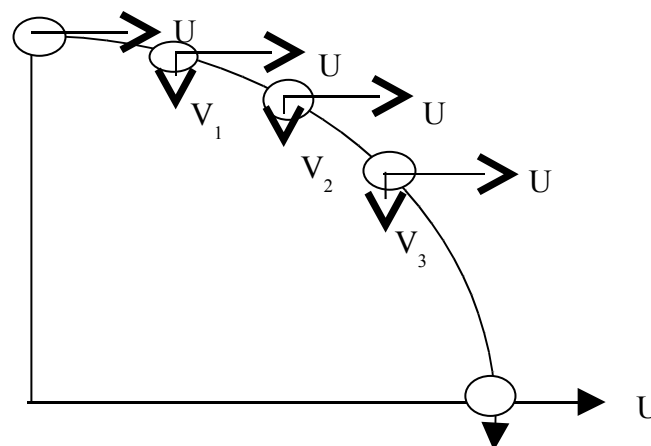


Fig. 4.1: Motion of a projectile

When you throw the stone forward initially, it has no vertical velocity. The vertical pull of gravity accelerates it downwards at a rate of about  $9.8 \text{ ms}^{-2}$ . The vertical or y - component of the velocity increases by  $9.8 \text{ ms}^{-1}$  every second.

Hence, you will observe that any projectile in flight is doing two things at once:

- i. it is flying horizontally with a constant speed.
- ii. It is moving up or down with an acceleration g.

Once you are able to recognise the above, solutions of projectile problems become easy for you. You simply split each problem into two parts. One involving horizontal motion at constant velocity the other is exactly the same as the free fall due to gravity.

### 3.3 Vertical and Horizontal Projections

#### 3.3.1 Vertical Projection

If you throw a body such as a piece of stone vertically upwards, the maximum height it will attain depends on the initial vertical velocity.

If H = maximum height reached by the object;

U = upward velocity of projection, i.e., initial velocity; and

g = acceleration due to gravity.

At the time t after projection,  
the upward velocity  $v = u - gt$ ; and the height  
reached  $h = ut - \frac{1}{2} gt^2$ .

At the maximum height reached,



$$V=0 \text{ and } h=H$$

$$\therefore 0 = u - gt,$$

$$\text{i.e. } t_1 = \frac{u}{g}$$

$$\text{Thus } H = ut_1 - \frac{1}{2}gt_1^2$$

$$= \frac{u^2}{g} - \frac{1}{2} \frac{u^2}{g} \quad (\text{substituting for } t_1 \text{ i.e. } t_1 = \frac{u}{g})$$

$$\text{Leading to } H = \frac{u^2}{2g} \text{ or } \frac{1}{2}gt_1^2 \quad (4.1)$$

Now, the body, starts from rest from the highest point (P) reached and falls to earth under constant acceleration  $g$  (Fig. 4.2)

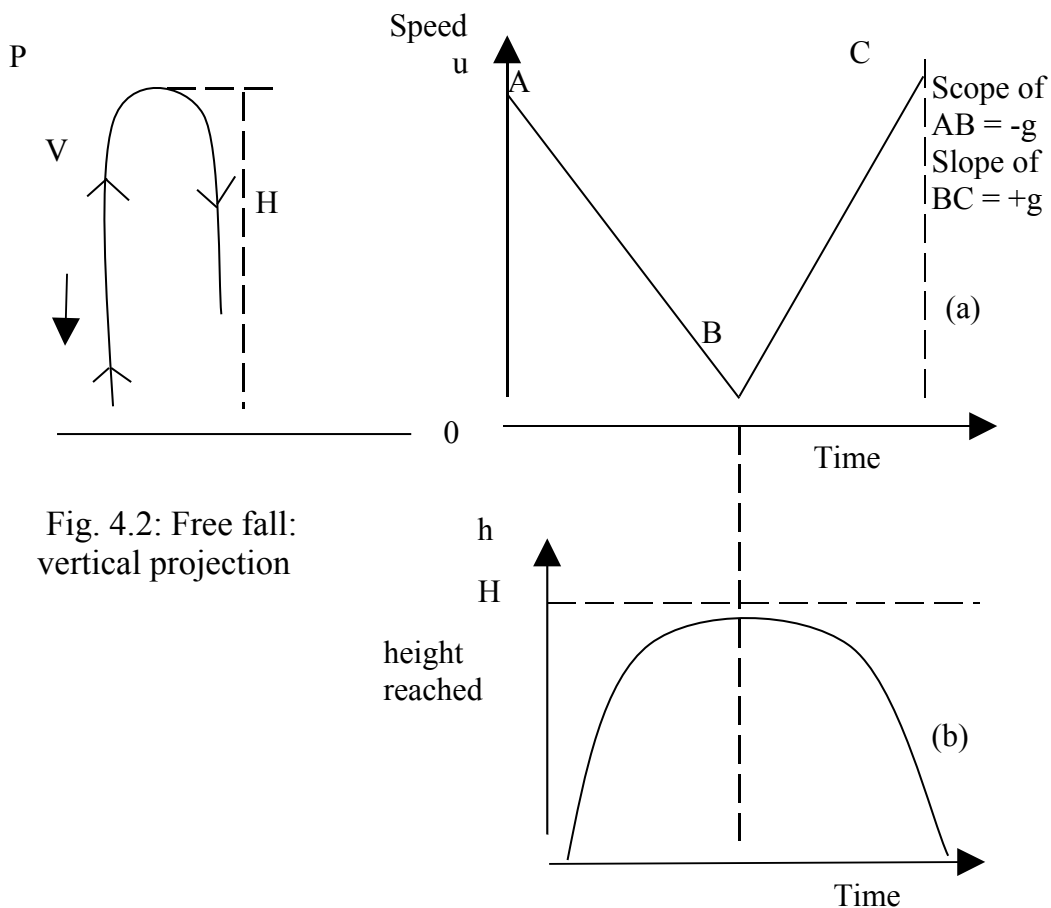


Fig. 4.2: Free fall:  
vertical projection

Fig. 4.3: Graphs of vertical projection

- You should be able to show easily that the body strikes the ground with a velocity which is equal in magnitude to the initial



velocity of projection but opposite in direction. The time the body takes to return to earth from the point P is also equal to  $t_1$ .

Therefore, total time of flight,  $T$ , of the body is  $2t_1$

$$= \frac{2u}{g};$$

$$\Rightarrow T = \frac{2u}{g} \quad (4.2)$$

Fig. 4.3: Illustrates the graphs of vertical projection

### 3.3.2 Horizontal Projection

When you roll a ball at fast speed along a horizontal table, after reaching the edge of the table it continues with its horizontal motion. Now it also experiences the pull of gravity. This downward pull deflects the ball. From its original direction and it finally strikes the floor at a distance which depends on the original horizontal speed and the height of the table. This is illustrated in Fig. 4.4.

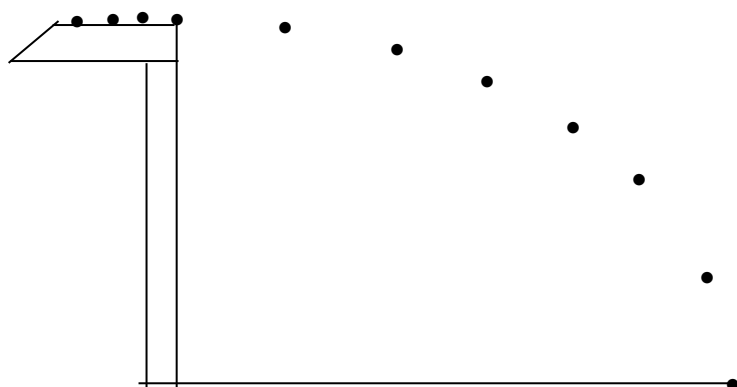


Fig. 4.4: Flight of a ball with initial velocity horizontal

You can regard the motion of the ball as the combination of two components:

- i. a horizontal motion at constant velocity, and
- ii. a vertical motion under constant acceleration.

Suppose the total time of flight of the body from the instant it was projected horizontally at A until it strikes the ground at B is  $T$ . Because the horizontal velocity,  $u$  is constant, the horizontal range  $R$  is given by  $R = uT$  as illustrated in Fig. 4.5



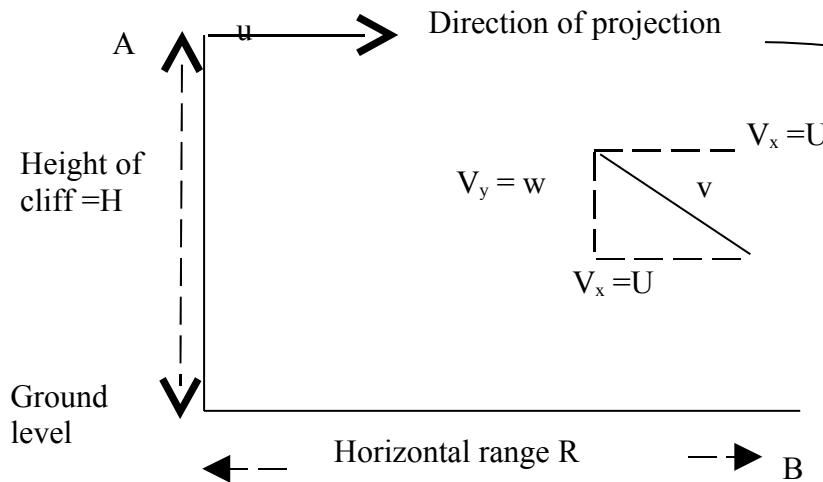


Fig. 4.5: Horizontal projection from the top of a cliff

You know that the vertical component of the motion has zero initial velocity, and acceleration  $g$ . This means that in time  $T$ , the vertical distance travelled, which is the same as the height of the cliff  $H$  is given by:

$$H = \frac{1}{2} gT^2$$

Therefore, the time of flight is

$$T = \sqrt{\frac{2H}{g}} \text{ and } R = u\sqrt{\frac{2H}{g}} \quad (4.3)$$

You should note that the last equation relates the horizontal range to the height of the cliff and the initial horizontal velocity  $u$ .

### 3.4 Resultant Velocity of a Projectile

You know that the velocity of the projectile during flight is made up of a horizontal component,  $u$ , and a vertical component  $w$ . This is shown in Fig. 4.5.

The vertical component  $w$  at a time  $t$  after the start of the motion is given by  $w = gt$

$$\begin{aligned} \text{The resultant velocity } v &= \sqrt{(u^2 + w^2)} \\ &= \sqrt{(u^2 + g^2 t^2)} \end{aligned} \quad (4.4)$$



The angle  $\phi$ , between the direction of motion of the body and the horizontal is given by

$$\begin{aligned}\tan \phi &= \frac{u_y}{u_x} = \frac{gt}{u} \\ \text{i.e. } \phi &= \tan^{-1} \left[ \frac{gt}{u} \right] \quad (4.5)\end{aligned}$$

### 3.5 Projection at an angle to the Horizontal

Here we consider the more general case of a body projected at an angle to the horizontal from the ground. The body moves along a parabolic path, reaches a maximum height and returns to the ground as shown in Fig. 4.6

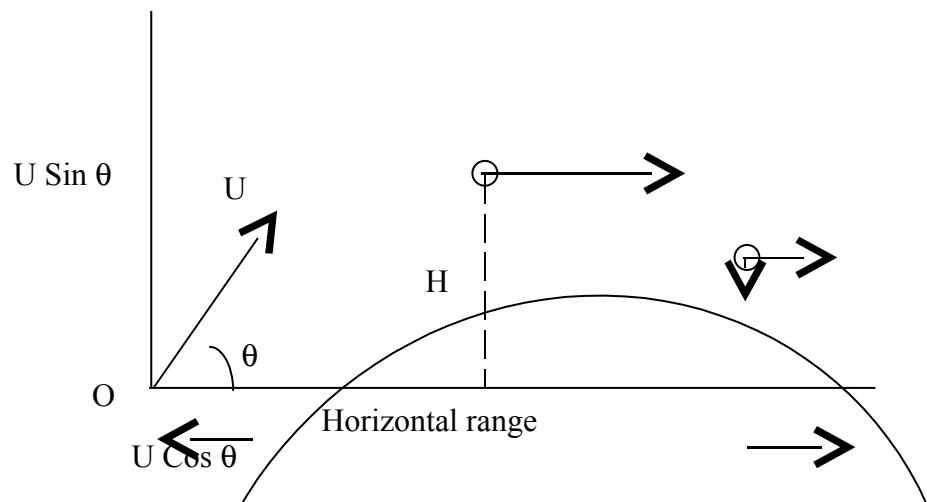


Fig. 4.6: Projecting at an angle to the horizontal

If you project the projectile with an initial velocity  $u$  at an angle  $\theta$  to the horizontal, then the initial horizontal component of the velocity,

$$V_x = U \cos \theta$$

Initial vertical component of the velocity  $V_y = U \sin \theta$

You will recall that the horizontal component of the velocity remains constant because no force acts in the  $x$  - direction. The vertical component of velocity changes with time because it is subject to gravity. At any instance,  $t$ , the vertical component  $V_y$  is given by

$$V_y = U \sin \theta - gt$$



when the ball is moving upwards, but,

$$V_y = U \sin \theta + gt$$

When the ball is moving downwards.

The height attained by the projectile at time  $t$  is given by

$$\begin{aligned} y &= U \sin \theta \cdot t - \frac{1}{2} gt^2 \quad (\text{from } s = ut - \frac{1}{2} gt^2) \\ &= Ut \sin \theta - \frac{1}{2} gt^2. \end{aligned} \quad (4.6)$$

### Worked examples

- A body projected vertically upwards with a velocity of  $30\text{ms}^{-1}$ . How high does it travel before it comes to rest momentarily at the highest point of its motion? What is the duration of the entire flight? (Take  $g = 9.8 \text{ ms}^{-2}$ ).

### Solution

Initial velocity of projection,  $u = 30 \text{ ms}^{-1}$  Acceleration of free fall due to gravity,  $g = 9.8 \text{ ms}^{-2}$

At time  $t$  after projection upwards, height reached is given by

$$V^2 = u^2 - 2gh$$

Where  $v$  = velocity at that instant, and  $v = 0$  at the maximum height

$$\Rightarrow h = H = \frac{u^2}{2g} = \frac{30^2 \text{m}^2 \text{s}^{-2}}{2 \times 9.8} = \underline{45.92 \text{ m}}$$

$$\begin{aligned} \text{Also, } v &= u - gt \\ &= 0 \text{ at the maximum height} \end{aligned}$$

$$\therefore t = t_1 = \frac{u}{g} = \frac{30 \text{ m/s}}{9.8 \text{ ms}^{-2}} = \underline{3.1 \text{ s}}$$

- Duration of the entire flight  $T = 2t_1$

$$= 3.1 \text{ s} \times 2 \\ \underline{6.2 \text{ s}}$$

- A boy throws a stone horizontally with a velocity of  $40\text{ms}^{-1}$  from the top of a building  $70 \text{ m}$  high. How far from the foot of the



building and with what speed does the stone strike the ground?  
(Take  $g = 9.8 \text{ ms}^{-2}$ )

### Solution

Horizontal velocity of projection,  $u = 40 \text{ ms}^{-1}$  Height of building,  $H = 70$

$$\text{Duration of flight, } T = \sqrt{\frac{2H}{g}}$$

$$\text{Horizontal range of projectile, } R = uT = u \sqrt{\frac{2H}{g}}$$

$$\therefore R = 40 \text{ ms}^{-1} \sqrt{\frac{2 \times 70 \text{ m}}{9.8 \text{ ms}^{-2}}} = 40 \times 3.78$$

$$= \underline{\underline{151 \text{ m}}}$$

At impact, vertical component of velocity =  $gT$

Horizontal component is constant =  $u$

=> speed of stone at impact

$$v = \sqrt{(u^2 + g^2 T^2)}$$

$$= \sqrt{(u^2 + 2gH)}$$

$$u = \sqrt{(40^2 + 2 \times 9.8 \times 70)}$$

$$= \sqrt{2972}$$

$$= \underline{\underline{54.52 \text{ ms}^{-1}}}$$

### SELF ASSESSMENT EXERCISE 1

1. What is a projectile motion?
2. A body is projected from the ground with a velocity of  $35 \text{ ms}^{-1}$  at an angle of  $60^\circ$  to the horizontal. Taking the acceleration of free fall due to gravity as  $9.8 \text{ ms}^{-2}$ , calculate:
  - (i) the time of flight
  - (ii) the horizontal range
  - (iii) the velocity with which the body strikes the ground



3. A young girl fired a stone from a catapult at an angle of  $45^\circ$  to the horizontal with a velocity of  $90 \text{ ms}^{-1}$ . Calculate:
- the time it will take to reach the highest point;
  - the maximum height reached; and
  - its horizontal distance from the point of projection at this instant.

### 3.6 Derivations of the Equations of Projectile Motion under Gravity

#### 3.6.1 Time of Flight (T)

You will recall from our earlier work that the height attained by the projectile at time  $t$  is given by

$$y = ut \sin \theta - \frac{1}{2} g t^2$$

If time of flight is  $T$ , then,

$$y = Ut \sin \theta - \frac{1}{2} g T^2 \quad (4.7)$$

- Time of flight ( $T$ ) is the time required for a projectile to return to the same level from which it was projected.

This implies as you know that after time  $T$ ,  $y = 0$ .

Hence, from equation (4.7), you obtain

$$0 = UT \sin \theta - \frac{1}{2} g T^2$$

$$T = 0 \text{ or } T = \frac{2u \sin \theta}{g}$$

$$\boxed{T = \frac{2u \sin \theta}{g}} \quad (4.8)$$

You may use an alternative method of obtaining the formula for time of flight by considering the vertical velocity at the maximum height.

Using  $V = U \sin \theta - gt$  gives you

$$0 = U \sin \theta - gt_1,$$

Because at maximum height  $v = 0$ ,  $t_1$  is the time to reach the maximum height.



Hence,

$$t_1 = \frac{u \sin \theta}{g}$$

(4.9)

You know that the time taken to rise from level of projection to maximum height is equal to the time taken to return from the height to the level of projection. Therefore, the total time of flight  $T$  is equal to  $2t_1$  or

$$T = \frac{2u \sin \theta}{g}$$

### 3.6.2 The Maximum Height (H)

- The maximum height (H) is the highest vertical distance attained as measured from the horizontal projection plane.

For you to find H, you will use the equation of motion under gravity  $V^2 = u^2 - 2gh$

recalling that the vertical component of velocity of projectile is  $V_y = U \sin \theta$

Hence,

$$V^2 = (U \sin \theta)^2 - 2gh$$

At maximum height,  $h = H$ ,  $v = 0$

Hence

$$0 = u^2 \sin^2 \theta - 2gH$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

.....(4.10)

### 3.6.3 The Range

- Range is defined as the horizontal distance from the point of projection to the point where the projectile hits the projection plane again.

You will recall that the time taken to travel this horizontal distance is  $T$ , the time of flight. Since the horizontal



component of the velocity of the projectile remains  $U \cos \theta$  throughout the flight, we have that:

$$R = U \cos \theta \times T$$

$$= U \cos \theta \times \frac{2u \sin \theta}{g}$$

Hence;  $R = 2u^2 \sin \theta \cos \theta$

from trigonometry, you know that Hence:

$$2 \sin \theta \cos \theta = \sin 2\theta$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

(4.11)

The range is maximum when  $\sin 2\theta$  of the above equation is maximum. The Sine of an angle is maximum when the angle is  $90^\circ$ . Thus, the maximum value of  $\sin 2\theta$  equals 1 when  $2\theta$  equals  $90^\circ$  or  $\theta = 45^\circ$ . This means that  $R =$

$R_{\max}$  at  $\theta = 45^\circ$  and

$$R_{\max} = \frac{u^2}{g}$$

(4.12)

That is, the maximum horizontal range is attained when the projection angle is  $45^\circ$ .

### 3.7 Applications of the Equations of Motion under Gravity in Solving Numerical problems

**3.7.1** With an initial velocity of  $36 \text{ ms}^{-1}$ , a gold player hits a ball at an angle of  $40^\circ$  from the horizontal thus making the ball to move in a vertical plane. Find the

- time taken by the ball to its highest point,
- maximum height of the ball;
- horizontal range of the ball;
- total time spent by the ball in air;
- velocity of the ball as it strikes the ground (Take value of  $g$  as  $9.8 \text{ ms}^{-2}$ ).
- Show that the motion of the ball is symmetrical



**Solution**

(a) The time taken by the ball to its highest point

$$t_1 = \frac{U \sin \theta}{g} : u = 36 \text{ m/s}, \theta = 40^\circ, g = 9.8 \text{ m/s}^2$$

$$\Rightarrow t_1 = \frac{36 \sin 40^\circ}{9.8} = 2.36 \text{ sec}$$

(b) Maximum height of the ball

$$H = \frac{u^2 \sin^2 \theta}{2g} \quad u = 36 \text{ m/s}, \theta = 40^\circ, g = 9.8 \text{ m/s}^2$$

$$\sin^2 40^\circ = 0.413$$

$$= \frac{36^2 \sin^2 40^\circ}{2 \times 9.8} = \underline{27.32 \text{ m}}$$

(c) Horizontal range of the ball

$$R = \frac{u^2 \sin 2\theta}{g} : u = 36 \text{ m/s}, \theta = 40^\circ, g = 9.8 \text{ m/s}^2$$

$$\sin (2 \times 40^\circ) = \sin 80^\circ = 0.95$$

$$= \frac{36^2 \sin (2 \times 40^\circ)}{9.8} = \underline{130.24 \text{ m}}$$

(d) Total time spent by the ball in air

$$2T = \frac{2u \sin \theta}{g} : u = 36 \text{ m/s}, \theta = 40^\circ, g = 9.8 \text{ m/s}^2$$

$$\sin 40^\circ = 0.643$$

$$= \frac{2 \times 36 \sin 40^\circ}{9.8} = \underline{4.72 \text{ sec.}}$$



(e) Velocity of the ball as it strikes the ground

$$\begin{aligned}
 V &= \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + w^2} = \sqrt{(u \cos \theta)^2 + 2gH} \\
 &= \sqrt{(36 \cos 40^\circ)^2 + 2 \times 9.8 \times 27.32} : u = 36 \text{ m/s}; \theta = 40^\circ \\
 &\quad g = 9.8, H = 27.3 \text{ m} \\
 &\quad \cos 40^\circ = 0.766 \\
 &= \sqrt{760.52 + 535.472} \\
 &= \sqrt{1296} = 36 \text{ m/s} \\
 \text{OR } V &= \sqrt{(36 \cos 40^\circ)^2 + (36 \sin 40^\circ)^2} = 36 \text{ m/s}
 \end{aligned}$$

(f) To show that the motion of the ball is symmetrical in direction of velocity

$$\tan \theta = \frac{v_y}{v_x} = \frac{\sqrt{535.472}}{\sqrt{760.52}} = \frac{23.14}{27.58}$$

$$\begin{aligned}
 \tan \theta &= 0.839 \\
 \therefore \theta &= \tan^{-1} 0.839 = 39.997^\circ \cong 40^\circ
 \end{aligned}$$

This shows that the motion of the ball is symmetrical as illustrated in Fig. 4.7.

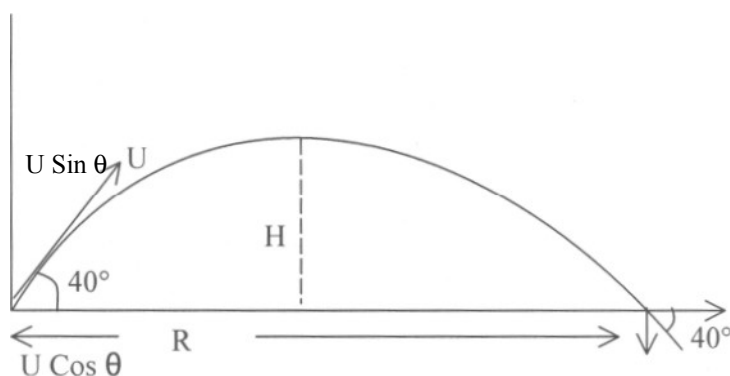


Fig. 4.7: Symmetrical motion of a projectile

### 3.8 Applications of Projectiles

From your knowledge and study of projectiles you can now deduce that the principle of projectiles is used in warfare for aiming accurately at targets, obtaining maximum height and obtaining maximum ranges. For instance, you know that for a bomber from an aeroplane to hit a target,



the bomb must be released when the target appears at a certain angle of depression ~ given by:

$$\tan \phi = \frac{1}{u} \sqrt{\frac{gh}{2}}$$

You also know that at certain angle of projection ( $\theta = 45^\circ$ ) a projectile would attain a maximum range.

In sports, the knowledge of projectile motions could be useful since maximum ranges can be obtained in sports in javelin, short put, football, golf, baseball etc. Also, attaining maximum height, maximum time of flight are sometimes required.

## SELF ASSESSMENT EXERCISE 2

- 1 Describe two situations where the knowledge of projectile motion is found useful.
- 2 What conditions are necessary for an object thrown into the air to describe a parabola?
- 3 A bullet leaves a gun with a velocity  $450\text{ms}^{-1}$  at  $38^\circ$  to the horizontal. Find the:
  - (a) time taken by the bullet to its highest point;
  - (b) the greatest vertical height reached by the bullet;
  - (c) the horizontal range of the bullet
  - (d) total time spend by the bullet in air
  - (e) velocity of the bullet as it strikes the target.

## 4.0 CONCLUSION

In this unit you have learned how to identify a projectile motion as motion under gravity. You have also learned how to derive the equations of projectile - range, maximum height and time of flight. Further more, you have learned how to apply the equations of motion under gravity in solving numerical problems and the importance of the study of projectiles in our everyday life.

## 5.0 SUMMARY

- A projectile is an object which given an initial velocity (usually obliquely or horizontally) is then allowed to move under the action of gravity and air resistance. Usually, in order to simplify the analysis air resistance is neglected.



- The flight of a projectile involves two motions - a constant horizontal motion and a vertical motion under gravity.
- The path or trajectory of a projectile is a parabola.
- Time of flight is the time required for a projectile to return to the same level from which it was projected.

$$T = \frac{2U\sin\theta}{g}$$

- The maximum height attained is the highest vertical distance attained by the projectile as measured from the horizontal projection plane.

$$H = \frac{U^2\sin^2\theta}{2g}$$

- The range is defined as the horizontal distance from the point of projection to the point where the projectile returns to the same level again.

$$R = \frac{U^2\sin 2\theta}{g}$$

- The range is maximum when  $\theta$  equals  $45^\circ$ .

## 6.0 TUTOR-MARKED ASSIGNMENT

1. Define the range of a projectile motion and write down an expression for it. At what angle of projection is the range of a projectile maximum?
2. Given that the height attained by a projectile at a time  $t$  is  $Y = ut \sin \theta - \frac{1}{2}gt^2$ . Show that the time of flight is

$$T = \frac{2U\sin\theta}{g}$$

3. A body is projected from the ground with a velocity of  $38 \text{ ms}^{-1}$  at an angle of  $60^\circ$  to the horizontal. Taking the acceleration of free fall due to gravity as  $9.8 \text{ ms}^{-2}$ , calculate:
  - (i) the time of flight,
  - (ii) the horizontal range, and
  - (iii) the velocity with which the body strikes the ground.



## **7.0 REFERENCES/FURTHER READINGS**

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## UNIT 5 SIMPLE HARMONIC MOTION (SHM)

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### 1.0 INTRODUCTION

In units 3 and 4 of this first Module we studied the motion of a body moving with uniform acceleration under the action of a constant force. It was easy and useful to derive expressions for its velocity, acceleration or displacement from a given point at any given time. In this unit, we will discuss the motion of a body under the action of a resultant force which is not constant but changes during the motion. Like the force, the acceleration also changes at each instant. We shall particularly consider the situation in which the body always returns to an equilibrium position when displaced because a "restoring force" acts on it. The restoring



force causes the body to oscillate 'to and fro' past the equilibrium position.

## **2.0 OBJECTIVES**

At the end of this unit, you should be able to:

- explain and define simple harmonic motion
- show the relationship between:
  - \* linear and angular speed,
  - \* linear and angular acceleration.
- show the relation between period and frequency of the oscillating body
- calculate the energy in the system
- explain forced vibration and resonance.

## **3.0 MAIN CONTENT**

### **3.1 Definition of Simple Harmonic Motion**

Simple Harmonic Motion (SHM) is the periodic motion of a body or particle along a straight line such that the acceleration of the body is directed towards a fixed point (or centre of motion) and is also proportional to its displacement from that point.

#### **3.1.1 Examples of Simple Harmonic Motion**

Examples of Simple harmonic motion include the:

- (i) Motion of a simple pendulum
- (ii) Heart beat
- (iii) Motion of a loaded test tube in a liquid
- (iv) Motion of a mass suspended from a spiral spring
- (v) Motion of the balance wheel of a watch
- (vi) Motion of the strings in a musical instrument e.g. guitar strings
- (vii) Motion of the pistons in a motor vehicle engine
- (viii) Motion of the prongs of a sounding tuning fork
- (ix) Motion of a child's swing
- (x) Motion of the diving board in a swimming pool.



### 3.2 Simple Harmonic Motion from Circular Motion

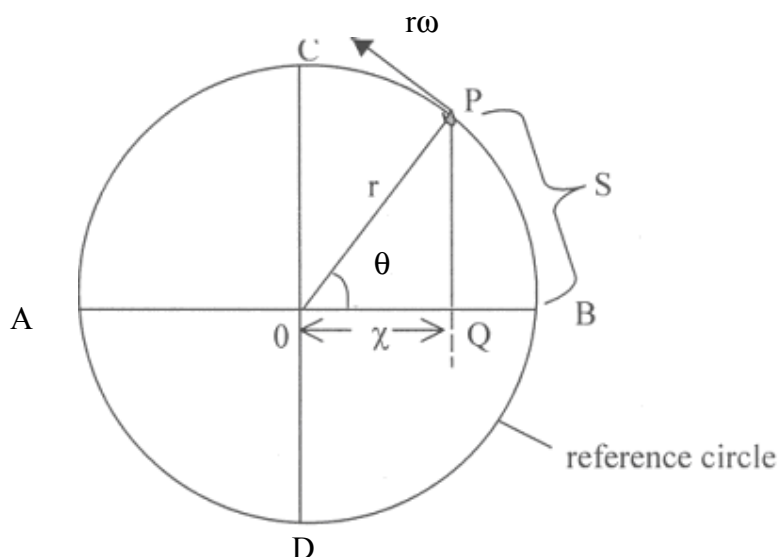


Fig. 5.1: SHM from circular motion

Consider a particle, P moving round a reference circle with centre O and radius  $r$  with a uniform speed  $v$ . If the angular velocity of the radius vector OP is  $\omega$ , then  $a = r\omega$ . Let us draw PQ perpendicular to the diameter AB at Q so that Q is the projection of P on AB. As P moves round the circle, Q moves 'to - and - fro' along AB, being momentarily at rest at A and B and moving with maximum speed as it passes through O, the centre of the path AB. Thus the motion of Q along AB is simple harmonic while the particle P moves with a constant speed  $v$  round the circle ADBCA.

From Fig. 5.1,  $x = r \cos \theta$ , where  $x$  is the horizontal distance of Q from the centre O at that point in time. As the particle moves from B through C, A, D and back to B (i.e. round the circle), the angle  $\theta$  swept through by the radius vector from B to P varies from  $0^\circ$  at B to  $90^\circ$  at C, to  $180^\circ$  at A, to  $270^\circ$  at D and to  $360^\circ$  or  $0^\circ$  back to B. Hence  $\cos \theta$  varies from 1 to 0 to -1 to 0 and back to 1 as  $x$  varies from  $r$  to 0 to  $-r$  and to  $r$  again. Thus the value of  $x$ , the horizontal displacement from the central position O, varies from the value  $r$  to  $-r$  and we can say that  $-r \leq x \leq r$ .

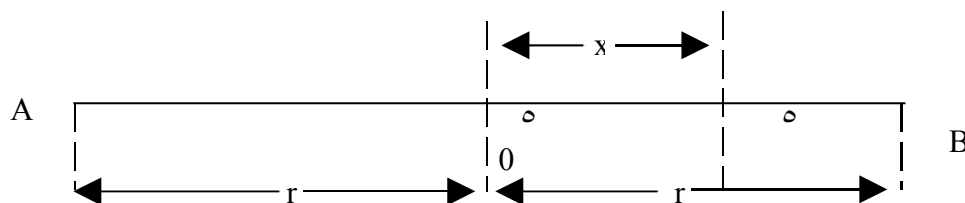


Fig. 5.2: Simple harmonic motion in a straight line



This is simple harmonic motion from the geometrical construction of the motion of a particle in simple circular motion. The circular path of the particle is called "reference circle" and, as we have seen, is quite useful for studying simple harmonic motion.

### 3.3 Amplitude, Period and Frequency of SHM

#### 3.3.1 Amplitude (A)

The maximum value of  $x$ , the displacement from the equilibrium (central) position 0, is known as the amplitude (A) of the SHM. In Fig. 5.2,  $A = r$ .

Definition: The amplitude (A) of a simple harmonic motion is the maximum displacement of the body performing simple harmonic motion from its equilibrium or central position 0.

#### 3.3.2 Period (T)

The total time taken by the point Q to move from B to A and back to B is called the "period" of the SHM. It is also the same time taken to move from 0 to B then to A and back to 0 (Fig. 5.1). This is the time taken by the body in SHM to make one complete oscillation from one point back to that same point but in the same direction.

Definition: The period (T) of a simple harmonic motion is the total time taken by an oscillating body to make one complete oscillation or cycle or revolution about a central reference point.

#### 3.3.3 Frequency (F)

If a body makes  $n$  oscillations in a given time  $t$  seconds then the number of oscillations per second is called the frequency of the SHM and is given by

$$f = \frac{n}{t}$$

Definition: The frequency ( $f$ ) of a simple harmonic motion is the number of complete oscillations per second made by the vibrating body.



### 3.3.4 Relationship between f and T

n complete oscillations or cycles are made in t seconds

∴ Number of complete oscillation or cycle in 1 second,

$$f = \frac{n}{t}$$

Time for complete oscillation or cycle,  $\frac{T}{n} = t$

$$= \frac{1}{n} = \frac{1}{f}$$

$$\therefore T = \frac{1}{f}$$

## 3.4 Speed and Acceleration of SHM

### 3.4.1 Speed of SHM

From Fig. 5.1, we note that as P moves round the circle once, it sweeps through an angle of  $\theta = 360^\circ$  or  $2\pi$  radians in the period T, the period of motion. The rate of change of the angle  $\theta$  with time t is called the angular velocity  $\omega$ . That is, angular velocity ( $\omega$ ) is defined as follows:

$$\omega = \frac{\text{angle turned through by the body}}{\text{time taken}}$$

$$\therefore \omega = \frac{\theta}{t}$$

$$\therefore \theta = \omega t$$

$\theta$  is measured in radians (rad), where  $2\pi \text{ rad} = 360^\circ$ .

$\omega$  is measured in radians per second (rad/s), since  $\omega = \frac{\theta}{t}$

As the angle  $\theta$  is changing (Fig. 5.1) with time, the arc length  $S = BP$  is also

$$\text{changing with time. } \frac{S}{2\pi r} = \frac{\theta}{2\pi} \therefore S = r\theta$$

By definition  $\theta$  (in radians) = s and hence  $S = r\theta$



where  $r = \text{amplitude (A)} = \text{radius of the circle}$ .

The angular velocity ( $\omega$ ) is given by

$$\begin{aligned}\omega &= \frac{\theta}{t} = \frac{s/r}{t} \\ &= \frac{s}{t} \cdot \frac{1}{r}\end{aligned}$$

But  $s/t = v$ , the linear velocity of the particle

Hence,  $\omega = v \cdot 1/r$

$$\therefore v = r\omega = A\omega$$

Thus the linear speed is equal to the product of the angular speed and the radius or amplitude of motion.

### Examples:

1. A small metal object is tied to the end of a string and whirled round a circular path of radius 30 cm. The object makes 20 oscillations in 5 seconds. Calculate the angular and linear speeds of the object.

### Solution

$$\begin{aligned}20 \text{ complete oscillations} &= 360^\circ \times 20 \\ &= 2\pi \times 20 \text{ rad.} \\ &= 40\pi \text{ rad}\end{aligned}$$

$$\text{Angular speed, } \omega = \frac{\theta}{t} = \frac{40\pi}{5\text{s}} \text{ rad} = 8\pi \text{ rad/sec}$$

$$\text{Linear speed, } v = r \omega = 30 \times \frac{8\pi}{\text{s}} \text{ cm}$$

2. A loaded test tube floating upright in water oscillates vertically. If it makes 25 complete oscillations in 5 seconds, calculate the period and frequency of the motion.



**Solution**

$$\text{Period, } T = \frac{\text{time taken}}{\text{number of vibrations}}$$

$$= \frac{5\text{s}}{25} = 0.2\text{s}$$

$$\text{frequency } f = \frac{1}{T} = \frac{1}{0.2} \text{ s}^{-1} \text{ or Hz}$$

$$= 5 \text{ Hz}$$

**3.4.2 Acceleration of SHM (Using Motion of Q)**

From Fig. 5.1, the linear velocity  $v$  at the point Q whose distance from the central point O is  $x$ , is given by

$$v = \omega PQ$$

$$\omega \sqrt{r^2 - x^2}$$

The maximum velocity  $V_m$  corresponds to the velocity at O where  $x = 0$ .  
Hence  $V_m = r\omega$

Thus the maximum velocity of SHM, occurs at the equilibrium or central position ( $x = 0$ ) while the minimum velocity of SHM occurs at the extreme position of motion ( $x = r$ ) and is numerically equal to zero.

An expression relating linear acceleration with the angular velocity can be obtained using differential calculus as follows:

From Fig. 5.1,  $x = r \cos \theta$

Where  $x$  is the displacement of the particle from the centre of motion (same as the distance of Q from O).

$$\text{But } \theta = \omega t, \text{ hence}$$

$$X = r \cos \omega t$$

Where  $\omega$  is the angular speed and  $r$  is the amplitude of vibration.  
Differentiating, we have

$$\frac{dx}{dt} = -r\omega \sin \omega t$$



But  $\frac{dx}{dt}$  = rate of change of displacement with time  
 = linear velocity  $v$

$$\therefore \frac{dx}{dt} = v = -r\omega \sin \omega t$$

We differentiate  $v$  in order to get the acceleration, as follows:

$$a = \frac{dv}{dt} = -r\omega^2 \cos \omega t \text{ since } x = r \cos \omega t.$$

$$= -\omega^2 x$$

Hence, acceleration,  $a = -\omega^2 x$

Thus the linear acceleration of simple harmonic motion is the product of the square of the angular speed and the displacement,  $x$  of the particle from the centre of motion.

The negative sign in the acceleration equation indicates that the acceleration ' $a$ ' is always directed towards the centre of motion while the displacement is measured from the centre outwards, i.e. the two vectors always act in opposite directions at any point in time. This is an essential feature of SHM, namely that (i) the (linear) acceleration of the vibrating object and hence the force tending to restore the object to its equilibrium position are always in a direction opposite to the displacement of the body, and (ii) the acceleration and force are always proportional to the displacement of the body from the equilibrium position.

### 3.4.3 Acceleration of SHM (Using Motion of P)

The equation ( $a = -\omega^2 x$ ) for the linear acceleration of SHM can also be obtained by considering the circular motion of (the particle) P around the reference circle (Fig..5. 1) as follows:

The centripetal acceleration ' $a$ ' of the particle is along PO (the radius of the circle) and its value is given by:

$$\text{Centripetal acceleration, } a = \frac{-v^2}{r}$$

Where  $r$  = radius of circle = amplitude of SHM

But  $v = r\omega$ , hence



$$\begin{aligned}\text{Acceleration, } a &= -\frac{v^2}{r} = -\frac{r^2\omega^2}{r} \\ &= r\omega^2 \\ \therefore a &= -r\omega^2\end{aligned}$$

Note that the component of this centripetal acceleration along OB ( $a_x$ ) is given by  $a_x = a\cos\theta = -\omega^2 r\cos\theta$

$$\therefore a_x = -\omega^2 x, \text{ since } x = r \cos \theta$$

Thus: (i) centripetal acceleration  $= \frac{v^2}{r} = -\omega^2 r$ , and

(ii) Linear acceleration  $= -\omega^2 x$ .

### 3.4.4 Relation between T, $\omega$ and f

It has been shown (section 3.4.1) that by definition,  $\omega = \theta/t$

When  $t = T$ , then  $\theta = 360^\circ$  or  $2\pi$  radians. Hence

$$\omega = \frac{2\pi}{T}$$

$$\text{Or } T = \frac{2\pi}{\omega}$$

Similarly, the frequency  $f$  is given by

$$f = \frac{1}{T} = \frac{1}{2\pi/\omega}$$

$$T = \frac{2\pi}{\omega}$$

$$\text{Hence } f = \frac{\omega}{2\pi}$$

$$\text{Or } \omega = 2\pi f$$

### Example:

A body in simple harmonic motion oscillates with a frequency of 40Hz and amplitude of 2 cm. Calculate.

- (i) the period.
- (ii) the acceleration at the middle and at the end of the path of motion.



- (iii) the velocities at the middle and at the end of the path of motion.
- (iv) The velocity and acceleration at a distance of 1.0 cm from the centre of motion.

### Solution

$$(i) \quad T = \frac{1}{f} = \frac{1}{40\text{s}^{-1}} = 0.025\text{sec}$$

$$= \frac{1}{40} \text{ s}$$

$$(ii) \quad \text{Acceleration, } a = -\omega^2 x$$

$$(a) \quad \text{At middle of path, } x = 0$$

$$\therefore a = -\omega^2 x = 0$$

$$(b) \quad \text{At end of path, } x = \text{amplitude} = 2 \text{ cm}$$

$$\therefore a = -\omega^2 x$$

$$\text{but } \omega = 2\pi f = 2\pi \times 40\text{s}^{-1} = 80\pi \frac{\text{rad}}{\text{s}}$$

$$\therefore a = \left( \frac{80\pi \text{ rad}}{\text{s}} \right)^2 \times 2\text{cm} = 128 \pi^2 \times \frac{10^2 \text{ cm}}{\text{s}^2}$$

$$= 128 \times 10^3 \text{ cm}^{-2}$$

$$= \underline{1.28 \times 10^3 \text{ ms}^{-2}}$$

$$(iii) \quad V = \omega \sqrt{r^2 - x^2}$$

$$(a) \quad \text{At } x = 0, v = \omega \sqrt{r^2 - 0} = \omega r = \frac{80\pi \times 2}{100} \text{ ms}^{-1}$$

$$\therefore V = \underline{1.6\pi \text{ ms}^{-1}}$$

$$(b) \quad \text{At } x = r, v = \omega \sqrt{r^2 - r^2} = \omega \times 0 = 0$$

$$(iv) \quad (a) \quad V = \omega \sqrt{r^2 - x^2}$$

$$= \omega \sqrt{2^2 - 1^2} \quad (x = 1\text{cm})$$

$$= 80\pi \sqrt{3} \text{ cm s}^{-1}$$

$$= 0.8\pi \sqrt{3} \text{ m s}^{-1}$$



$$(b) \quad a = -\omega^2 x \\ = (80\pi)^2 \times 1.0 \text{ cms}^{-2}$$

$$= (80\pi)^2 \times \frac{1}{100} \text{ ms}^{-2}$$

$$= 6400\pi^2 \times \frac{1}{100} \text{ ms}^{-2}$$

$$= 640 \text{ ms}^{-2}$$

### 3.4.5 Angular Acceleration ( $a_\theta$ )

Definition: The angular acceleration ( $a_\theta$ ) of a body in simple harmonic motion is the time rate of change of its angular velocity ( $\omega$ ).

Unit of  $a_\theta$ : Angular acceleration ( $a_\theta$ ) is expressed in radians per second ( $\text{rad s}^{-2}$ )

Suppose the angular velocity of a body in SHM changes uniformly from  $\omega_1$  to  $\omega_2$  in  $t$  seconds, then angular acceleration ( $a_\theta$ ) is given by

$$a_\theta = \frac{\text{change in angular velocity}}{\text{Time taken for the change}}$$

$$= \frac{\omega_2 - \omega_1}{t}$$

But  $\omega = \frac{v}{r}$  or  $v = r\omega$ , hence

$$a_\theta = \frac{\frac{V_2 - V_1}{r}}{t}$$

$$= \frac{1}{r} \frac{(v_2 - v_1)}{t}$$

$$= \frac{a}{r}$$

where  $a$  is the linear acceleration of the body

$$\therefore a = ra_\theta$$



Thus linear acceleration ( $a$ ) is equal to the product of angular acceleration ( $a_\theta$ ) and the radius ( $r$ ) (or the displacement of the particle from its equilibrium or central position,  $x$ ).

### 3.5 Energy in Simple Harmonic Motion

When a body is in SHM, there is always a restoring force which tends to return the body to its central or equilibrium position. Since "force" and "displacement" are involved, it follows that "work" and "energy" are also involved in SHM. In simple harmonic motion, the total energy of the system is made up of only two types of energy, namely Potential and Kinetic energies. At any instant of the motion, the energy of the system may be only Potential energy (PE) or only Kinetic energy (KE) or a combination of both PE and KE. Thus in SHM, there is a constant transformation of energy from Potential energy to kinetic energy and from Kinetic energy to potential energy as the body oscillates about its equilibrium position. The total energy of a body in SHM is always constant and at any point in time is given by

Total energy of SHM = Potential energy (PE) + Kinetic energy (KE)

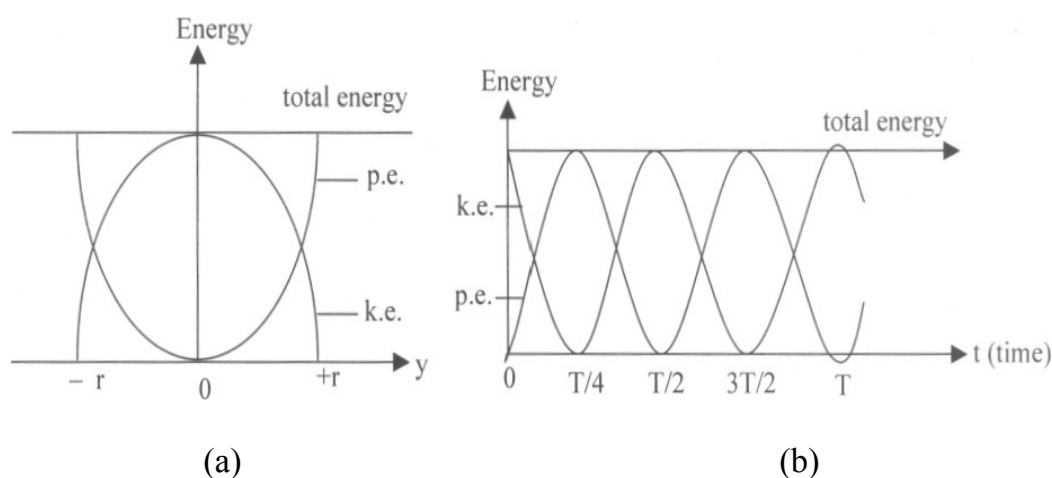


Fig. 5.3: Kinetic and potential energy variation

Fig. 5.3 (a) shows how the PE and KE vary with the displacement  $x$  from the centre of motion  $r$  is the maximum displacement or amplitude. Fig. 1.5.3. (b) shows how the P.E. and KE vary with time  $t$ , when  $t$  is measured from the centre of motion  $0$ . If  $T$  is the period, then  $T/4$  is the time when the object is at the end of its oscillation,  $T/2$  is the time when it returns to the centre, and so on. The graphs have the shape of sine curve.



We shall illustrate energy in SHM using two SHM system as examples:

- (i) motion of a simple pendulum, and
- (i i) motion of a loaded spiral spring.

### 3.5.1 Energy in an Oscillating Simple Pendulum

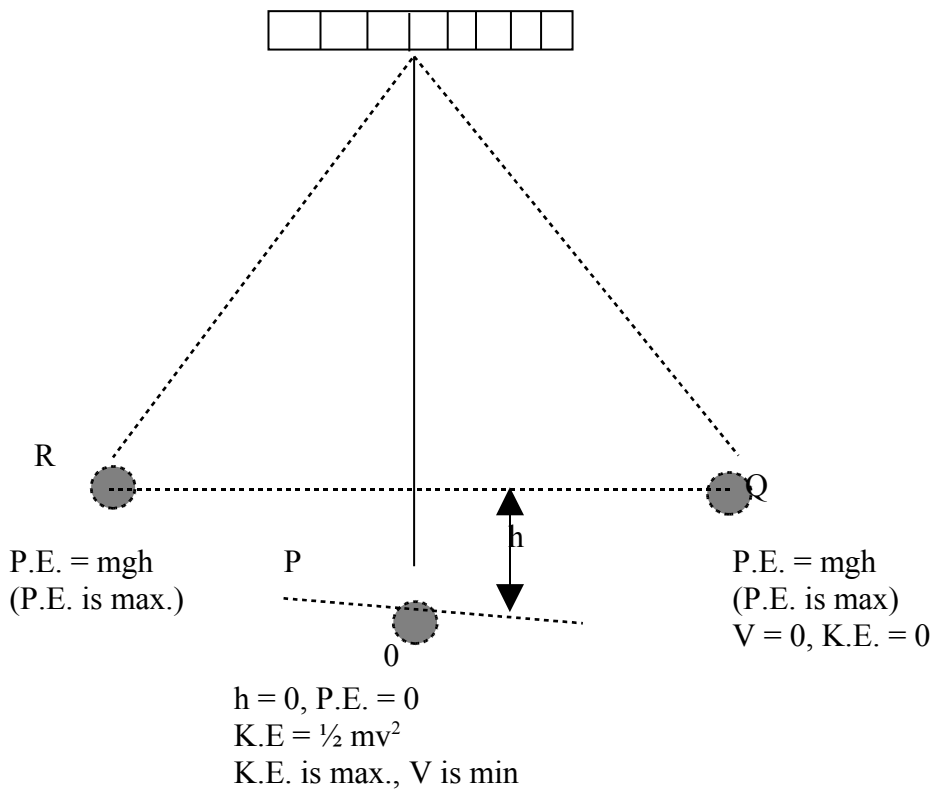


Fig. 5.4: Energy transformations of a simple pendulum

Fig.5.4 shows the energy transformations in a system comprised of an oscillating simple pendulum. The energy at the extreme positions R and Q is all PE and is equal to  $mgh$  where  $m$  is the mass of the bob,  $h$  its height above the lowest level and  $g$  is the acceleration due to gravity. At 0,  $h = 0$  and  $PE = 0$ , but velocity of the moving bob is maximum ( $V_m$ ). Hence the KE is

maximum and is equal to  $\frac{1}{2}mV_m^2$

Thus,

$$mgh = \frac{1}{2}mV_m^2$$

$$\text{or } V_m = \sqrt{2gh}$$

At any instant of the oscillation of the simple pendulum,  
Total energy = Potential energy + Kinetic energy



$$\text{i.e. Total energy} = mgh + \frac{1}{2} Mv^2 = mgh = \frac{1}{2}mV_m^2$$

where  $h_m$  = maximum height reach of by the pendulum bob and  
 $V_m$  = maximum velocity attained by the bob.

### 3.5.2 Energy in an Oscillating Loaded Spiral Spring

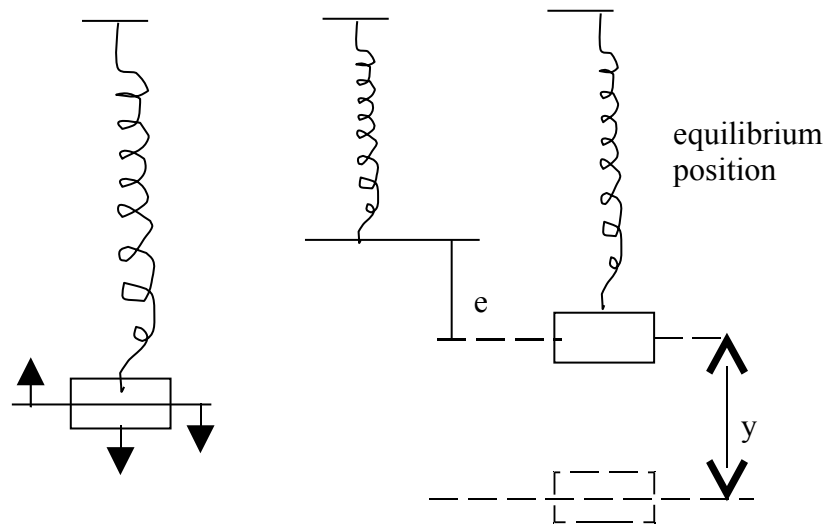


Fig. 5.5: Energy in a vibrating loaded spring

If a mass suspended from the end of a spiral spring is stretched vertically downwards and released, it oscillates up and down in SHM. The force that tends to restore the spring to its centre of motion as it oscillates is the elastic (restoring) force and is given by

$$F = -ky$$

where  $k$  is the force constant,  $y$  is the displacement of the spring from the equilibrium position or centre of motion. The total work ( $W$ ) done in stretching the spring through the distance  $y$  is given by

$$W = \text{average force} \times \text{displacement}$$

$$= \frac{1}{2} ky \times y = \frac{1}{2} ky^2$$

Thus the maximum total energy stored in the spring is given by

$$W = \frac{1}{2} kr^2$$

where  $r$  is the amplitude of motion or maximum displacement from equilibrium position. This maximum energy is conserved throughout the oscillation of the SHM system.



At any point in time during the oscillation, the total energy is

$$W = \frac{1}{2} kr^2 = \frac{1}{2} mv^2 + \frac{1}{2} ky^2$$

(KE)      (PE)

where  $v$  is the velocity of the suspended mass at a distance  $y$  from the equilibrium position.

### 3.5.3 Period of Oscillation of a Loaded Spring

We have seen that the total energy ( $W$ ) in a loaded spring in SHM is given by

$$W = \frac{1}{2} kr^2 = \frac{1}{2} mv^2 + \frac{1}{2} ky^2$$

$$\therefore v^2 = \frac{k}{m} (r^2 - y^2)$$

Thus the linear velocity ( $v$ ) of the suspended mass is given by

$$V = \sqrt{\frac{k}{m} (r^2 - y^2)}$$

Comparing the two equations above, we have that

$$w = \sqrt{\frac{k}{m}}$$

$$w^2 = \frac{k}{m}$$

Hence the period  $T$  is given by

$$T = \frac{2\pi}{w} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$\therefore T = 2\pi \sqrt{\frac{m}{k}}$$

The period ( $T$ ) therefore depends only on the mass ( $m$ ) and the force or elastic constant ( $k$ ) of the spring. It does not depend on the amplitude of vibration or on the value of  $g$  at the place of vibration. The value of  $k$  can be obtained from Hooke's law using  $F = mg = ke$

where  $e$  is the extension produced in the spring by the mass  $m$ . Hence

$$K = \frac{mg}{e}$$



In obtaining the formula for the period  $T$  of the oscillating loaded spring, the mass ( $m_s$ ) of the spring has been assumed negligible. If the mass of the spring is taken into account, then the true value of the period  $T$ , is given by

$$T = 2\pi \sqrt{\frac{m + m_s}{k}}$$

### 3.5.4 Period of Oscillation of a Simple Pendulum

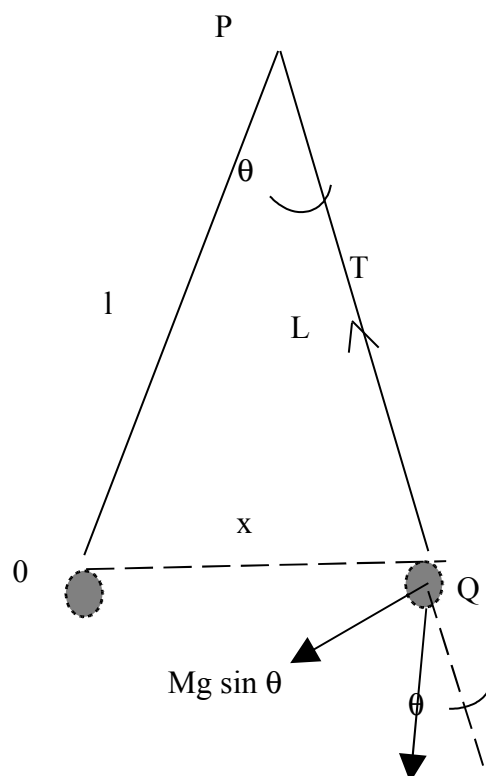


Fig. 5.6: Theory of simple pendulum

To derive the formula for the period ( $T$ ) of an oscillating simple pendulum, all that is needed is to obtain the value of the angular velocity ( $\omega$ ) of the system in terms of the parameters of the system, since

$$T = \frac{2\pi}{\omega}$$

Consider a simple pendulum oscillating in such a way that at a time  $t$ , the bob is at  $Q$  where  $OQ = x$  and angle  $OPQ = \theta$  ( Fig.5 .6) With the bob at  $Q$  the restoring force which pulls it towards  $O$ , the centre of motion is equal to  $mg \sin \theta$ .  $PQ$  is perpendicular to the tangent at  $Q$ , hence the tension in the spring has no component along  $QO$ . Therefore.

$$\text{Restoring force} = ma = - mg \sin \theta$$



where  $a$  is the acceleration along the arc OQ and the negative sign indicate that the force towards and the displacement  $x$  of the bob from O are in opposite directions.

If  $\theta$  is very small, then  $\sin\theta = \theta$  in radians. Again from Fig.5.1; we have that

$$\text{Using the sector OPQ } \frac{x}{2\pi l} = \frac{\theta}{2\pi}; \Rightarrow \theta = \frac{x}{l}$$

Hence the above equation for the restoring force becomes:

$$\text{Restoring force} = ma = -mg\sin\theta = -mg \frac{x}{l}$$

$$\begin{aligned} \therefore a &= -\frac{g}{l}x \\ &= -\omega^2 x \\ \therefore \omega^2 &= \frac{g}{l} \end{aligned}$$

$$\text{Or } \omega = \sqrt{\frac{g}{l}}$$

The period of oscillation (T) of the simple pendulum is given by

$$T = \frac{2\pi}{\omega}$$

$$\text{Hence } \frac{T}{\sqrt{g/l}} = 2\pi \quad = 2\pi \sqrt{L/g}$$

$$\therefore T = 2\pi \sqrt{L/g}$$

Thus since  $g$  is constant at a particular place the period of a simple pendulum depends only on the length  $l$  of the pendulum. Since  $g$  varies from place to place, the period (T) of a simple pendulum will also vary from place to place.

### Example

A mass of 25 g is suspended from the end of a spiral spring whose force constant is  $0.5 \text{ Nm}^{-1}$ .

If the system is set into simple harmonic motion with amplitude 20cm, calculate the



- (i) Period of the motion,
- (ii) Frequency of the motion,
- (iii) Angular speed,
- (iv) Total energy of the system,
- (v) Maximum velocity of the motion, and
- (vi) Maximum acceleration of the system

**Solution**

$$(i) \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{m/k}}$$

$$= \frac{2\pi}{\sqrt{\frac{0.025}{0.5}}}$$

$$= \underline{1.41 \text{ sec}}$$

$$(ii) \quad F = \frac{1}{T} = \frac{1}{1.41}$$

$$= 0.71 \text{ Hz}$$

$$(iii) \quad \omega = 2\pi f = 2\pi \times 0.71 = 4.46 \text{ rad s}^{-1}$$

$$(iv) \quad \text{Total energy, } E = \frac{1}{2} kx^2$$

$$= \frac{1}{2} \times 0.5 \times (0.2)^2 \quad \text{J}$$

$$= \underline{0.01 \text{ J}}$$

- (v) Let maximum velocity of motion be  $V_m$ , then

$$\frac{1}{2} m v_m^2 = \frac{1}{2} k x^2 = 0.01$$

$$\therefore v_m^2 = \frac{2 \times 0.01}{0.025} \frac{\text{m}^2}{\text{s}^2}$$

$$V_m = \sqrt{0.08} \frac{\text{m}^2}{\text{s}^2}$$

$$\therefore V_m = 0.89 \text{ ms}^{-1}$$

- (vi) Maximum acceleration  $a$  is given by

$$a = -\omega^2 r$$

$$= (4.46)^2 \times 0.2 \frac{\text{m}}{\text{s}^2}$$

$$= \underline{3.98 \text{ ms}^{-2}}$$



### 3.6 Force Vibration and Resonance

#### 3.6.1 Free, Forced and Damped Vibration

When a system is performing simple harmonic motion, it gradually loses its energy owing to friction within its parts as well as air resistance. The amplitude of such a motion reduces gradually but steadily until it eventually decreases to zero. Such a motion is known as “damped” motion. If there is no damping, the motion is said to be free and the amplitude remains constant with time. If the free oscillation of a body in SHM is to be maintained for a long time, an “external period force” would be required to force the body to continue its oscillatory motion in spite of the damping. The oscillation of such a body is then said to be a “force vibration”.

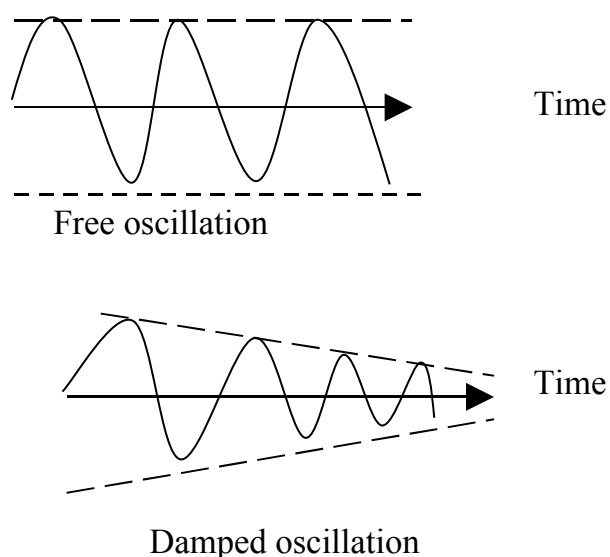


Fig.5.7 above illustrates the sketch displacement – time graphs of a body performing (a) free simple harmonic oscillations, and (b) damped simple harmonic oscillation.

#### Definition

Forced vibrations are oscillations resulting from the action of an external periodic force on an oscillating body.

#### 3.6.2 Resonance in a Vibrating System

Every vibrating object possesses a frequency ( $f$ ) of vibration. The frequency ( $f_0$ ) with which the object would oscillate if it is left undisturbed after being set into vibration is called the “natural frequency” of vibration. If the object is subjected to an external periodic



force, the frequency of such an external force is called the “forcing frequency ( $f_e$ )”.

If the frequency of a vibrating system which acts on another system coincides with the natural frequency of the second system, then the second system is set into vibration with a relatively large amplitude. This phenomenon is known as “resonance”

### Definition

Resonance in a system performing SHM is said to occur when the forcing frequency ( $f$ ) of an external periodic force coincides with the natural frequency ( $f_0$ ) of a body with which it is in contact, causing the body to vibrate with a large amplitude.

At resonance, maximum energy is transferred from the periodic external force to the natural vibrations of a system.

### Examples of Resonance in vibration systems

- (i) A vibrating tuning fork, pressed against a table top, is heard to give a louder sound.
- (ii) Divers make use of resonance to obtain a maximum lift –off from diving boards. They do this by tuning the frequency of their bounce on the diving board with the natural frequency of the diving board.
- (iii) Glass windows may vibrate as a result of the high- pitched notes emitted from a nearby radio set.
- (iv) Car bodies rattle at certain car speeds as a result of the resonant vibrations, induced in parts of the car when the natural frequency of vibration coincides with the frequency of vibration of the car engine.

### SELF ASSESSMENT EXERCISE 1

1. What do you understand by simple harmonic motion? Give three examples of simple harmonic motion.
2. Explain the meaning of (i) amplitude, (ii) complete oscillation of an object in simple harmonic motion.
3. An object in simple harmonic motion has an amplitude of 0.05m and a frequency of 30 Hz. Calculate
  - (i) the period of oscillation ,
  - (ii) the accelerations at the middle and at the end of an oscillation,



- (iii) The velocities at the middle and at the end of an oscillation.
4. Describe the energy transformations of the bob of a simple pendulum Oscillating about its equilibrium position .

Calculate the period of oscillation of an object which vibrates at 10 cycles per second.

## 4.0 CONCLUSION

In this unit you learned about simple harmonic motion – Definition, example and terms associated with simple harmonic motion. You also learned speed and acceleration of simple harmonic motion and energy in simple harmonic motion. Finally, you learned forced vibration and resonance in simple Harmonic Motion. (SHM).

## 5.0 SUMMARY

- Simple harmonic motion is characterized by regularity and repetition of vibrational (to – and –fro) motion.
- In SHM the acceleration is always directed towards a fixed point (the centre of motion) and is directly proportion to the distance of the particle from the fixed point.
- The following relations hold in SHM

(i) Acceleration,  $a = -\omega^2 x$ ,  $a_{\max} = -\omega^2 r$

(ii) Period,  $T = \frac{2\pi}{\omega}$

(iii) Frequency,  $f = 1/T = \frac{\omega}{2\pi}$

(iv) Linear velocity,  $v = \pm \omega \sqrt{r^2 - x^2}$   
 $v = \pm \omega r$

(v) Angular velocity,  $\omega = \theta/t$

where  $r$  = amplitude of motion, and  $v$  is the velocity when the displacement from the centre of motion is  $x$ .

(vi) Linear acceleration  $a = a_0 r$ , where  $a_0$  is the angular acceleration.

4 a For a simple pendulum of length  $l$ , the period of oscillation is given by  $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{l/g}}$

b. For a loaded spring in SHM  
 $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{m/k}}$



where  $K$  is the spring constants

- 5 Resonance occurs in a vibrating system when the forcing frequency of an external periodic force coincides with the natural frequency of a vibrating body.

## 6.0 TUTOR-MARKED ASSIGNMENT

1. By clearly defining each term, distinguish between, (i) angular velocity and linear velocity, (ii) angular acceleration and linear acceleration of a body in simple harmonic motion.
2. Define simple harmonic motion. A body in simple harmonic motion has an amplitude of 10 cm and a frequency of  $100\text{Hz}$ . Calculate:
  - (a) The period of oscillation,
  - (b) The acceleration at the maximum displacement
  - (c) The velocity at the centre of motion.
3. Write the equation for the total energy of an oscillating loaded spring in terms of the potential energy (PE) and the Kinetic energy (KE) at any stage of the motion, explaining all the Symbols used.

Hence or otherwise, derives an expression for the period,  $T$  of the System.

## 7.0 REFERENCES/FURTHER READINGS

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## MODULE 2

Unit 1	Force, Centre of Gravity and Equilibrium
Unit 2	Friction in Solids and Liquids
Unit 3	Linear Momentum
Unit 4	Simple Machines

### UNIT 1      FORCE,      CENTRE      OF      GRAVITY      AND EQUILIBRIUM

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- 7.0 References/Further Readings

## 1.0 INTRODUCTION

In units 3 and 4 of Module 1, the concepts of motion, types of motion, and the parameters used in describing motion were considered. No mention was made of what causes motion. This unit discusses the cause of motion, its nature and types. The unit also treats the concepts of centre of gravity, equilibrium of bodies and moment or turning effect of a force about a point.

## 2.0 OBJECTIVES

At the end of the unit you should be able to:

- define force and explain its nature
- identify, explain and distinguish between various types of force.
- explain the meaning of centre of gravity and identify its position for some regular uniform and irregular bodies.
- explain the concept of equilibrium and distinguish between (a) static and dynamic equilibrium (b) resultant and equilibrant forces
- explain the conditions that must be satisfied if an object is to be kept in equilibrium under the action of:
  - parallel
  - non-parallel forces.
- explain what is meant by moment of force about a point
- take moments of given forces about any point and show their directions
- solve simple problems on objects kept in equilibrium by a number of given forces
- explain the effect of cg on the stability of a body



- identify the forces acting on a body partially or completely immersed in a liquid and state the conditions for the body to float in the liquid.

### **3.0 MAIN CONTENT**

#### **3.1 Force**

##### **3.1.1 Concept of Force**

Force is the agent which causes objects to move. It usually manifests in the form of a “push” or a “pull”.

An object, e.g a book which is placed on a table is (macroscopically) at rest unless it is caused to move visibly by

- i. Pushing it
- ii. Pulling it or
- iii. Lifting it.

When a body is moving in a straight line with uniform velocity, it will continue in that manner unless a push or a pull is applied to it to change its speed or direction, or both speed and direction.

Force may also change (or tend to change) the shape of a body. It is difficult to describe force because it is not a visible entity. However, its effects can be seen and therefore be described.

##### **3.1.2 Definition and Nature of Force**

###### **Definition of Force**

Force may be defined as a push or a pull which changes or tends to change the shape or state of rest or uniform motion of a body in a straight line.

###### **Nature of Force**

The nature of force may be summarized as follows:

- i. Force is a push or a pull
- ii. It is the thing or entity that causes objects to move.
- iii. It is a vector quantity:-it always has both magnitude and direction.
- iv. Force is not a visible entity and can be described through its effects which are easy to see.



- v. When a body A exerts a force (action) on a body B, B also exerts an equal force (reaction) in the opposite direction.

## 3.2 Types of Force

There are two main types of forces:

- i. Contact forces
- ii. Non-contact forces or force fields.

### 3.2.1 Contact Forces

This is a force which is exerted when two bodies or surfaces are in contact with each other, directly or indirectly. Examples include a push, a pull, reaction forces, friction, tension forces, thrust forces.

### 3.2.2 Non-contact Forces or Force Fields

This is a force which is exerted between objects which are not in contact with each other. A force field makes itself felt without the presence of any material connection between the bodies involved. Force fields are of the action at a distance type. Suppose an object X sets up force field, it is observed that within a fixed region or neighbourhood of X, the influence of X on Y can be felt or detected and we talk about Y being in the field of X. In some cases, it can also be considered that Y sets up a force field and that a force is accordingly exerted on X in the field of Y.

Examples of force fields are gravitational, magnetic, electrostatic or electric force fields and nuclear force fields. A brief description of each of these force fields is presented below.

#### a. Gravitational Force Field

Every object in the universe attracts other objects with a gravitational force. This gravitational force is responsible for the orbital motion of the moon round the earth or the planets round the sun. it was discovered by Sir Isaac Newton (1642-1727).

#### b. Electrostatic or Electric Force Field

Electrostatic force is the force between two electrical charges. A force of attraction is experienced if the two charges are unlike (positive and negative) while a force of repulsion is experienced if the charges are alike (positive and positive or negative and negative). The law governing the force between electrical charges was discovered by a French scientist, Charles Coulomb (1736-1806).



### c. Magnetic Force Field

A permanent magnet or an electromagnet (a wire or coil carrying current) sets up a force field in a region or neighbourhood surrounding it. Another magnet or electromagnet at any point within this region or neighbourhood experiences a force of attraction or repulsion due to the magnet/electromagnet. Such a force is called a magnetic force field. A **moving charge** also sets up a magnetic force field in addition to the electric field which surrounds it. A second charge in these two combined fields will experience a force due to the magnetic field **only if it is in motion**.

### d. Nuclear Force Field

There exists, in the nucleus of an atom, the strongest force in nature which holds together the protons and neutrons that make up the nucleus. If this force were not there, then the electrostatic forces of repulsion between the protons which carry positive electrical charges would cause the nucleus to disintegrate. A very important feature of nuclear forces is that they are short ranged- the nuclear force between two nucleons is negligible if the separation between the nucleons is greater than about  $10^{-15}\text{m}$ .

Another important feature or property of nuclear forces is that they are independent of charge, i.e the nuclear force between two protons is the same as that between two neutrons and the same as that between a proton and a neutron.

## 3.3 Concepts and Types of Equilibrium

### 3.3.1 Concepts of Equilibrium

A body is said to be in equilibrium if its acceleration is zero, i.e there is no net force acting on it in any direction. Therefore, the body may be at rest or moving with constant velocity.

#### Definition

A body is said to be in equilibrium if:

- i. The body as a whole remains at rest
- ii. The body is moving in a straight line with constant velocity and
- iii. The body is rotating at a constant angular velocity.

### 3.3.2 Types of Equilibrium



There are two types of equilibrium: static equilibrium and dynamic equilibrium.

### i. Static Equilibrium

If a body has zero acceleration by virtue of being at rest, the body is said to be in static equilibrium.

An example is a book resting on a table.

### ii. Dynamic Equilibrium

If a body is moving with a constant velocity in a straight line or is rotating with a constant angular velocity about a fixed point or axis through its centre of mass, the body is said to be in dynamic or kinetic equilibrium.

An example is a car whose engine is running at constant speed for a considerable length of time.

### 3.3.3 Resultant and Equilibrium Forces

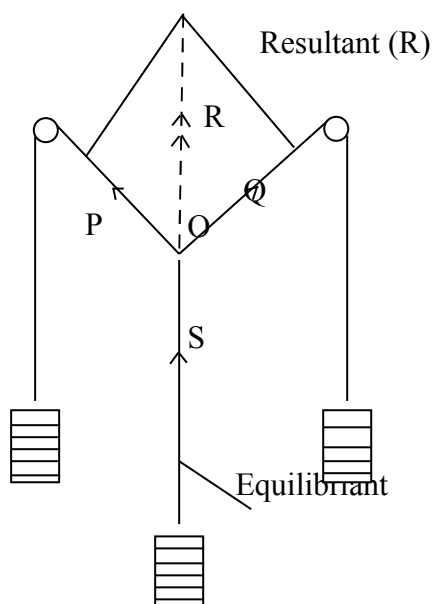


Fig.1.1: Resultant and equilibrium forces

Consider the force board arrangement shown in Fig 1.1 above. Suppose the force  $P$ ,  $Q$  and  $S$  are in an equilibrium state. The resultant  $R$  of  $P$  and  $Q$  acting at  $O$  can be obtained by the principle of parallelogram of forces. The resultant  $R$  would be found to be equal in magnitude but exactly opposite in direction to the third force  $S$  which keeps the point  $O$  in equilibrium. This third force is known as the equilibrant force and



counterbalances the other two forces  $P$  and  $Q$  while  $R$  is called the resultant (force) of  $P$  and  $Q$ .

### Definitions

- i. A resultant force is that single force which, acting alone, will have the same effects in magnitude and direction as two or more forces acting together.
- ii. The equilibrium of two or more forces is that single force which will balance all the other forces taken together. It is equal in magnitude but opposite in direction to the resultant force.

**Note:** If three forces  $F_1$ ,  $F_2$ ,  $F_3$  acting at a point are in equilibrium (Fig. 1.2), the resultant of two of these forces is equal but opposite in direction to the third force.

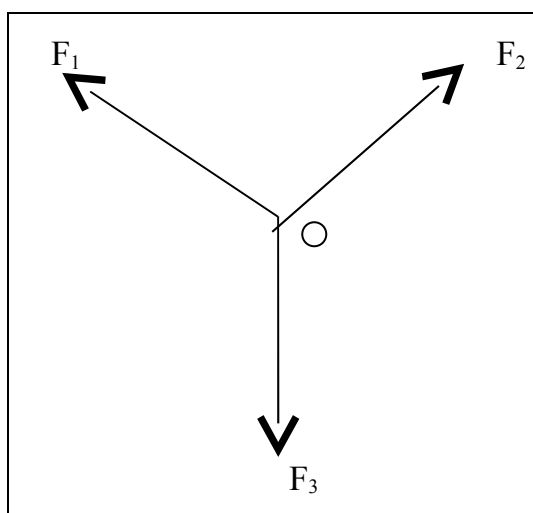


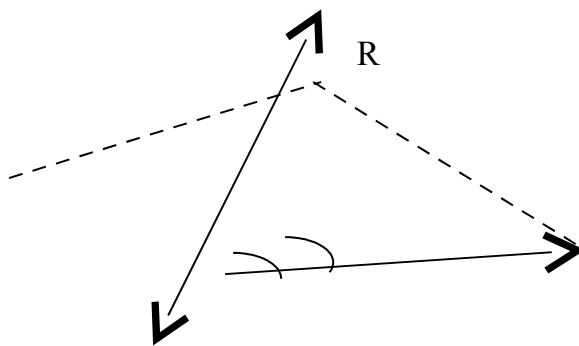
Fig. 1.2: Three forces in equilibrium

Any of these forces is said to be the equilibrant of the other two. Again, the lines of action of  $F_1$ ,  $F_2$  and  $F_3$  must all pass through common point ( $O$  in this case).

### Worked example

Two forces  $5\text{N}$  each are inclined to each other at  $120^\circ$ . Find the single force that will

- a. Replace the given force system
- b. Balance the given force system







E

The resultant force (R) is given by

$$\begin{aligned}
 R^2 &= 5^2 + 5^2 + 2 \times 5 \times 5 \cos 120^\circ \\
 &= 25 + 25 + 50 \cos (90^\circ + 30^\circ) \\
 &= 25 + 25 - 50 \sin 30^\circ \\
 &= 25 \\
 \therefore R &= 5\text{N}
 \end{aligned}$$

Using the Sin rule, we have

$$\begin{aligned}
 \frac{5}{\sin \alpha} &= \frac{R}{\sin 60^\circ} = \frac{5}{\sin 60^\circ} \\
 \therefore \frac{5}{\sin \alpha} &= \frac{5}{\sin 60^\circ} \\
 \therefore \sin \alpha &= \sin 60^\circ \\
 \therefore \alpha &= 60^\circ
 \end{aligned}$$

The resultant (R) is a 5N force at  $60^\circ$  with each of the given forces.

### 3.3.4 Equilibrium of Three Forces Acting at a Point (Parallelogram and Triangle of Forces)

#### i. Parallelogram of Forces

The principle of parallelogram of forces states that if two inclined forces are represented in magnitude and direction by the adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through the point of intersection of the two sides.

#### ii. Triangle of Forces



The principle of triangle of forces states that if three forces are in equilibrium, they can be represented in magnitude and direction by the three sides of a triangle taken in order.

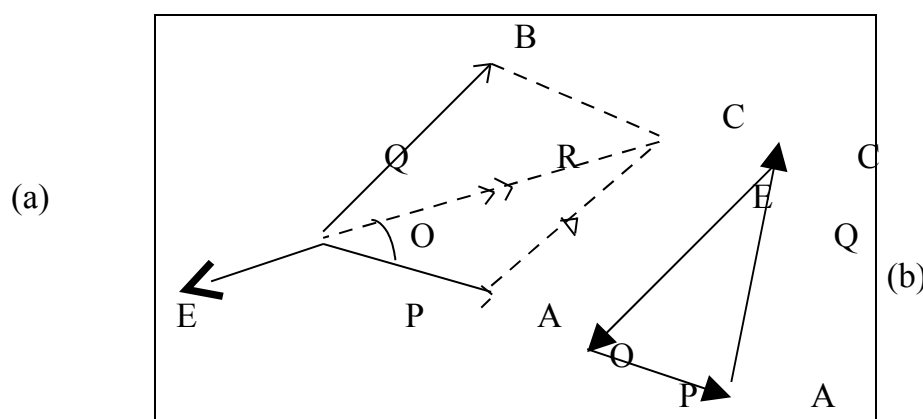


Fig.1.3: Parallelogram and triangle of forces

Fig.1.3 (a) Shows three forces P, Q and E which keep the point O in equilibrium. OACB is the parallelogram of forces in which OA represents P in magnitude and direction and OB represents Q in magnitude and direction. The diagonal of the parallelogram OC represents the resultant R of P and Q in magnitude and direction. R is counterbalanced by the equilibrant force E.

Triangle OAC in Fig 1.3 (b) is the triangle of forces for P, Q and E (Note that the arrows are taken in a cyclical order.

The resultant R of P and Q is given by

$$R^2 = P^2 + Q^2 + 2PQ \cos Q$$

where Q is the angle at which P and Q are inclined.

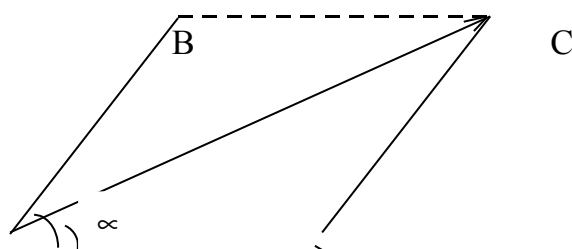
### Worked examples

- Two forces of 3N and 4N acting at point O are inclined at an angle of  $60^\circ$  with each other. Find the resultant forces.

#### Solution

#### By scale drawing

Choose a suitable scale and draw OA and OB to represent 3 units and 4 units at an angle of  $60^\circ$  with each other.





3 units

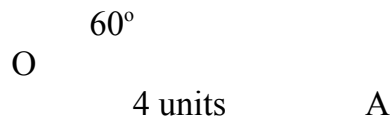
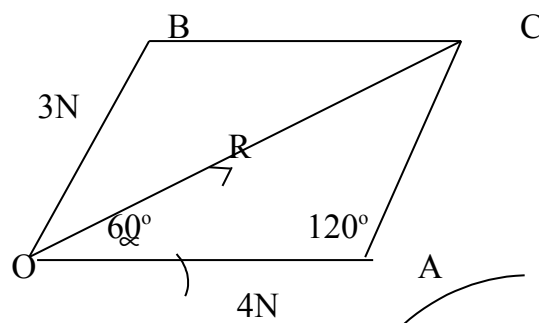


Fig.1.4

Complete the parallelogram by drawing AC parallel to OB and BC parallel to OA. Join and measure OC which represents the resultant in magnitude and direction. Measure also angle AOC ( $\alpha$ ), the angle between the resultant force and the 4N force. You will find that R is approx 6.1 N and  $\alpha = 25^\circ 18'$ .

By analytical method



Using the cosine rule, we have that

$$R^2 = 3^2 + 4^2 + 2 \times 3 \times 4 \cos 60$$

$$R = \sqrt{9 + 16 + 12} = \sqrt{37}$$

$$\therefore R = 6.08\text{N}$$

Using the sine rule, we have that

$$\frac{3}{\sin \alpha} = \frac{6.08}{\sin 120^\circ}$$

$$\therefore \sin \alpha = \frac{3 \times \sin 120^\circ}{6.08} = 0.4273$$

$$\therefore \alpha = \sin^{-1}(0.4273) = 25^\circ 18'$$



2. A mass of 2.5 kg is supported by two chords which make angles of  $30^\circ$  and  $50^\circ$  with the vertical. Calculate the tensions in the two chords.

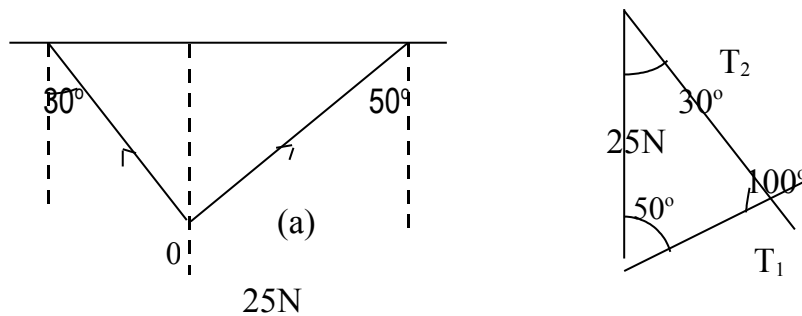


Fig. 1.5

### Solution

#### Analytical method

The problem is illustrated in Fig.1.5 (a)

Mass of 2.5 kg = 25N force

Sketch the triangle of forces

Fig. 1.5 (b) (not to scale), then use the sine rule as follows:

$$\begin{aligned} \frac{25}{\sin 100^\circ} &= \frac{T_1}{\sin 30^\circ} = \frac{T_2}{\sin 50^\circ} \\ \therefore T_1 &= \frac{25 \sin 30^\circ}{\sin 80^\circ} = 12.69 \text{ N} \\ T_2 &= \frac{25 \sin 50^\circ}{\sin 80^\circ} = 19.45 \text{ N} \end{aligned}$$

(You should attempt the solution by graphical method and compare the values obtained by the two methods).

Triangle OAC in fig. 2.5 (b) is the triangle of forces for the forces P, Q and E (Note that the arrows are in a cyclical order around the triangle).

In fig 2.3 (a) O A C B O is the parallelogram of forces. OC is resultant force.



### 3.4 Moment of a Force

The turning effect of a force is called its moment. Examples of the turning effects of forces in everyday life include:

- i. Opening a door about its hinges by exerting a force on the door handle or door edge
- ii. Turning a tap on or off
- iii. Force exerted on the pedals when riding (i.e pedaling) a bicycle
- iv. Opening of the metal cap of a soft drink bottle
- v. Driving a screw in or out of wood with a screw driver
- vi. Tightening or loosening a nut with a spanner etc.

#### Definition

The moment of a force about a point or axis is the turning effect of the force about that point or axis. It is equal to the product of the force and the perpendicular distance of its line of action from the point or axis.

Mathematically,

Moment = Force  $\times$  perpendicular distance of pivot to the line of action of the force.

Unit of moment

Since force is in newtons (N) and distance is in metres (M), the SI unit of moment is Newton-metre (Nm).

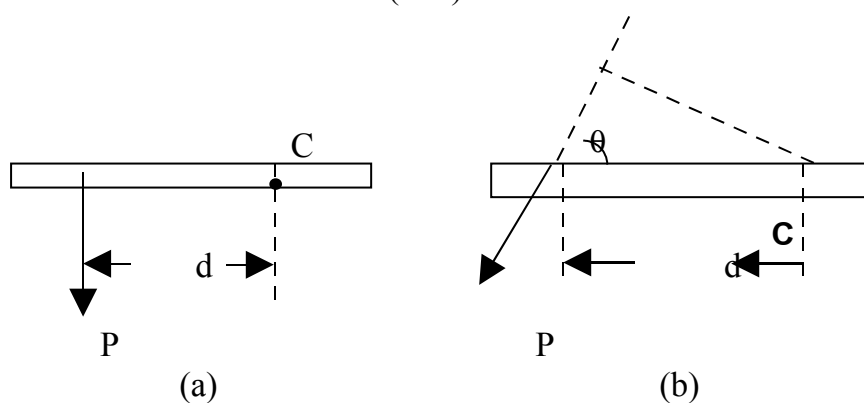


Fig. 1.6: Moment of a force

In Fig. 1.6 above, the moment of the force P about the point C is given by

$$\text{Moment} = P \times d \text{ (Fig 1.3 (a))}$$

$$\text{Moment} = P \times d \sin Q \text{ (Fig.1.3 (b))}$$



### Worked examples

Weight of 20 N and 30 N are hung on a light beam as shown in Fig.1.7. Find the moments of the forces about O.

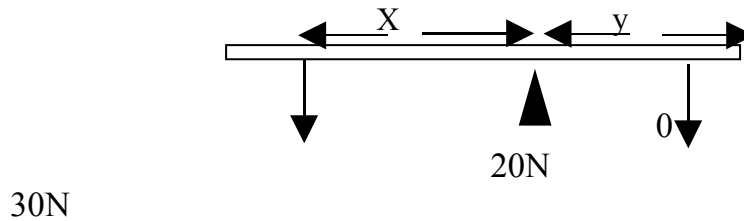


Fig. 1.7

### Solution

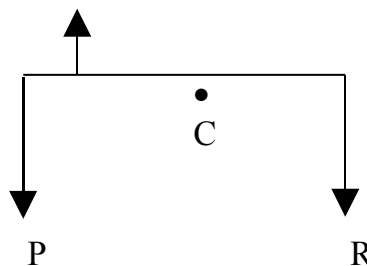
Moment of 20N force =  $20 \times X = 20x \text{ Nm}$

Moment of 30N force =  $30 \times y = 30y \text{ Nm}$

### 3.4.1 Principles of Moments

The principle of moments states that if a body is in equilibrium under the action of a number of coplanar forces, then the sum of the clockwise moment about any point on the body is equal to the sum of the anticlockwise moment about same point.

### 3.4.2 Clockwise and Anticlockwise Moments



In Fig 1.8: The forces P and R do to turn the body in anticlockwise direction.

In Fig. 1.8 the force P tends to turn the body in anticlockwise direction. Its moments are called anticlockwise moment. On the other hand the forces  $\infty$  and R tend to turn the body in clockwise direction. Their moments about C are thus clockwise moments. If clockwise moments are taken to be positive, and then anticlockwise moments must be taken as negative and vice versa.

Its moment is called anticlockwise moment. On the other hand the force  $\alpha$  and R tends to turn the body in a clockwise direction. Their moments about C are thus clockwise moments.



### 3.4.3 Couples

#### Definition

A couple is a pair of equal but oppositely directed and parallel forces which are not acting in a straight line.

#### Characteristics of a couple

- (i) The couple has a zero resultant force but has a turning effect on the body on which it acts.
- (ii) A couple can only cause a body on which it acts to rotate but not to move linearly.
- (iii) The moment of a couple is the product of one of the forces and the perpendicular distance between the lines of action of the two forces.
- (iv) The moment of a couple is also called a “torque”.
- (v) The distance between the two equal parallel forces of the couple is called the “arm of the couple”.
- (vi) A couple can be cancelled or neutralized by applying an equally similar couple of opposite moment to the body experiencing the couple.

#### Examples (Application) of couples

Uses, application or examples of (the effect of) couples in everyday life include:

- (i) Turning a tap on or off. It is easier to turn a tap on or off by applying two equal and opposite parallel forces (couple) rather than by applying a single force.
- (ii) It is easier to turn a steering wheel of a vehicle by applying a couple with our two hands instead of applying a single force with one arms.
- (iii) Loosening a bolt. It is easier to loosen or tighten a bolt by applying a couple at both ends of the bolt using two spanners.

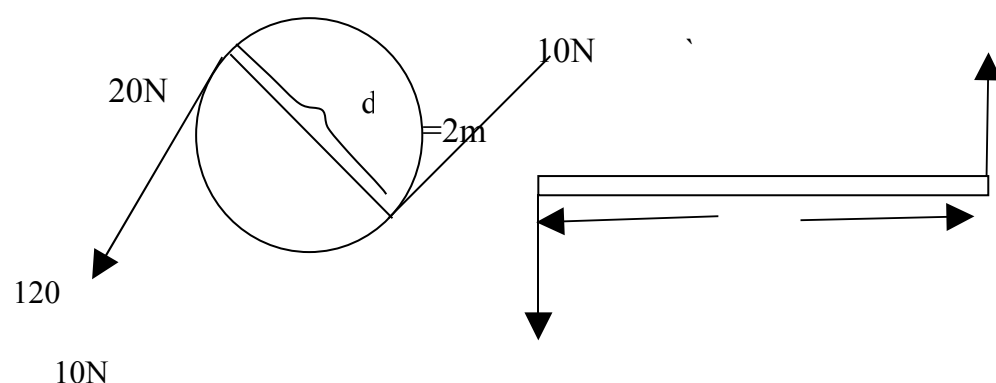






Fig.1.9: Moments of a couple

In Fig. 1.9: Above, the moments of the couples are:

- (i) In Fig. 1.9 (a) moments =  $10^{\text{N}} \times 2\text{m} = 20\text{Nm}$

In Fig. 1.9 (b) moments =  $20^{\text{N}} \times 1.5\text{m} = 30\text{Nm}$

### 3.5 Conditions of Equilibrium

#### 3.5.1 Conditions of Equilibrium under the Action of Parallel Coplanar Forces

Coplanar forces are forces that lie in the same plane. Parallel forces are forces whose lines of action are all parallel to each other.

Consider a body which is under the action of several forces which are coplanar and parallel. For the body to be in equilibrium (i.e. it does not move or rotate), then the following two conditions must be satisfied:

- (i) **Forces:** The algebraic sum of the forces in any given direction must be zero. This means the sum of forces acting in one direction must be equal to the sum of forces working in the opposite direction.
- (ii) **Moments:** the algebraic sum of the moments of all forces about any point on the body must be zero, or the total clockwise moments of the forces about any point on the body must be equal to the total anticlockwise moments of the forces about the same point.

#### Worked examples

1. Fig. 1.10 shows a beam balanced at P under the action of the forces shown. Calculate the value of the weight W.



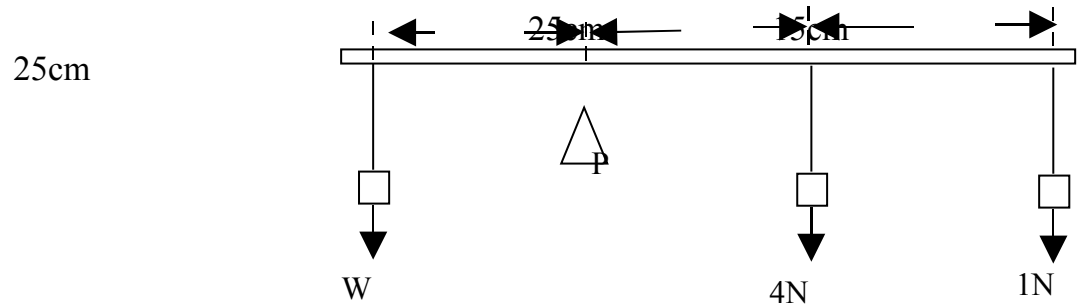


Fig. 1.10: A beam balanced at P

**Solution**

For equilibrium,

Anticlockwise moments about P = clockwise moment about P

$$\therefore (W \times 25) = (4 \times 15) + (1 \times 40)$$

$$\therefore (W \times 25) = (60 + 40)$$

$$\therefore W = \frac{1}{25} = \frac{100}{25} = 4\text{N}$$

2. In Fig.1.11 - a beam 1m long is balanced at its midpoint. Calculate the magnitude of the force P required for the beam to be in equilibrium

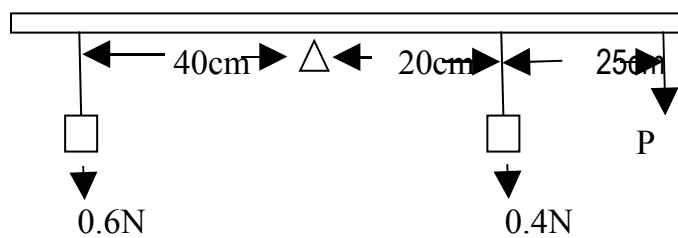


Fig.1.11: A 1m- long beam balanced at its mid point

**Solution**

For equilibrium,



Clockwise moments about O = anticlockwise moments about O

$$\begin{aligned}\therefore P \times 0.35 + 0.4 \times 0.2 &= 0.6 \times 0.4 \\ \therefore P &= \frac{(0.6 \times 0.4) - (0.4 \times 0.2)}{0.35} \\ P &= 0.5\text{N}\end{aligned}$$

Measurement of weight of a meter rule using principle of moments

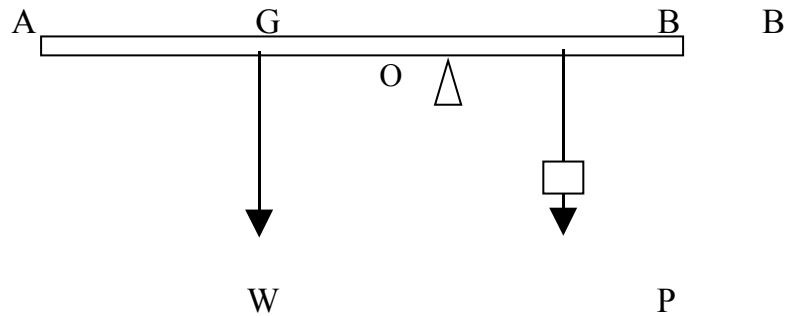


Fig. 1.12: Determining weight of a metre rule

Find the rough position of the centre of gravity (G) of the ruler by balancing it on the knife edge O. Suspend the ruler at a whole number O (e.g. 70cm) near one end. Hang a suitable weight P on the opposite side of O and adjust the position until the ruler balances horizontally. By taking moments about O, we have that,

$$\begin{aligned}W \times GO &= P \times BO \\ \therefore W &= \frac{P \times BO}{GO}\end{aligned}$$

The experiment can be repeated using different values of P and obtaining different balance points. An average value of W is then calculated.

### 3.5.2 Conditions of Equilibrium under the Action of Non-Parallel Coplanar Forces

A set of non-parallel coplanar forces can act on a body to keep it in equilibrium. The non-parallel forces can easily be resolved into horizontal and vertical components, giving rise to two parallel forces at right angles to each other. The problem can then be treated as in section 3.1.5(a) with the method of that section applied separately to the horizontal and vertical components of the forces. The two conditions of equilibrium for a body under such non-parallel forces would then be:

#### (i) Forces



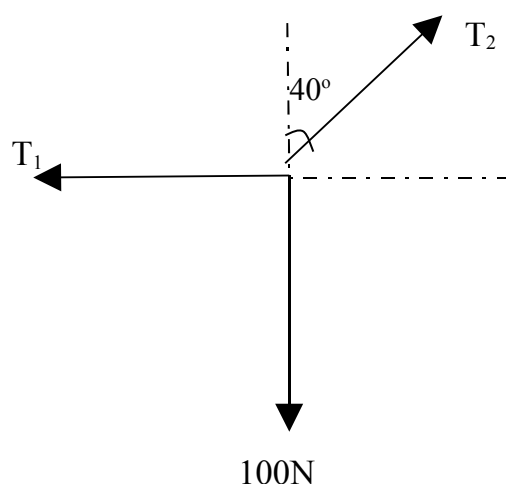
The vector sum of all the forces acting on the body must be zero. This means that the algebraic sum of the forces or components of the forces acting on the body on any direction must be zero, i.e.  $\Sigma F_x = 0$  or any algebraic sum of the horizontal must be zero and  $\Sigma F_y = 0$  or the algebraic sum of the vertical components must be zero.

## (ii) Moments

The algebraic sum of the moments of all the forces about any point on the plane or axis perpendicular to the plane of the forces must be zero.

### Worked examples

A force of 100N is in equilibrium by two ropes as shown in Fig.1.13. If one rope pulls the body in a horizontal direction and the other in a direction of  $40^\circ$  with the vertical, calculate the tension in each rope.



### Solution

#### Method 1: Resolving into vertical and horizontal components

Let  $T_1$ ,  $T_2$  be the tensions in the rope. For equilibrium sum of horizontal components are equal

$$\begin{aligned} \therefore T_2 \cos (90-40)^\circ - T_1 &= 0 \\ \therefore T_2 \sin 40^\circ - T_1 &= 0 \\ \therefore T_1 &= T_2 \sin 40^\circ \quad \text{————— (1)} \end{aligned}$$

Also sum of vertical components are equal for equilibrium,

$$\therefore T_2 \cos 40^\circ - 100 = 0 \quad \text{————— (2)}$$



$$\therefore T_2 = \frac{100}{\cos 40} = 130.54\text{N}$$

$$\text{From (i), } T_1 = T_2 \sin 40^\circ = 130.54 \times \sin 40^\circ \\ = 83.91\text{N}$$

### Method 2: Using triangle of forces

Sketch the triangle of forces for the system (not to scale)

From the Sin rule,

$$\frac{100}{\sin 50} = \frac{T_1}{\sin 40} = \frac{T_2}{\sin 90}$$

$$\therefore T_1 = \frac{100 \sin 40}{\sin 50} = 83.91\text{N}$$

$$T_2 = \frac{100 \sin 90}{\sin 50} = 130.54\text{N}$$

(You should also solve the problem by graphical method and compare the values of tension obtained).

### 3.5.3 Equilibrium under the Action of Three Non-Parallel Forces (Special Case)

For such a body to be in equilibrium, the following conditions must be satisfied:

- (i) The three forces must lie in a plane (i.e. they must be coplanar forces).
- (ii) The line of action of the forces must intersect in a common point.
- (iii) The vectors representing the three forces can be arranged to form a closed triangle with side respectively parallel to the direction and proportional in length to the magnitude of the forces.

## 3.6 Centre of Gravity

### Definitions

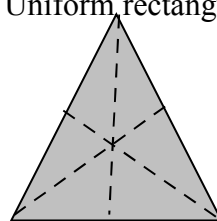
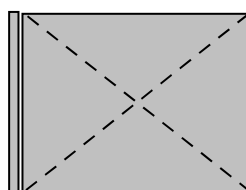
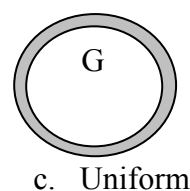
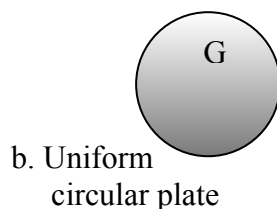
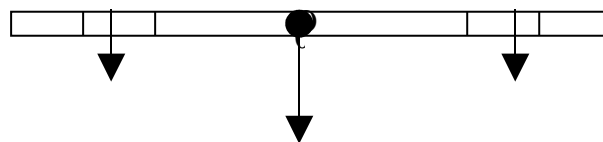


- (i) The centre of gravity (C.G) of a body is defined as the point through which the line of action of the weight of the body always passes irrespective of the point at which the entire weight of the body appears to be concentrated.
- (ii) The centre of mass of a body is defined as the point at which the total mass of the body appears to be concentrated.

**Note:** For small objects the centre of gravity (C.G) and the centre of mass coincide.

### 3.6.1 Position of C.G for some Regular Uniform Bodies

- (i) The C.G of a uniform rod is at the midpoint (G) of the rod. (Fig 1.14)
- (ii) The C.G of a uniform circular plate is at its centre (Fig.1.14 (b))
- (iii) The C.G. of a uniform circular ring is at its centre (Fig. 1.14( c ) )
- (iv) The C.G. of a uniform square or rectangle sheet is at the point of intersection of its diagonal (Fig. 1.14(d)).
- (v) The C.G. of a uniform triangle plate is at the intersection of the Medians (Fig. 1.14 (e))





- a. Uniform rod

uniform triangular plate

Describe an experiment to locate the C.G. of a lamina of any shape  
Fig. 1.14 C.G. of objects

### 3.6.2 An Experiment to Locate the Centre of Gravity of a Lamina to locate the C.G. of a Lamina

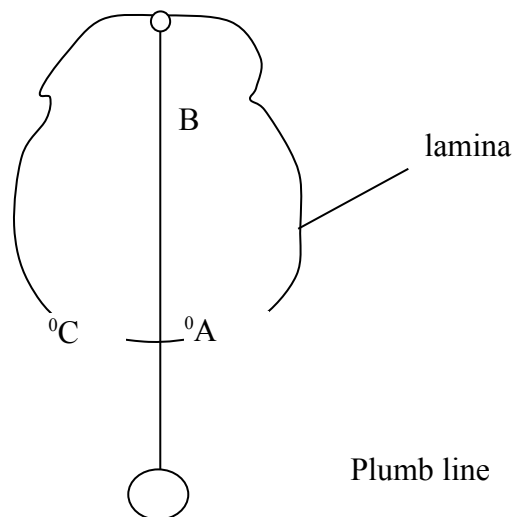


Fig 1.15: Locating C.G. of a lamina

You will make use of a plumb line (i.e. a thread with a small heavy weight tied to one end). Make holes near the edge of the lamina at about three different locations A, B, C, as shown in fig. 1.15 Hang the lamina



from a smooth horizontal support at A so that it can swing freely. Suspend the plumb line from the same point A on the lamina Repeat the procedure by suspending the lamina at B and C. if you have done the experiment carefully you will observe that the three lines intersect at a point which is the C.G. of the lamina. This is because the lamina will always come to rest with its C.G. vertically below the point of suspension when it is free to move.

### 3.7 Types of Stability of Equilibrium

There are three types of stability of equilibrium which a body at rest may experience:

- (a) Stable equilibrium
- (b) Unstable equilibrium
- (c) Neutral equilibrium.

#### 3.7.1 Stable Equilibrium

##### Definition

A body is said to be in stable equilibrium if it tends to return to its original position when slightly displaced.

Examples of bodies in stable equilibrium

- (i) A rectangular block resting in a horizontal surface.
- (ii) A cone resting on its base
- (iii) A racing car with wide base and low C.G.

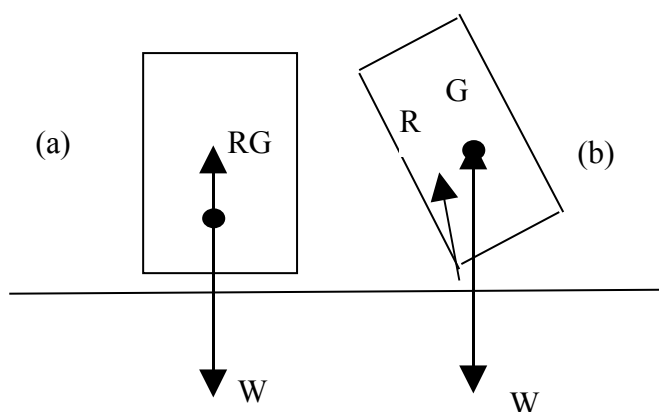


Fig. 1.16: Stability of rectangular block

Consider a rectangular block resting on a horizontal surface (Fig 1.16). In Fig.1.16 (a) the weight of the block  $W$  and the reaction  $R$  of the surface of the table are equal but opposite in direction and the same line



of action. In Fig. (b) the block has been given a slight tilt. The line of action of  $W$  still falls within the base of the block.  $W$  and  $R$  are still equal and opposite but act along different lines as shown. The two forces therefore constitute a couple which acts in a clockwise sense restore the block to its original position. Hence the block is in stable equilibrium.

### Requirements for Stable Equilibrium

Generally,

- (i) a low C. G; and
- (ii) a wide base

### 3.7.2 Unstable Equilibrium

#### Definition

A body is said to be in unstable equilibrium if it tends to move further away from its original position or topples over when slightly displaced.

#### Examples of bodies in unstable equilibrium

- (i) a cone resting on its apex
- (ii) an egg resting on its pointed end.
- (iii) A tight-rope walker (i.e. a person walking on a tight rope).

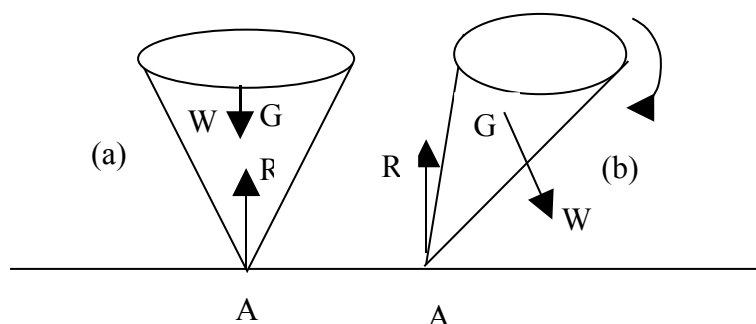


Fig. 1.17: Unstable equilibrium

Consider a cone resting on its apex A (Fig.1.17) A. In Fig. 1.17 (a)  $W$  and  $R$  are equal and opposite and have the same line of action. As the cone is slightly displaced (Fig 1.17(b)), the line of action of  $W$  quickly falls out of the apex.  $W$  and  $R$  now constitute a couple with clockwise moment whose effect causes the cone to topple over or move further away from its original position. Hence the cone is in unstable equilibrium.

### 3.7.3 Neutral Equilibrium

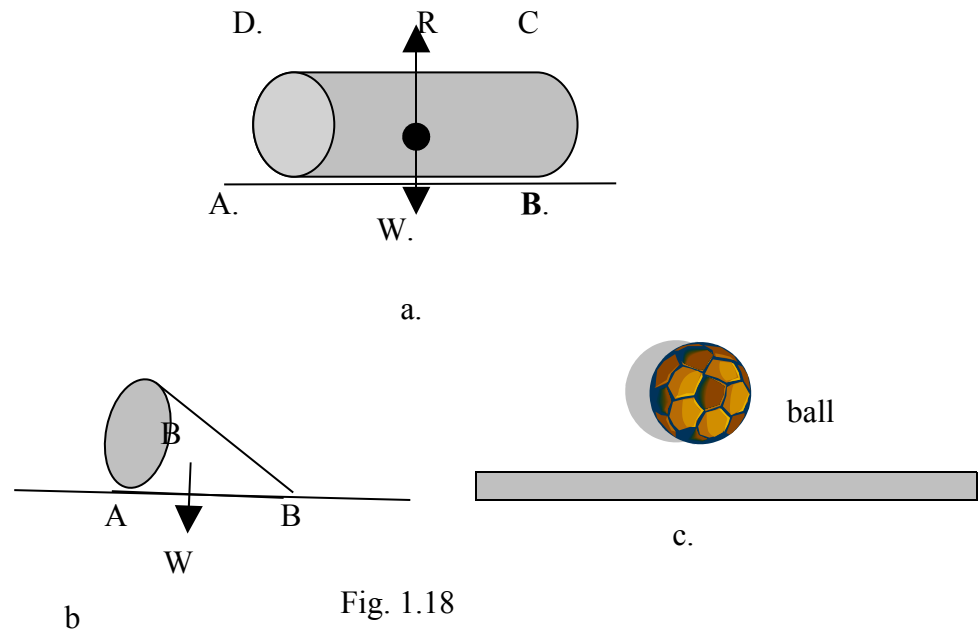


### Definition

A body is said to be in neutral equilibrium if it tends to come to rest in its new position when slightly displaced.

### Examples:

- (i) A cone, or cylinder or an egg resting on its side.
- (ii) A ball or sphere on a smooth horizontal table (or surface).



Consider the cylinder or cone resting on its side or the ball resting on the ground (Fig.1.18). When the ball is slightly displaced, the weight of the ball has no moment about the point of contact with the ground. The position of the C.G. and the height of the C.G. above the ground remain unchanged. The bodies are therefore in neutral equilibrium.

### 3.7.4 Effect of Height of C.G. on Stability of Equilibrium

As already pointed out, the C.G. of a body above the ground level and the width of the base of the object affect the stability of the object.

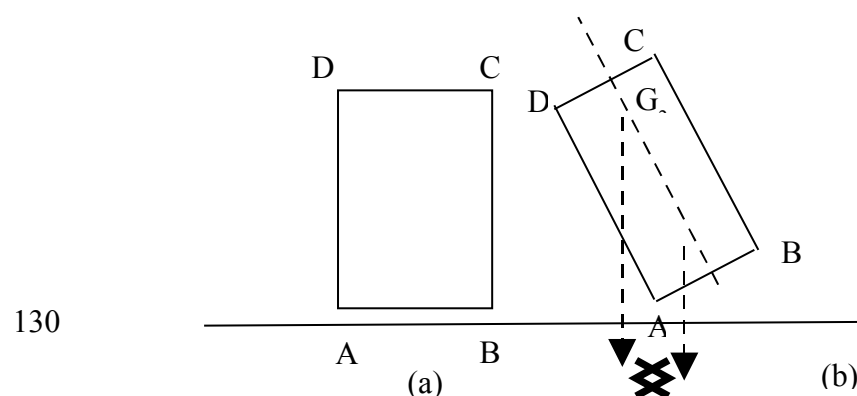




Fig.1.19: Height of C.G. and stability of an object  
y

Consider a body in the form of a rectangle ABCD resting on a table (Fig.1.19 (a)). Suppose the body is slightly lifted as in Fig.1.19 (b) if the body has a low C.G. at  $G_1$ , the body will not topple over because the line of action of the weight of the body passes within the base AB. On other hand, if the body has a high C.G. at  $G_2$ , a different situation will result if the body is slightly titled about A. the line of action of the weight of the body ( $W$ ) falls outside its base of support. Therefore a wide base and low C.G. enhance the stability of an object.

### 3.7.5 Application of the Effect of Height of C.G. on Stability of Object

- (i) Racing cars are built low and with a wide wheel base to prevent them from overturning while negotiating bends at highspeed
- (ii) Cars engine are always located at the bottom of the car and not on the roof.
- (iii) A tight rope walker often carries a weighted pole or umbrella
- (iv) Lorries that carry heavy loads usually have the loads stacked on the floor but not near the roof or on top of the roof.
- (v) Certain toys which are difficult to topple are designed in such a way that they have low C.G. and large base area.

## 3.8 Equilibrium of Bodies in Liquids

### 3.8.1 Concept of Upthrust

Common experience shows that when a box is totally or partially immersed in a liquid such as water, the body appears to be lighter. Such an object appears heavier when completely out of water. This suggested there s an upward force exerted by a liquid on an object completely or partially immersed in it. As a result of this upward force there is thus an apparent loss in weight of the immersed object. This upward force is called the ‘upthrust’ of the liquid on the object.

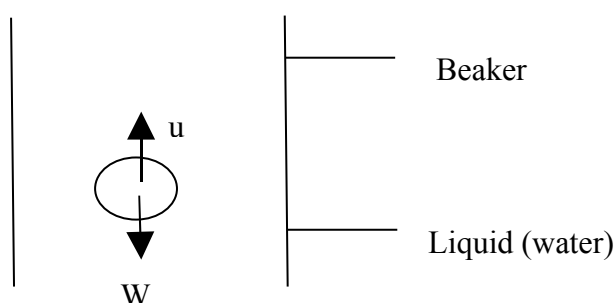




Fig. 1.20: Upthrust of liquids on an object

Fig 1.20 shows an object completely immersed in water. The weight of object in air is  $W$ , while weight in water is  $W_2$ . The upthrust of the water  $U$  is given by  $U = W_1 - W_2$ . Hence the apparent weight in water  $W_2$  is given by  $W_2 = W - U$ . Hence  $W_2$  is less than  $W_1$ . The value of the upthrust depends on the volume of the object immersed and the density of the liquid in which it is immersed. The magnitude of the upthrust is given by Archimedes principle.

### 3.8.2 Archimedes Principle (statement)

Archimedes principle states that when a body is totally or partially immersed in a fluid (liquid or gas), it experiences an upthrust which is equal to the weight of fluid displaced.

#### Formula for upthrust

An object completely immersed in a liquid displaces a volume of liquid equal to its own volume. The upthrust is the weight of this displaced volume of liquid and is given by = volume of object  $\times$  density of liquid  $\times g$  where  $g$  is acceleration due to gravity. If upthrust is  $U$ , volume of object  $V$  and density of liquid  $\rho$ , then

$$U = V\rho g$$

where,  $V$  = volume of object submerged,  $\rho$  = density of the fluid and  $g$  = acceleration of free fall.

If the object is partially immersed in the liquid (e.g. only  $\frac{2}{3}$  of the object is immersed in the liquid), then

$$U = \frac{2}{3}V \times \rho \times g$$

$$\therefore U = \frac{2V\rho g}{3}$$

### 3.8.3 Verification of Archimedes Principle



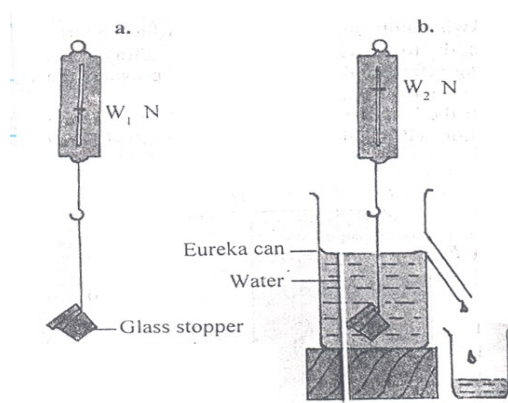


Fig. 1.21: Verification of Archimedes principle

Fig. 1.21 shows the apparatus for an experiment to verify Archimedes principles. A solid object (e.g. a piece of stone) is freely suspended from a spring balance and its weight in air ( $W_1$ ) read off. The object is then gently lowered into the water in the Eureka can filled to the level of the spout as shown.

The weight of the object in water ( $W_2$ ) is read from the spring balance. Water displaced by the object is collected in the beaker which has been previously weighed. The beaker with water is reweighed to obtain the weight of water displaced. The readings are recorded as follows:

- Weight of object in air =  $W_1$
- Weight of object in water =  $W_2$
- Weight of empty beaker =  $W_3$
- Weight of beaker + water displaced =  $W_4$
- Apparent loss of weight of object =  $W_1 - W_2$
- Weight of water displaced =  $W_4 - W_3$

It will be found by comparing the values that

$$W_1 - W_2 = W_4 - W_3$$

Thus the apparent loss in weight of object (upthrust) is equal to weight of water displaced showing that an Archimedes principle is true. The principle will be similarly confirmed if the experiment is repeated with the object only partially immersed in the water.

### Worked example

A spring balance reads 12.0N when a metal block is suspended from it and 10.0N when the block is completely immersed in water. Calculate the upthrust on the block, the mass of water displaced, and the volume of the block.



**Solution**

$$(i) \quad \text{upthrust on block} = \text{loss in weight in water} \\ = 12.0\text{N} - 10.0\text{N} = 2.0\text{N}$$

$$(ii) \quad \text{weight of water displaced} = \text{upthrust} = 2.0\text{N}$$

$$\therefore \quad \text{mass of water displaced} = \frac{\text{weight}}{g} \\ = \frac{2.0}{10} \text{ kg} = 0.2 \text{ kg} \\ = 200 \text{ g.}$$

$$(iii) \quad \text{volume of block} = \text{volume of water displaced}$$

$$= \frac{\text{mass of water displaced}}{\text{density of water}} \\ = \frac{0.2 \text{ kg}}{1000 \text{ kg/m}^3} = 2 \times 10^{-4} \text{ m}^3 \\ = 200 \text{ cm}^3$$

**3.8.4 Principle of Floatation**

The Principle of floatation states that an object will float in a fluid (liquid or gas) when the upthrust exerted by the fluid in which it floats is equal to the weight of the object.

From the principles of floatation it can be said that:

- (1) A floating body displaces its own weight of the fluid in which it floats.
- (2) Any floating body is in equilibrium under two forces, namely its own weight acting downwards and the upthrust of the liquid acting upwards.
- (3) A body will float in a liquid if the density of the object is less than the density of the liquid.
- (4) A body denser than a liquid can still float in the liquid, if the body is shaped in such a way that its volume can displace its own weight of the liquid. For this reason metal ship floats in water because the large volume of the ship displaces a large volume of liquid whose weight counter balances the huge weight of the ship.



**Worked example**

A cube of wood of side 10 cm floats in water with 4.5 of its depth below the surface, and with its sides vertical. What is the density of the wood?

**Solution**

$$\begin{aligned}\text{Volume of wood submerged} &= (10 \times 10 \times 4.5) \text{ cm}^3 \\ &= 450 \text{ cm}^3\end{aligned}$$

$$\therefore \text{volume of water displaced} = 450 \text{ cm}^3$$

$$\therefore \text{mass of water displaced} = 450 \text{ cm}^3$$

$$(\text{Density of water} = 1 \text{ gcm}^{-3})$$

Since a floating object displaces its own weight of fluid, then

$$\begin{aligned}\text{Mass of wood} &= 450 \text{ g} \\ \text{Volume of wood} &= 1000 \text{ cm}^3 \\ \therefore \text{density of wood} &= \frac{450 \text{ g}}{1000 \text{ cm}^3} \\ &= 0.45 \text{ g cm}^{-3}\end{aligned}$$

**3.9 Density and Relative Density**

Equal volumes of different substances have different masses or weights. This is due to differences in the property of the substances known as density.

**Definition**

The density of a substance is defined as the mass per unit volume of the substance.

**S.I** unit of density is kilogram per metre cubed ( $\text{kg/m}^3$  or  $\text{kg m}^{-3}$ ) and its symbol is  $\rho$  ( rho).

**Definition**

The relative density (R.d) of a substance is defined as:



$$\text{R.d.} = \frac{\text{mass (or weight) of a substance}}{\text{Mass (or weight) of equal volume of water}}$$

Alternative definition of relative density is:

$$\text{R.d.} = \frac{\text{density of substance}}{\text{Density of water}}$$

Relative density has no unit since from the two definitions above, mass or weights are compared.

### 3.9.1 Measurement of Density and Relative density

- 1(a) The density of a regular solid (e.g. wooden block) can easily be obtained by measuring its mass using a chemical balance. The volume is obtained by measuring its dimensions.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

- (a) The density of an irregular solid (e.g. a piece of stone) can be obtained by first measuring the mass using a chemical balance. The volume is obtained by immersing the solid completely in a measuring cylinder containing water. The difference in the levels of water before and after the immersion of the solid gives the volume of the solid.
- (b) Relative density of a solid which floats in water (e.g. cork) is best measured by means of Archimedes principles as follows:

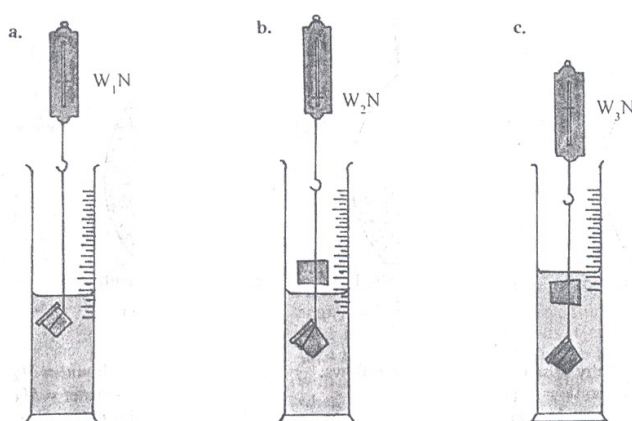


Fig. 1.22: Relative density of a solid which floats

The cork cannot sink in water of its own accord, therefore a “sinker” is used to make it sink completely in water when immersed. The sinker (e.g. a glass stopper or brass weight) is first suspended from the hook of



a spring balance and its weight when completely immersed in water is noted from the spring balance (Fig 1.22(a)). The cork is then tied to the thread carrying the sinker in such a way that the cork does not touch the water. The weight of sinker in water and cork in the air is obtained (Fig. 1.22(b)). The cork is then tied near the sinker and the weight a sinker and cork, both completely immersed in water is noted. The relative density of cork is calculated as follows:

Weight of sinker in water  $= W_1$

Weight of sinker in water and cork in air  $= W_2$

Weight of cork in air  $= W_2 - W_1$

Upthrust on cork = Apparent loss in weight of cork

$? = W_3 - (W_2 - W_1)$

Relative density of cork =  $\frac{\text{Weight of cork in air}}{\text{Weight of equal volume of water}}$

$\frac{\text{Weight of cork in air}}{\text{Upthrust on cork}}$

$$= \frac{W_2 - W_1}{W_2 - W_3}$$

- 3 (a) Relative density of solid in form of particles or powder (e.g. sand) is best measured using a relative density bottle and a chemical balance. This should be read up from physics texts by students.
- (b) Relative density of solid particles or powder which dissolves in water (e.g. table salt) is similarly measured using a relative density bottle, a chemical balance and a liquid in which the solid (powder) does not dissolve but whose density is known. This should also be read up from physics texts by students.
- 4 (a) Relative density of a liquid using relative density bottle.

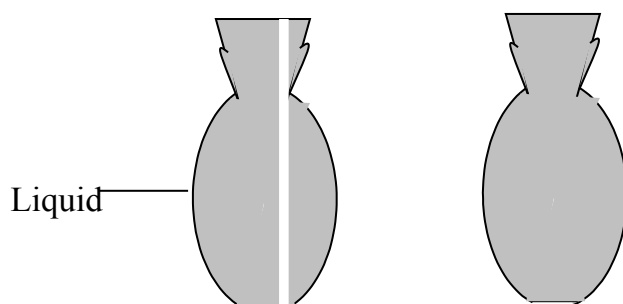


Fig. 1.23: Relative density of liquid

The relative density bottle is cleaned, dried and weighed empty. The bottle is filled with the liquid whose relative density is required, cleaned and reweighed. The bottle is often emptied of the liquid, cleaned, dried



and reweighed when filled with water. The relative density of the liquid is calculated as follows:

$$\begin{aligned}
 \text{Mass of empty bottle} &= M_1 \\
 \text{Mass of bottle + liquid} &= M_2 \\
 \text{Mass of bottle + water} &= M_3 \\
 \text{Mass of liquid} &= M_2 - M_1 \\
 \text{Mass of equal volume of water} &= M_3 - M_1
 \end{aligned}$$

$$\begin{aligned}
 \text{Relative density of liquid} &= \frac{\text{mass of liquid}}{\text{Mass of equal volume of water}} \\
 &= \frac{M_2 - M_1}{M_3 - M_1}
 \end{aligned}$$

Relative density of a liquid can also be measured easily using Archimedes principle. In this, case a solid (e.g. glass stopper) is used. The solid is suspended from the hook of a spring balance and weighed in air. It is then weighted when completely immersed in a liquid whose relative density is required. The solid is then cleaned, dried and reweighed when completely immersed in water. The relative density if the liquid is calculated as follow:

$$\begin{aligned}
 \text{Mass of objects in air} &= M_1 \\
 \text{Apparent mass of objects in liquid} &= M_2 \\
 \text{Apparent mass of object in water} &= M_3
 \end{aligned}$$

Upthrust in liquid = weight of liquid displaced by the objects

$$= M_1 - M_3$$

Upthrust in the water = weight of water displaced by the object

$$= M_1 - M_3$$

$$\begin{aligned}
 \text{Relative density of liquid} &= \frac{\text{upthrust in liquid}}{\text{Upthrust in water}} \\
 &= \frac{M_1 - M_2}{M_1 - M_3}
 \end{aligned}$$

### Worked examples



1. A body weighs 0.52N in air. Totally immersed in water it weighs only 0.32N while its weight when immersed in another liquid is 0.36N. The density of water is  $1000 \text{ kg/m}^3$ . What is the density of the other liquid?

**Solution**

$$\begin{aligned}\text{Upthrust in water} &= \text{loss of weight in water} \\ &= 0.52\text{N} - 0.32\text{N} = 0.20\text{N}\end{aligned}$$

$$\begin{aligned}\text{Mass of water displaced} &= \frac{\text{upthrust}}{g} \\ &= \frac{0.20\text{N}}{10\text{N/kg}} = 0.02 \text{ kg}\end{aligned}$$

$$\begin{aligned}\text{Volume of water displaced} &= \frac{\text{Mass}}{\text{Density}} \\ &= \frac{0.02\text{ kg}}{1000 \text{ kg/m}^3} = 0.02 \times 10^{-3} \text{ m}^3\end{aligned}$$

$$\begin{aligned}\therefore \text{Volume of water displaced} &= \frac{\text{Mass}}{\text{Density}} \\ &= \frac{0.02 \text{ kg}}{1000 \text{ kg/m}^3} = 0.02 \times 10^{-3} \text{ m}^3\end{aligned}$$

$$\begin{aligned}\text{Upthrust in liquid} &= \text{loss of weight in liquid} \\ &= 0.52\text{N} - 0.36\text{N} = 0.16\text{N}\end{aligned}$$

$$\therefore \text{Mass of liquid displaced} = \frac{0.16\text{N}}{10\text{N/kg}} = 0.016 \text{ kg}$$

$$\begin{aligned}\text{Volume of liquid displaced} &= \text{volume of water displaced} \\ &= 0.02 \times 10^{-3} \text{ m}^3\end{aligned}$$

$$\begin{aligned}\therefore \text{Density of liquid} &= \frac{\text{mass of liquid displaced}}{\text{Volume of liquid displaced}} \\ &= \frac{0.016 \text{ kg}}{0.02 \times 10^{-3} \text{ m}^3} \\ &= 0.8 \times 10^3 \text{ kg/m}^3 \\ &= 800 \text{ kg/m}^3\end{aligned}$$

2. An empty relative density bottle has a mass of 15.0 g. When completely filled with water its mass is 39.0 g. What will be its mass if completely filled with acid of relative density 1.20?



**Solution**

$$\begin{aligned}\text{Mass of water which fills bottle} &= 39\text{g} - 15\text{ g} \\ &= 24\text{ g}\end{aligned}$$

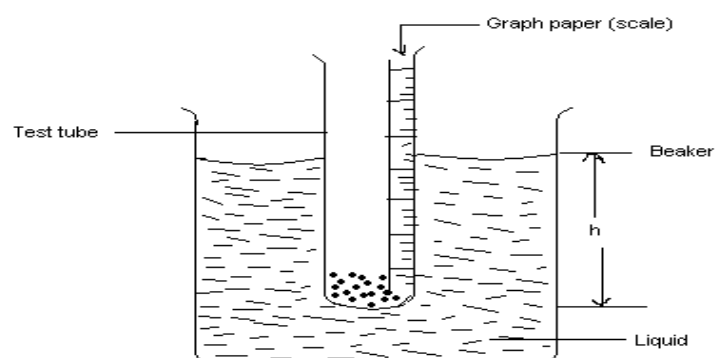
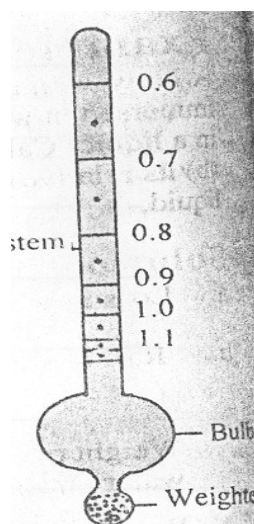
$$\begin{aligned}\text{Volume of water which fills bottle} &= \frac{\text{mass}}{\text{density}} \\ (\text{Density of water} &= 1\text{gm /cm}^3) \\ &= \frac{24\text{g}}{1\text{g/cm}^3} = 24\text{ cm}^3\end{aligned}$$

$$\text{Relative density of liquid} = \frac{\text{mass of liquid}}{\text{Mass of equal volume of water}}$$

$$\therefore 1.20 = \frac{\text{mass of liquid}}{24\text{ g}}$$

$$\therefore \text{mass of liquid which fills bottle} = 1.20 \times 24\text{ g} = 28.8\text{ g}$$

$$\begin{aligned}\therefore \text{mass of bottle completely} \\ \text{filled with liquid (acid)} &= 28.8 + 15.0\text{ g} \\ &= 48.3\text{ g}\end{aligned}$$

**3.9.2 The Hydrometer**

Principle of the hydrometer



Fig. 1.24: Simple hydrometer

The hydrometer is an instrument used to measure the relative density of liquids.

### The Principle of the Hydrometer

Consider a test tube weighted by means of lead shots so that it can float upright in a beaker of water as shown in Fig. 1.24. If the tube is floated in different liquids it will be noted that the length of the tube submerged in liquid is different for different liquids. This is due to the differences in the relative densities of the liquids.

Suppose the weight of the test tube (with the lead shots) is  $W$ . Each time the tube floats in a liquid, the weight of liquid displaced is  $W$  Newton's

$$\therefore \text{Volume of liquid displaced} = \frac{\text{mass}}{\text{Density}} = \frac{W}{\rho g}$$

where  $\rho$  is the density of the liquid and  $g$  is the acceleration due to gravity. If the cross-sectional area of the tube is  $A$  and the length of the tube submerged is  $l$ , then:

$$\text{Volume of liquid} = A \times l$$

$$\therefore A l = \frac{W}{\rho g}$$

$$\therefore L = \frac{W}{A \rho g}$$

$$\therefore L \propto \frac{1}{\rho}$$

Since  $W$ ,  $A$  and  $g$  are constants.

The above relationship implies that the length of the test-tube submerged in the liquid is inversely proportional to the density of the



liquid, i.e. the higher the density of liquid, the shorter the length of the tube immersed and vice versa.

The working of the hydrometer is used on this principle, that is, the less the density of a liquid, the more the hydrometer sinks in it.

### 3.9.3 The Practical Hydrometer

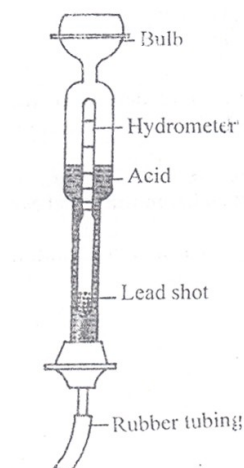


Fig. 1.25: A battery hydrometer

Fig. 1.25 shows a practical hydrometer which is commonly used in daily life to measure the relative densities of liquids. The real hydrometer is housed (as shown) in an endorser with a glass chamber into which the liquid (whose relative density is required) can be drawn so that the hydrometer can float in it and the value of the relative density can be read directly.

Uses of the practical hydrometer

- (i) Checking the relative density of batteries
- (ii) Testing the purity of liquids whose densities are know
- (iii) Testing the quality of milk

### SELF ASSESSMENT EXERCISE

1. Explain what you understand by the term “force”. Define the terms “resultant” and “equilibrant” with respect to the forces



acting at a point and explain the differences between the two terms. A force of 10N acts in the direction E  $60^\circ$  N while another force of 15N in the easterly direction if both forces act at a point, find (i) the resultant, and (ii) the equilibrant of the two forces.

2. State the law of triangle of forces and briefly describe an experiment to illustrate it. An object of mass 700 g hangs from the end of a long string attached to a support. A horizontal string attached to the object pulls it in such a way that the first string makes an angle  $30^\circ$  with the vertical. Find the tensions in the two strings.
- 3 Explain what is meant by the “moment of a force”. State the principle of moments and give two examples of the practical application of the principle in everyday life.

A uniform plank AB has length 5 m and mass 80 kg. It is supported horizontal on two points C and D where C is 1m from the end A and D is 1 m from the end B of the plank. Find the magnitude and direction of the minimum force which would be applied at; A (i) to lift the plank just off the pivot at C, (ii) to lift the plank just clear of the pivot at D

- 4 State the Principle of Archimedes and explain what is meant by “upthrust” exerted by a fluid on an object immersed in it.

How is a submarine made (i) to sink to a point below the surface of water (ii) to rise to the surface of water from beneath it?

A floating crane has mass 20600 kg and floats in sea water of density  $1.03 \text{ g/cm}^3$ . If the base of the crane is a rectangular block 4m square and 2.5m deep, what is the maximum mass which can be lifted by the crane when the top of the base is just awash with water.

## 4.0 CONCLUSION

In this unit you learned about cause of motion – force. You also learned resultant force and forces in equilibrium. Furthermore, you learned about laws governing bodies in equilibrium. In addition you learned Archimedes principle, the principle of floating and relative density. Finally, you learned the applications of the laws and principles in solving numerical problem.

## 5.0 SUMMARY



1. Motion is caused by a “force” which is essentially a “push” or “pull”, acting on the moving object. There are four major groups or fields of force, namely; gravitational, electric or electrostatic, magnetic and nuclear force.
2. The “resultant force” is that simple force which, acting alone will have the same effect in magnitude and direction as two or more forces acting together.
3. A body is said to be in equilibrium if
  - (a) The body as a whole either remains at rest or moves in a straight line with constant speed, or
  - (b) The body is rotating with a constant angular velocity or is not rotating at all.
4. The “equilibrant force” is that single force which will balance all the other forces taken together. The equilibrant force is equal in magnitude but opposite in direction to the resultant force.
5. The "law of triangle of forces” states that when three forces are in equilibrium, they can be represented in magnitude and direction by the three sides of a triangle taken in order.
6. The “moments of a force” about a point or axis is the turning effect of the force about that point or axis. It is equal to the product of the force and the perpendicular distance of the line of action of the points from the point.
7. For a body in equilibrium, the sum of the clockwise moments at a point equals the sum of the anticlockwise moments about the same points. This is also called the “principle of moments”.
8. Two equal but opposite parallel forces which are not acting in a line is called a “couple”
9. The “centre of gravity” of a body is the point through which the line of action of the weight of the body always acts, or it is the point at which the entire weight of the body appears to be concentrated.
10. A body is said to be:
  - (i) In “stable equilibrium” if it tends to return to its original position when slightly displaced.



- (ii) In “unstable equilibrium” if when slightly displaced, it tends to move further away from its original position.
  - (iii) In “neutral equilibrium” if when slightly displaced, it tends to come to rest in its new position
11. For adequate stability, a body requires (i) a wide base of support, and a low centre of gravity.
  12. “Archimedes principle” states that when a body is totally or partially immersed in a fluid, it experiences an upthrust which is equal to the weight of fluid displaced
  13. “The principle of flotation” states that an object floats in a fluid when the upthrust exerted upon it by the fluid is equal to the weight of the object.
  14. “Relative density (R.d)” of a substance is defined by:

$$\begin{aligned}
 \text{R.d} &= \frac{\text{mass of substance}}{\text{Mass of equal volume of water}} \\
 &= \frac{\text{density of substance}}{\text{density of water}} \\
 &= \frac{\text{upthrust in liquid}}{\text{upthrust in water}}
 \end{aligned}$$

Relative density has no units. It can be measured with a “hydrometer”.

## 6.0 TUTOR- MARKED ASSIGNMENT

1. What do you understand by the “resultant” of two or more forces acting at a point? Explain briefly how such a resultant may be obtained. Three forces of magnitude  $5\sqrt{2}\text{N}$ ,  $15\sqrt{2}\text{N}$  and  $20\sqrt{2}\text{N}$  act at a point in the west, north and east direction respectively. Another force of magnitude  $10\text{N}$  acts at the same point in a direction  $\text{S } 45^\circ\text{W}$ , that is  $45^\circ$  to the  $5\sqrt{2}\text{N}$  force. Draw a sketch of the arrangement and determine the result of the system in magnitude and direction.
2. Explain briefly the meaning of (i) moment of force, (ii) a couple. A uniform plank  $600\text{ cm}$  long rests on two supports A and B which are  $100\text{ cm}$  apart. Weights of  $100\text{N}$  and  $80\text{N}$  are hung from the ends of the beam near A and B. Calculate the reaction forces at the supports A and B.



3. State Archimedes principle and describe how you will use the principle to determine the relative density of a given liquid. An Object weighs 0.30N in air, and 0.25N when fully immersed in water. If the object weighs 0.27N in a liquid, calculate (i) its relative density (ii) the relative density of the liquid

## **7.0 REFERENCES/FURTHER READINGS**

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## **UNIT 2 FRICTION IN SOLIDS AND LIQUIDS**

### **CONTENTS**



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- 7.0 References/Further Readings

## 1.0. INTRODUCTION

In this unit you will learn about friction in solids and liquids. Whenever one moves or tends to move over another surface, frictional forces are called into play. Friction is unavoidable because no surface is perfectly smooth. When one object rubs against another, the roughness of their surfaces prevents them from sliding freely over another. Frictional forces in liquids (viscosity) explain why some liquids flow easily and others do not.

## 2.0. OBJECTIVES

By the end of this unit, you should be able to:



- identify a force resisting the motion between two surfaces in contact and moving relative to each other.
- define friction and viscosity
- state the advantages and disadvantages of friction.
- reduce friction in a given situation
- classify liquids according to their viscous properties.
- give at least two applications of viscosity.

### **3.0 MAIN CONTENT**

#### **3.1. Concept of Friction**

You are aware that when a solid is in contact with another, the roughness of their surfaces prevents them from sliding freely over one another. Such action is brought about by the effect of friction.

Frictional forces act tangential to the surface of separation between two bodies in contact. It does not appear unless there is relative motion or a force tending to produce motion, then it always appears to oppose the motion or the force tending to produce motion.

You have to overcome some frictional force before an object can move over another. The frictional force continues to act as long as one body moves over another. When you withdraw the force producing the motion, friction slows down the moving body until it eventually comes to rest.

- ❖ Friction is defined as a force which acts at the surface of separation between two objects in contacts and tends to oppose the motion of one over the other.

#### **3.2. Advantages and Disadvantages of Friction**

##### **3.2.1. Advantages of Friction**

We desire friction for the following reasons:

- i. Without friction, it would be impossible for us to walk or to stop walking after getting started. Friction protects us from slipping while walking.



- ii. Friction enables the automobile tyres to make a firm grip with the road thus aiding their movement. The threading pattern of vehicle tyres is designed to give the tyre a good grip on the road.
- iii. Friction enables the locomotive engine to pull the train on the rail.
- iv. Friction is utilized in fan belts used over wheels or pulleys in machinery. The belting that drives the wheels in machines depends on friction to make the wheels turn without slipping.
- v. Friction enables a screw or a nail to remain in place after being screwed into position. Such nails thus hold pieces of wood together because of the friction between the nails and the wood.
- vi. Friction is used in the grindstone to sharpen knives and chisels. The grindstone's rough surface wears away the metal surface to sharpen them.
- vii. Friction enables the brakes to stop the car and train

### **3.2.2. Disadvantages of Friction**

- i. Friction leads to wear and tear on the moving parts of machines.
- ii. Friction reduces the efficiency of machines because it causes loss of energy. Much useless work is therefore done by machine in overcoming friction which opposes their motion.
- iii. It causes heating of engines. Energy that must be used to overcome friction is converted into heat. This results in the bearing of machines getting hot when they are running. Sometimes, the energy is converted to sound.

### **3.3. Methods and Reasons for Reducing Friction**

Here we consider three major ways of reducing friction. You may have been using these methods in reducing friction in the past and you may also know other methods. These are:

- i. The use of lubricants like oil, grease, air and graphite. You can use lubricants to keep two metal surfaces from coming into direct contact. You can also do this by maintaining a layer of air or other gases between them.



- ii. The use of ball or roller bearing. Where a wheel is turning on an axle, the use of ball or roller bearings reduces friction. These are like wheels within wheels and are precisely shaped and have specially hardened surfaces. In roller bearings, the surfaces do not slide but roll over the others and rolling friction is much less than sliding friction.
- iii. The streamlining of body shapes of moving objects. You can reduce friction by shaping a body to the streamlines of the fluid through which it is moving. This reduces the frictional force or retarding force. You will notice that modern cars, ships and aero- planes are made in such shapes as to lesson the friction between them and the fluid or the medium through which they move.

We try to reduce friction in machines for two reasons

- i. it causes wear and tear
- ii. it uses up energy and reduces efficiency.

### **3.4. Types of Friction**

There are two types of frictions which we are going to define here. These are:

- i. Static or Limiting Friction  
This is the maximum force that must be overcome before a body can just start to move over another.
- ii. Kinetic or Dynamic Friction

This is the force that must be overcome so that a body can move with uniform speed over another body.

- Usually dynamic friction is slightly less than the static friction.

## **3.5 Laws of Friction and Coefficient of Static Friction**

### **3.5.1 Laws of Friction**

At this point you will learn the laws of friction. They are:



- i. Friction opposes the relative motion between two surfaces in contact. It acts in a direction opposite to that of the motion.
- ii. The force of friction increases to the same extent as the force which tends to start the motion.
- iii. Frictional force depends on the two surfaces in contact- the nature of the substance and the conditions of the surfaces such as a rough, smooth, polished, wet, dry and others.
- iv. It is independent of the area of the surfaces in contact.
- v. It varies directly with the normal force pressing the surfaces together. That is, it is proportional to the normal reaction R.

### 3.5.2 Coefficient of Friction

Results of earlier experiments show that the frictional force  $F$  is roughly proportional to the normal reaction  $R$  between the two surfaces in contact.

That is:

$$\text{Or } F \propto R \quad \text{or} \quad F = \mathcal{M} R \quad \text{-----} \quad (1)$$

The constant  $\mathcal{M}$  is known as the coefficient of friction.

$$\mathcal{M} = \frac{F}{R} \quad \text{-----} \quad (2)$$

Equation (2) defines

- The coefficient of friction ( $\mathcal{M}$ ) as the ratio

$$\mathcal{M} = \frac{\text{Frictioned force } F}{\text{Normal reaction between the two surfaces in contact } R}$$

You will note that:

- i. For static friction, the frictional force  $F$  required to determine  $\mathcal{M}$  in the above equation is the maximum force required to start the body moving and  $\mathcal{M}$  is called the coefficient of static or limiting friction.



- ii. For kinetic friction, the frictional force used is the force required to keep the body in uniform motion and  $\mathcal{M}$  in the above equation is known as coefficient of dynamic or kinetic friction.
- iii. The coefficient of static friction is greater than the coefficient of dynamic friction.
- iv. From experience you know that more force is required to start a body sliding along a surface than is required to keep it in motion once it is started.

### 3.6 Determination of the Coefficient of Static Friction Using:

#### 1. Horizontal Plane.

To determine the coefficient of static friction using a horizontal plane you will need the apparatus set up as shown in Fig. 2.1.

You will now follow these procedures.

- Weigh the block of wood B with a spring balance and record its weight ( $W_B$ ).
- Place the block of wood on the horizontal board and connect it to a cord adjusted in such a way that it pulls parallel to the surface of the board.
- Pass the cord over a pulley and attach it to a scale pan to which known weights can be added.
- Suspend known weights in the scale pan until the block is just about to move.
- Tap the block gently to prevent sticking each time a weight is added into the pan.
- Place a known weight on the block B and increase the weight on the scale pan until the block is just about to move again after gentle tapping.
- Repeat the experiment five more times by varying  $M$  and noting the corresponding values of  $W$  in the scale pan.
- The total weight pulling the block also includes the weight of the scale pan ( $W_o$ )
- Tabulate your readings as shown in Table 2.1.

$M$	$R = W_B + M$	$F = W + W_o$
152		



Table 2.1

- Plot a graph of force  $(W + W_0)$  against normal reaction  $R = (W_B + M)$  and determine slope of the graph which is the coefficient of static friction.

The graph of an a earlier experiment is as shown in Fig. 2.2 below.

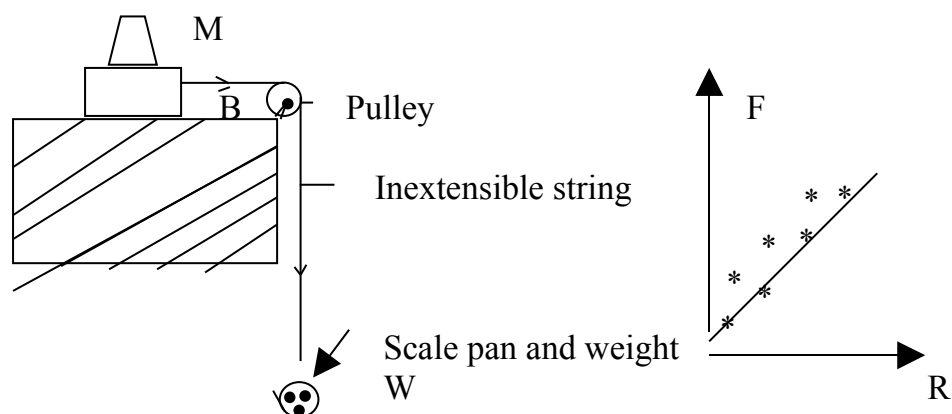


Fig 2.1: Measurement of coefficient of friction. Fig. 2.2: Graph of  $F$  against

## ii. Inclined Plane

You can also determine the coefficient of static friction  $M$  by using inclined plane.

Here, you will obtain the value of  $M$  for the two surfaces by placing the block A on B as shown in Fig. 2.3 and gradually increasing the angle  $\theta$  of inclination of B to the horizontal unit A is just about to move down the plane.



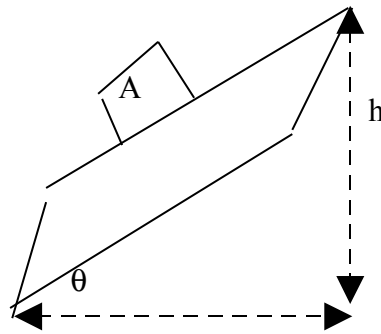


Fig. 2.3: Block A on an inclined plane with angle  $\theta$

- Measure the values of  $x$  and  $h$  using your metre rule. You will then obtain the value of  $\mathcal{M}$  (the coefficient of friction) by evaluating the expression relating  $\mathcal{M}$  to the values of  $x$  and  $h$ . That is:

$$\tan \theta = h/x = \mathcal{M} \quad (3)$$

### Precautions

You will take the following precautions while determining the coefficient of friction

- Take repeated readings.
- The block of wood should be started at the same spot each time.
- The inclined plane should be gradually increased.
- The weight in the scale pan should be gradually increased.
- It is more accurate to measure  $\theta$  by measuring  $h$  and  $x$ .

### Worked example

1. A brass block weighs 25N. The coefficient of limiting friction between the block and the horizontal surface of a table is 0.61
  - a. What is the horizontal force required to just move the block?
  - b. If another 25N brass block is stacked on this block, what is the minimum horizontal force that must be applied to move the blocks?



**Solution**

- a. Coefficient of limiting friction,  $\mathcal{M} = 0.61$  Normal reaction,  $R =$  weight of block.  $= 25\text{N}$

$$\text{Using } \mathcal{M} = \frac{F}{R}$$

$$F = \mathcal{M} R = 0.61 \times 25\text{N} = 15.25\text{N}$$

The minimum horizontal force required to move the block is 15.25N.

- b. A 25 N brass block is stacked on the given block coefficient of limiting friction remaining the same.

$$\text{Normal reaction, } R = 25\text{N} + 25\text{N} = 50\text{N}$$

$$\text{Using } F = \mathcal{M} R$$

$$F = 0.61 \times 50\text{N} = 30.5\text{N}$$

2. A 250N weight can just rest on a plane inclined at  $35^\circ$  to the horizontal when a 49N force, parallel to the plane, is applied on it.

Calculate:

- the value of the frictional force present.
- the coefficient of limiting friction.

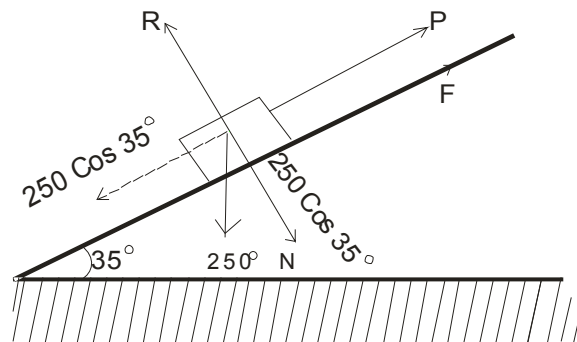
**Solution**

Fig. 2.4: Forces on weight resting on an inclined plane



Let the frictional force be  $F$ , and the applied force of 49N be  $P$  (Fig. 2.4).

Restoring the 250N weight into two perpendicular directions, we have  
Component weight parallel to the plane

$$= 250\text{N} \sin 35^\circ = 143.79\text{N}$$

Component weight perpendicular to the plane  
 $= 250\text{N} \cos 35^\circ = 204.79\text{N}$

$$\therefore \text{Normal reaction } R = 204.79\text{N}$$

Since the block is at rest,

$$P + F = 143.39\text{N}$$

$$\therefore F = 143.39 - P = (143.39 - 49)\text{N} = 94.39\text{N}$$

The friction force is 94.39N

$$(b) \text{ using } \mathcal{M} = \frac{F}{R},$$

$$\mathcal{M} = \frac{94.39\text{N}}{204.79\text{N}} = 0.46$$

The coefficient of limiting friction is 0.46

### SELF ASSESSMENT EXERCISE 1

1.
  - a. What is friction?
  - b. Name and define the two types of friction.
  - c. Distinguish between coefficients of static and dynamic friction.
2. State
  - i. the laws of friction between the surfaces of two bodies in contact
  - ii. two advantages of friction
  - iii. two disadvantages of friction
  - iv. methods of reducing friction
3. A body of mass 5 kg rest in an inclined place whose angle of inclination to the horizontal can be varied. If the coefficient of friction between the body and the surface of the inclined place is 0.4, at what angle does the body start to slide down the plane?



4. A block of wood weighing 5N is placed on a horizontal table. It is then pulled by means of a spring balance attached to one of its ends. The block just begins to move when the spring balance records a horizontal force of 2.5N.
- i what is the coefficient of static friction?
  - ii what is the frictional force when a horizontal force of 2.0N is recorded by the spring balance.

### 3.7 Viscosity

#### 3.7.1 Concept of Viscosity

Frictional forces also exist between different layers of liquid surfaces. We refer to such frictional force as viscosity. The viscosity of a liquid is low if the frictional force is low and vice versa. For instance, if you throw an object into water and a similar object into palm oil, the object moving through water will experience less resistance than when it moves through palm oil because the viscosity of water is Lower than that of palm oil. This makes the object to move faster in water.

The attraction between molecules in neighboring layers of a liquid is responsible for the viscosity of the liquid.

Viscosity can be defined as the property of a fluid which tends to prevent motion of one part over another.

It is the internal friction between layers of a liquid or gas in motion.

The viscosity of a liquid rapidly decreases as its temperature rises. You will have noticed that tar which is used for making roads is first heated so that it will flow readily.

You can compare the viscosity of liquids roughly by dropping a small steel ball –bearing into a tall wide cylindrical vessel filled with the various liquids whose viscosity you want to compare. You will then measure the time it will take the ball bearing to fall through the liquids.

You will notice that with water, the ball bearing fall rapidly. It falls fairly slowly in oil, and very slowly in glycerine (Fig. 2.5).



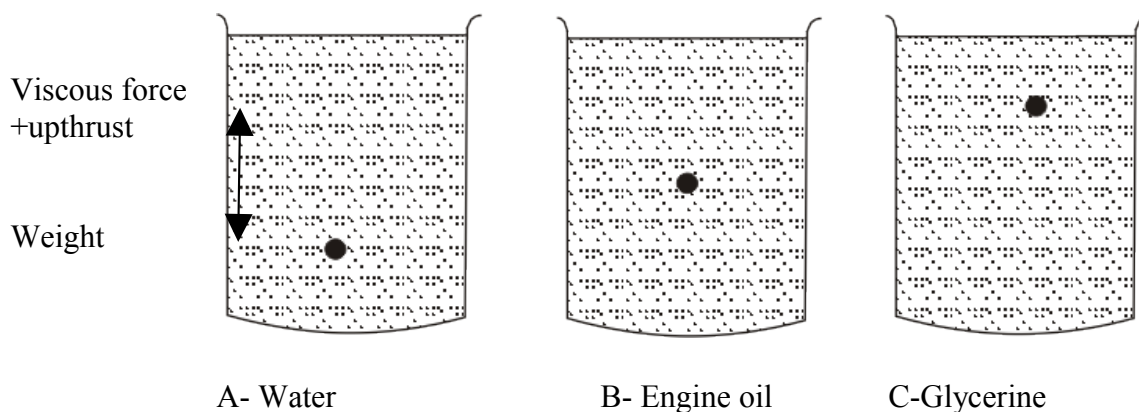


Fig. 2.5: Motion in a viscous liquid

### 3.7.2 Effects of Viscosity

Here we look at some important applications of viscosity. You may have identified some of this yourself in our earlier discussions although you may not have categorized them under this heading.

The viscosity of oil is very important in selecting suitable lubricants for the various sections of the machines. We use more viscous oils for heavier machines than in light ones.

Oils, grease and air are used as lubricants because of their viscosity.

We use engine oils in lubricating engines to keep metal surfaces from rubbing against each other.

Water is not used as a lubricant because it has low viscosity.

The viscosity of oils and greases decrease with temperature. Therefore, their lubricating effects are lowered at high temperature.

The viscosity of the air makes the bob of a swinging pendulum come to rest more quickly with a cord attachment to the string than without a cord.

The wider the area of the cord the greater the viscous force opposing the pendulum's motion and the pendulum bob comes to rest quicker.



## 3.8 Laws of Viscosity and Coefficient of Viscosity

### 3.8.1 Laws of Viscosity

- It opposes motion (like solid friction).
- It depends on the area of the surface in contact (unlike solid friction).
- It depends on the relative motion between the layers (unlike solid friction).
- It depends on the nature of the liquid.  
It is independent of normal reaction (unlike solid friction).

### 3.8.2 Coefficient of Viscosity

The coefficient of viscosity of a fluid is a measure of the viscosity of the fluid.

To define this coefficient, let us consider a thin layer of liquid between two flat parallel plates (Fig. 2.6). If the lower plate is held in a fixed position,

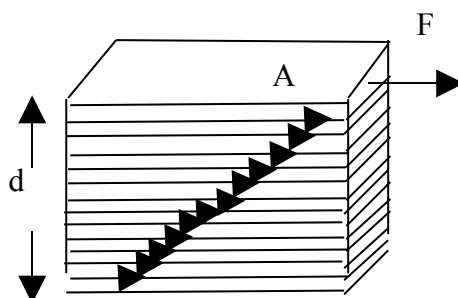


Fig. 2.6: Viscosity of a fluid

Each layer moves with a small velocity relative to an adjacent layer.

a force  $F$ , is required to move the upper plate at a constant speed. This force is observed to be proportional to the area  $A$ , and the speed  $V$ , of the moving plate, and inversely proportional to the thickness  $d$ , of the liquid between the plates, that is,

$$F \propto \frac{AV}{d}$$

Or  $F = \eta \frac{AV}{d}$



Where  $\eta$ , the constant of proportionality, is the coefficient of viscosity of the liquid,

We have that

$$\eta = \frac{F d}{A V} = \frac{F/A}{V/d}$$

The ratio; i,  $F/A$  is force per unit area (pressure)  
Unit: Pa

ii,  $V/d$  is velocity gradient  
Units:  $S^{-1}$

$\therefore$  S. I .unit of coefficient of friction is Pascal second (PaS).

Table 2.2: Typical values of viscosity

Temperature $^{\circ}C$	Coefficient of Viscosity		
	Castor oil Pas	Water $10^{-2}$ Pas	Viscosity Air $10^{-6}$ Pas
<b>0</b>	<b>53.00</b>	<b>1.792</b>	<b>171</b>
<b>20</b>	<b>9.86</b>	<b>1.005</b>	<b>181</b>
<b>40</b>	<b>2.31</b>	<b>0.656</b>	<b>190</b>
<b>60</b>	<b>0.80</b>	<b>0.469</b>	<b>200</b>
<b>80</b>	<b>0.30</b>	<b>0.357</b>	<b>209</b>
<b>100</b>	<b>0.17</b>	<b>0.284</b>	<b>218</b>

Table 2.2 shows that the viscosities of substances vary with temperature, but while those of liquid decrease with temperature, those of gases rise as the temperature rises.

### 3.9 Motion of a Ball in a Viscous Liquid

When you throw a steel ball to fall through a viscous liquid, the ball is subject to three forces:

- Its weight ( $w$ ) acting downwards;
- The upthrust ( $u$ ) of the liquid on the stone acting upwards, and
- The viscous force acting opposite to the motion of the ball i.e. upwards. This is shown in Fig. 2.7 below.



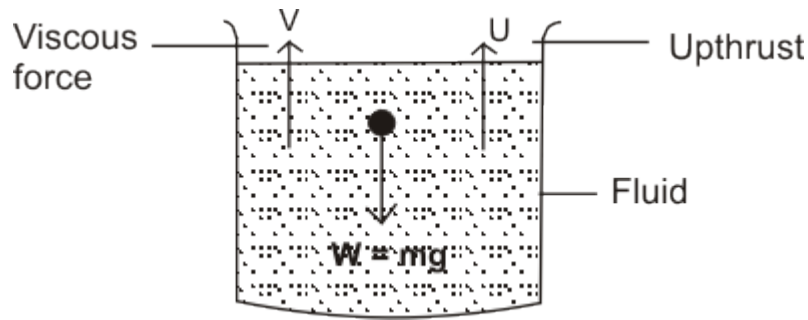


Fig. 2.7: A ball falling through a viscous liquid

The ball accelerates at first but finally attains a uniform velocity known as terminal velocity.

At first the equation of motion is given by  $mg - v - u = ma$

Where  $v$  = viscous force,  $u$  = upthrust,  $m$  = mass of the ball,  $g$  = accn due to gravity and  $a$  = acceleration of the ball in motion.

When the terminal velocity is reached,  $a = 0$ ,  
The equation becomes;

$$\begin{aligned} Mg - V - U &= 0 \\ \Rightarrow mg &= V + U. \quad (Mg = w) \\ \text{Or} \\ V &= W - U \end{aligned}$$

This constant velocity ( $v$ ) is termed the terminal velocity.

Fig. 2.8 shows the variation of velocity with time as an object dropped inside a viscous liquid attains terminal velocity.

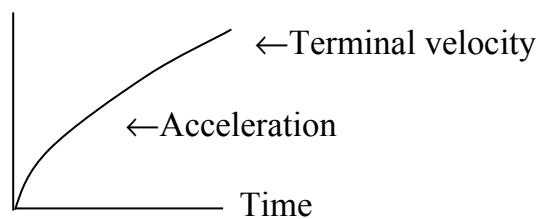


Fig. 2.8: Viscous fluid terminal velocity in viscous fluid



### 3.10 Similarities and Differences between Viscosity and Friction

You must have observed that the following similarities and differences exist between viscosity and friction.

#### Similarities

- Both forces oppose relative motion between surfaces.
- Both depend on the nature of the materials in contact.

#### Differences

- Friction does not depend on area of surfaces in contact; viscosity depends on areas of surfaces in contact.
- Friction is dependent on normal reaction, viscosity is not.
- Friction occurs in solids, viscosity takes place in liquids and gases (fluids).
- Friction does not depend on the relative velocities between two layers; viscosity depends on the relative velocity between layers.

### SELF ASSESSMENT EXERCISE 2

- 1 Explain the terms
  - i. Viscosity
  - ii. Terminal velocity
- 2 Draw a diagram to show the forces acting on steel ball falling through a viscous liquid.
3. Sketch the graph of velocity against time of the steel ball and explain its nature.
4. What are the differences and similarities between friction and viscosity?

### 4.0 CONCLUSION

In this unit you learned about friction in solids and liquids. You also learned about its advantages and disadvantages; why it is necessary to reduce friction as well as the laws governing it. You also learned how to determine the differences between friction and viscosity, and the applications of viscosity.



## 5.0 SUMMARY

- Friction is the force which acts to oppose motion at the surface separating two objects in contact.
- Static or limiting friction is the maximum force that must be overcome before a body can just start to move over another.
- Kinetic or dynamic friction is the force that must be overcome so that a body can move with uniform speed over another.
- Coefficient of static friction is the ratio of the frictional force (F) to the normal reaction R.

$$\mu = \frac{F}{R}$$

- Frictional force is independent of the areas of surfaces in contact.
- Friction has both desirable and undesirable effects.
- Friction is reduced by lubrication, ball bearing and by streamlining
- Viscosity is the internal friction between layers of a liquid or gas in motion.

Viscosity decreases with temperature.

- Viscous oils and air under pressure are used as lubricants.

## 6.0 TUTOR-MARKED ASSIGNMENT

- 1 Define :
  - i, Friction.
  - ii, Viscosity.
- 2 List the three major ways of reducing friction.
- 3 State :
  - i, four differences,
  - ii, two similarities
 between friction and viscosity.
- 4 Draw a diagram to show the forces acting on steel – ball falling on a viscous liquid.



- 5 A metal box weighs 50N. The coefficient of limiting friction between the metal box and the horizontal surface of a table is 0.3.
- What is the horizontal force required to just move the metal box?
  - If another 50N metal box is stacked on this metal box, what is the minimum horizontal force that must be applied to move the metal boxes?

## 7.0 REFERENCES/FURTHER READINGS

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## **UNIT 3      LINEAR MOMENTUM**

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### **1.0 INTRODUCTION**

In this unit we will discuss the influence of unbalanced forces on bodies, the motion of objects, and the laws governing such objects. We will also study the collision between bodies that are virtually free from forces other than those which they exert on each other at impact and the laws governing such collisions.



## 2.0 OBJECTIVES

At the end of this unit, you should be able to:

- State and explain in your own words, each of Newton's laws of motion
- Show that Newton's second law of motion gives
  - a measure of force as the product of mass and acceleration;
  - an operational definition of force as rate of change of momentum.
- Solve simple problems based on Newton's laws of motion and the principle of momentum.
- State and explain the meaning of the law of conservation of linear momentum.
- Solve simple problems based on conservation of linear momentum.
- Explain
  - Why walking is possible
  - Why a gun recoils when fired
  - How a rocket is propelled
  - How a jet plane is propelled
- Explain inertial mass and the relationship between mass and weight.
- Explain the variation in the weight of a body from place to place.

## 3.0 MAIN CONTENT

### 3.1 Momentum and Impulse

#### 3.1.1 Momentum

##### Definition

The momentum of a body is defined as the product of its mass and its velocity.

If a body of mass  $m$  moves with a velocity  $v$ , then its momentum  $p$  is given by:

$$P = mv$$

Unit of momentum is kilogram metre per second ( $\text{kgms}^{-1}$ ).

Momentum is a vector, its magnitude is the numerical value of the product  $m \times v$  and its direction is the direction of  $v$ .



Bodies moving in a straight line possess “linear momentum” while rotating bodies have “angular momentum”.

### 3.1.2 Impulse

#### Definition

Impulse is defined as the product of the average force on the particle and the time during which it acts.

If a force  $F$  acts for a short time  $t$ , the impulse  $I$  is given by

$$I = F \times t$$

Impulse is a vector with magnitude equal to the product  $F \times t$ , and direction same as that of the force  $F$ .

The unit of impulse is Newton – second (Ns).

#### Worked examples

- 1 A body of mass 4.0 kg moves with a velocity of  $15 \text{ ms}^{-1}$ . Calculate the momentum of the body?

#### Solution

$$\begin{aligned} P &= mv \\ \therefore p &= 4\text{kg} \times \frac{15\text{m}}{\text{s}} \\ &= 60 \text{ kg ms}^{-1} \end{aligned}$$

- 2 An average force of 30N hits a stationary ball for a time of 0.025. What is the impulse experienced by the ball?

#### Solution

$$\begin{aligned} I &= Ft \\ \therefore I &= 30 \text{ N} \times 0.02\text{s} \\ &= 0.6 \text{ Ns} \end{aligned}$$



## 3.2 Newton's Laws of Motion

### 3.2.1 Newton's First Law of Motion

#### Statement

Newton's law of motion states that every object continues in its state of rest or of uniform motion in a straight line unless acted upon by an external force. Importance of Newton's first law

- (i) The law identifies inertia as an inherent property of a body at rest or moving with a constant velocity. Inertia is thus a property of matter and may be defined as the reluctance of a body to change its state of rest or of uniform motion in a straight line unless an external force acts on it.

The mass of a body is a measure of its inertia. The greater the mass of a body, the greater its inertia and vice versa.

- (ii) Newton's first law explains why passengers in a fast moving vehicle tend to move forward when the vehicle stops suddenly, or jers backwards when the vehicle suddenly speeds off, since there is little or no force to restrain them. For this reason motorists are advised to use seat belts (or safety belts) while traveling in their cars.
- (iii) Newton's first law also explains what force does, although it does not suggest how it should be measured.

### 3.2.2 Newton's Second Law of Motion

#### Statement

Newton's second law states that the rate of change of momentum is proportional to the force causing motion and takes place in the direction of that force.

Importance of Newton's Second Law

- (i) It enables us to define an absolute unit of force which remains constant under all conditions.
- (ii) It gives a measure of force as product of mass and acceleration.
- (iii) It gives an operational definition of force as the rate of change of momentum.



According to Newton's Second Law,

$$F \propto \frac{\text{change in momentum}}{\text{time taken for the change}}$$

$$\text{i.e. } F \propto \frac{mv - mu}{t}$$

where  $m$ ,  $u$ ,  $v$ ,  $t$  are mass, initial velocity, final velocity and time respectively of motion of the body acted upon by a force,  $F$ .

$$\therefore F \propto \frac{mv - mu}{t} \text{ or } \frac{m(v - u)}{t}$$

$$\therefore F \propto ma, \text{ where } \frac{v - u}{t} = \text{acceleration, } a.$$

$$F = k ma, \text{ where } k = \text{constant.}$$

In the S.I. unit system,  $F$  is in Newton,  $m$  in kilogram, and the acceleration,  $a$  in meter per sec squared ( $\text{ms}^{-2}$ ). The Newton is chosen such that  $k$  has a value of 1.

$$\therefore F = ma$$

### Definition

The Newton is defined as the unit of force which gives a mass of 1kg an acceleration of  $1\text{ms}^{-2}$ .

### Worked examples

1. A mass of 5.0 kg is acted upon by an unbalanced force of 25N. What acceleration does the force give the mass?

### Solution

$$\begin{aligned} F &= ma \\ 25\text{N} &= 5 \text{ kg} \times a \\ \therefore a &= 25\text{N}/5 \text{ kg} \\ &= 25\text{kgms}^{-2}/5 \text{ kg} \\ &= 5\text{ms}^{-2} \end{aligned}$$

2. A stone of mass 900 g is pushed along a floor by a horizontal force of 20N. A frictional force of 8N opposed the motion of the stone. Calculate the acceleration given to the stone.



**Solution**

$$\text{Unbalanced force} = (20 - 8) \text{ N} = 12 \text{ N}$$

$$F = ma$$

$$12 \text{ N} = \frac{900 \text{ kg}}{1000} \times a$$

$$\therefore a = \frac{12 \times 1000}{900} \text{ ms}^{-2}$$

$$= 13.33 \text{ ms}^{-2}.$$

3. A steel ball of mass 0.1 kg fell from a height of 8 m onto a hard surface and bounced back to a height of 2 m. Calculate the change of momentum. If the ball was in contact with the hard surface for 0.15s, find the force exerted on the surface ( $g = 10 \text{ ms}^{-2}$ )

**Solution**

Velocity  $v$  of the ball on hitting the floor is given by

$$v^2 = u^2 + 2gh$$

$$\text{i.e. } v = \frac{\sqrt{0 + 2 \times 10 \times 8 \text{ m}}}{\text{s}^2} = \frac{\sqrt{160 \text{ m}^2}}{\text{s}^2}$$

$$\therefore v = 12.65 \text{ ms}^{-1}$$

let the velocity with which the ball left the surface after impact be  $U_1$ ,  
At maximum height the (final) velocity is zero. Hence

$$0 = u_1^2 - 2gh$$

$$\therefore 0 = \frac{u_1^2 - 2 \times 10 \text{ m} \times 2 \text{ m}}{\text{s}^2}$$

$$\therefore u_1^2 = \frac{40 \text{ m}^2}{\text{s}^2}$$

$$\text{i. } u_1 = \frac{\sqrt{40 \text{ m}^2}}{\text{s}^2} = 6.32 \text{ ms}^{-1}$$



$$\begin{aligned}
 \text{Change in momentum} &= mv + mu_1, \text{ i.e. } (mv - (-mu_1)) \\
 &= 0.1(12.65 + 6.32) \\
 &= 1.897 \text{ kgms}^{-1} \text{ (or Ns)}
 \end{aligned}$$

(Note that if the momentum before hitting the surface is positive, then momentum after hitting the surface is negative. Hence change in momentum is  $mv - (-mu_1) = mv + mu_1$ )

$$\begin{aligned}
 \text{Force} &= \frac{\text{Change in momentum}}{\text{time}} \\
 &= \frac{1.897 \text{ Ns}}{0.15 \text{ s}} \\
 &= 12.646 \text{ N} \\
 &= 12.6 \text{ N}
 \end{aligned}$$

### 3.2.3 Newton's Third Law

#### Statement

Newton's third law of motion states that to any action there is an equal and opposite reaction.

#### Example of Newton's Third Law in operation

1. If you hit your head against the wall, you exert some force on the wall and the wall in turn exerts an equal and opposite force on your head. If the force you exerted on the wall is sizeable, then pain you would feel on your head is a confirmation of Newton's third law. This implies that forces are always paired. No single force exists in isolation.

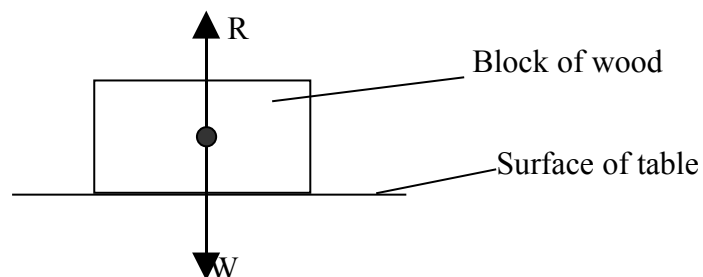


Fig.3.1: Wooden block resting on a table

- ii. If you place an object (e.g. a wooden block) on a table, it will remain at rest at the spot you left it (Fig.3.1). The block exerts on



the table a force equal to its weight. The table exerts on the block a reaction force,  $R$ , equal and opposite of  $W$ . since the two forces balance, the block remains at rest on the table.

- iii. The recoil of a gun, If you fire a bullet from a gun, you will observe that the gun jerks backwards or recoils (Fig.3.2). The force propelling the bullet forward (action) must be equal to the recoil force (reaction) acting on the gun (i.e. opposite). The jerking of the gun (which you will feel in your hand) is actually due to the conservation of momentum.

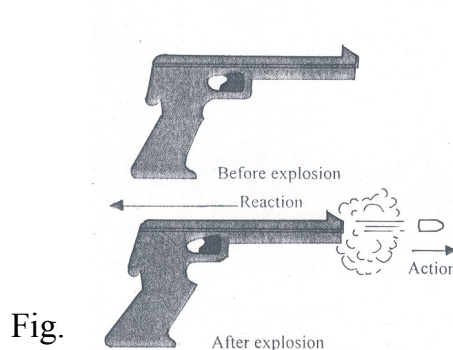


Fig.

2.3.2 Recoil of a gun

### 3.3 Conservation of Linear Momentum

#### 3.3.1 Statement of Principle of Conservation of Linear Momentum

The principle of conservation of linear momentum may be stated in different ways as follows:

- i. In any system of colliding objects, the total momentum is always conserved provided that there is no net external force acting on the system.

OR

- ii. The total momentum of an isolated or closed system of colliding bodies remains constant

OR

- iii. If two or more bodies collide in a closed system, the total momentum after the collision is equal to the total momentum before the collision.

#### **Closed System:**

A closed or isolated system is a system on which no external forces act. The only forces



acting in a closed system of colliding bodies are those due to the collisions.

The principle of conservation of linear momentum follows from Newton's second and third laws of motion.

### 3.3.2 Illustration of the Principle

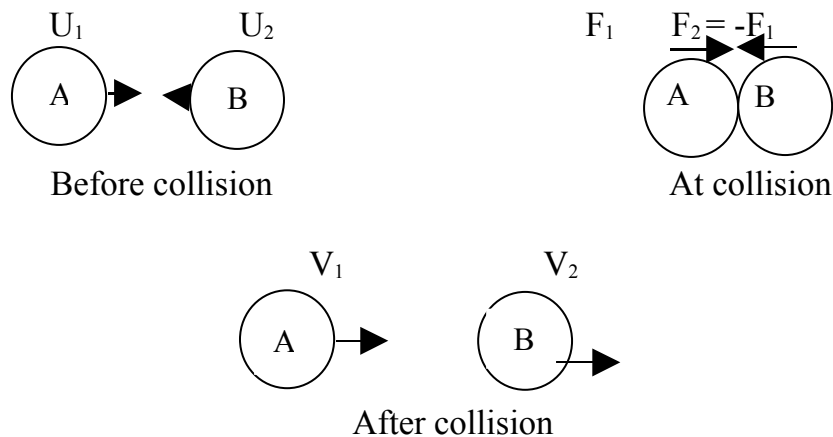


Fig.3.3: Conservation of linear momentum

Consider the bodies A and B (Fig.3.3). of masses  $m_1$ ,  $m_2$  respectively moving towards each other with velocities  $u, u_2$  respectively. Let their accelerations be  $a_1$ ,  $a_2$  respectively when the bodies collide, the force,  $f_1$  on A is equal and opposite to the force,  $f_2$  on B. from Newton's third law

$$F_1 = - F_2 \dots\dots\dots (1)$$

From Newton's second law,  $F=ma$

$$\therefore M_1 a_1 = m_2 a_2 \dots\dots\dots (2)$$

We recall that acceleration,  $a = \frac{v-u}{t}$ , hence

$$\therefore m_1 \left[ \frac{v_1 - u_1}{t} \right] = - m_2 \left[ \frac{v_2 - u_2}{t} \right] \dots\dots\dots (3)$$

where  $t$  is the contact time and  $u$  is the initial velocity before and  $v$  is the final velocity after impact. Eliminating  $t$  from equation (3) we have

$$m_1 (v_1 - u_1) = m_2 (v_2 - u_2)$$

$$\text{ie. } m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \dots\dots\dots (4)$$



Since we have defined momentum as mass x velocity equation (4) means that the total momentum of the system before the impact is equal to the total momentum after the impact.

### Worked examples

1. A bullet of mass 10 g is fired from a gun of mass 5 kg at a velocity of  $250 \text{ m s}^{-1}$ . Find the velocity of recoil of the gun.

#### Solution

Mass of gun,  $m_1 = 5 \text{ kg}$

Mass of bullets,  $m_2 = 10\text{g} = 0.01 \text{ kg}$

Let final velocity of gun  $= v_1$  (in  $\text{ms}^{-1}$ )

Final velocity of bullet  $= v_2 = 250 \text{ ms}^{-1}$

Total momentum of gun and bullet before firing is zero since both were at rest.

Total final momentum  $= m_1 v_1 + m_2 v_2$

Since momentum is conserved,

$$\begin{aligned} m_1 v_1 &= m_2 v_2 = 0 \\ \text{ii. } v_1 &= - \frac{m_2 v_2}{m_1} \\ &= \frac{- 0.01 \times 250}{5} \text{ ms}^{-1} \\ &= - 0.5 \text{ ms}^{-1} \end{aligned}$$

The negative sign indicates that the velocity of recoil of gun is in the opposite direction to the velocity of the bullet.

A body of mass 5 kg moving with a velocity of  $30 \text{ ms}^{-1}$  collides with another body moving in the opposite direction with a velocity of  $20 \text{ ms}^{-1}$ . if both of them now move in the direction of the first body (of mass 5 kg), at a common velocity of  $10 \text{ ms}^{-1}$ . Calculate the mass of the second body.

#### Solution

A sketch of the diagram will help us



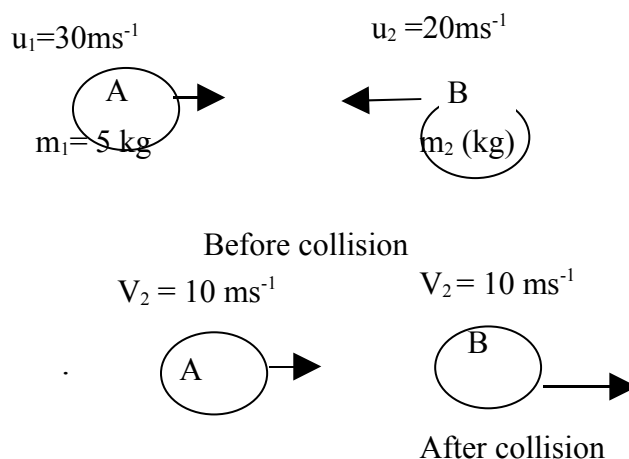


Fig.3.4

Let mass of B =  $m_2$  kg

$$\begin{aligned} \text{Total momentum before collision} &= (5 \times 30) + (-m_2 \times 20) \\ &= 150 - 20 m_2 \end{aligned}$$

$$\begin{aligned} \text{Total momentum after collision} &= (5 \times 10) + m_2 \times 10 \\ &= 50 + 10 m_2 \end{aligned}$$

Since linear momentum is conserved, we have

$$\begin{aligned} 150 - 20m_2 &= 50 + 10m_2 \\ 30m_2 &= 100 \quad \text{i.e. } (150 - 50 = 10m_2 + 20m_2) \\ m_2 &= 3.33 \text{ kg} \quad \Rightarrow 30m_2 = 100 \\ m_2 &= \frac{100}{30} \end{aligned}$$

### 3.3.3 Elastic and Inelastic Collision

There are two major types of collision:

- (ii) elastic collision and,
- (iii) inelastic collision

(a) A collision is said to be “perfectly” elastic if both of the momentum and the kinetic energy (k.e) of the system are conserved i.e. constant ,

Hence,

$$(i) \quad m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2; \text{ and}$$

$$(ii) \quad \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$



$$^2 \quad \quad \quad ^2 \quad \quad \quad ^2 \quad \quad \quad ^2$$

- b In an inelastic collision only the linear momentum is conserved. The kinetic energy after collision is less than the kinetic energy before collision

Hence,

$$(i) \quad m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$(ii) \quad E_{k1} > E_{k2}$$

Where  $E_{k1}$  is the total K.E before impact and  $E_{k2}$  is the total K.E after impact.

OR

$$\frac{1}{2} m_1 U_1^2 + \frac{1}{2} m_2 U_2^2 > \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\text{i.e } \sum \text{KE before} > \sum \text{KE after}$$

where

$\sum \text{EK before}$  is the total Kinetic Energy before impact

and

$\sum \text{EK after}$  is the total Kinetic Energy after impact

In inelastic collision, the colliding bodies usually stick together and move as a unit after collision, so as that  $v_1 = v_2 = v$ .

Hence

$$\text{Total kinetic energy before} = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \text{ and}$$

$$\text{Total kinetic energy after} = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

$$\text{or } EK_2 = \frac{1}{2} (m_1 + m_2) v^2$$

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 > \frac{1}{2} (m_1 + m_2) v^2$$



## SELF ASSESSMENT EXERCISE 1

Study explosion and compare it with elastic and inelastic collision

### 3.4 More Applications (Examples) of Newton's Law and the Conservation of Linear Momentum Law

#### 3.4.1 How Walking is Possible

A person walks by pushing with his/her foot against the ground and the ground exerts an equal and opposite force on the person (Fig.3.5)

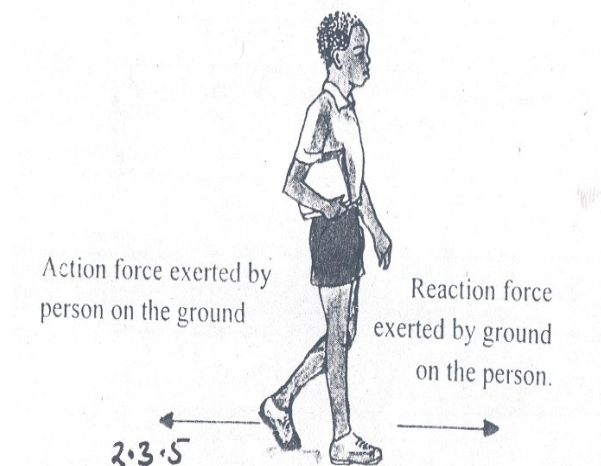


Fig...3.5:  
How

walking is possible

It is this reaction force of the ground on the person walking which moves him or her forward.

Thus a person walking is actually pushed forward by the reaction force of the ground on him and not by his on her own push.

#### 3.4.2 Jet and Rocket Propulsion

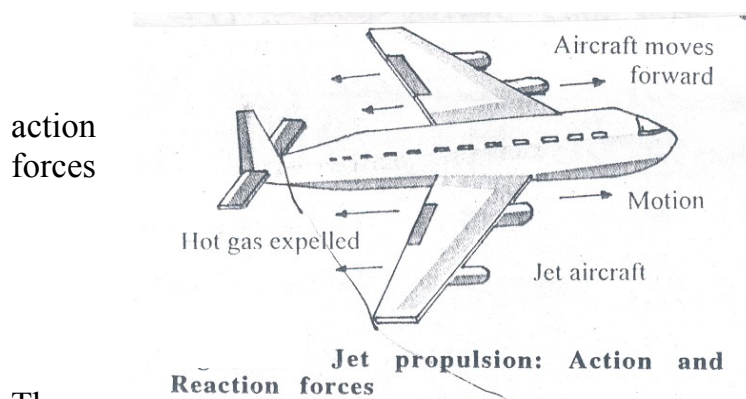


Fig. 3.6: Jet  
propulsion:  
and reaction

The principle of conservation of linear momentum and Newton's third laws are also in



the propulsion of jet aircraft and the rockets used for launching satellites. Gases are burnt in the combustion chambers of the engine. When jets of hot gases are expelled downwards through the tail nozzle at high speeds, from rocket or aircraft, an equal opposite momentum is given to the rocket or aircraft causing it to move.

### 3.4.3 The Lawn Sprayer

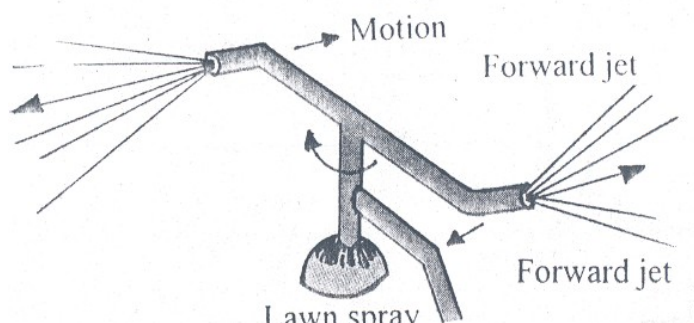


Fig.3.7: the lawn sprayer

The lawn sprayer is a device used to water a lawn or garden (Fig.3.7). It combines Newton's third law, the law of conservation of momentum and the principles of a couple in its operation. Water issues from the forward and backward nozzles of the sprayer at the same speed. The reaction forces due to the thrust of the water from the two jets constitute a couple whose moment causes the sprayer to rotate and so water the lawn or garden.

#### Worked example

A rocket burns fuel at the rate of  $200 \text{ g s}^{-1}$  and is ejecting all the gas in one direction at the rate of  $400 \text{ ms}^{-1}$ . Find the maximum weight the rocket can have if it is going to move vertically upwards.

#### Solution

$$\text{Mass of gas per second} = \frac{200}{1000} \text{ kg s}^{-1}$$

$$\text{Velocity of expulsion} = 400 \text{ ms}^{-1}$$

$$\text{Momentum change per second} = \frac{200}{1000} \times 400 \text{ kg ms}^{-2}$$

From Newton's second law, we have



$$F = w$$

$$F = \frac{200 \times 400}{1000}$$

$$\therefore F = 80\text{N}$$

### 3.5 Inertial Mass and Weight

#### 3.5.1 Inertial Mass

Mass was defined by Newton as the “quantity of matter in a body”. We have seen from Newton’s first law that “inertia”, the reluctance of a body to change its state of rest or uniform motion in a straight line, is an inherent property of matter. The mass of a body can also be defined as “a quantitative measure of the inertia of a body”. The more the mass a body has, the greater the force required to change its state of or uniform motion in a straight line and the greater the force required to give an acceleration  $\left( M \propto \frac{F}{a} \right)$ . Thus

this inherent property of matter is also called “inertial mass”.

#### Definition

The inertial mass of a body is a property of matter which represents the resistance of the body to any kind of force whatever.

The mass of a body gives a quantitative measure of its inertial mass.

#### 3.5.2 Weight

#### Definition

The weight of a body is the force acting on the body due to the earth’s gravitational pull.

The weight ( $w$ ) of a body is given by

$$W = mg$$

where  $m$  is the mass of the body and  $g$  is the acceleration due to gravity.

#### 3.5.3 Weight of a Body Inside a Lift



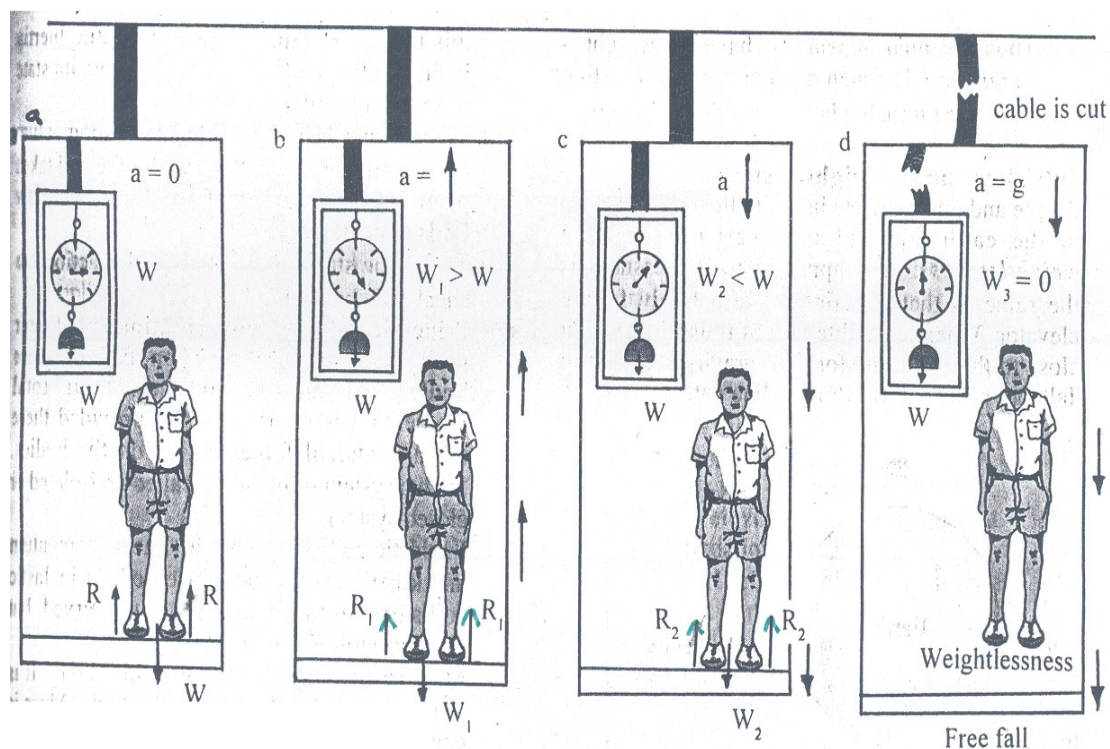


Fig.3.8: Weight of an object in a lift

Suppose you are standing in a lift or elevator.

Two forces act on you:

- (i) your true weight ( $w = mg$ ) acting vertically downwards, and
- (ii) the reaction  $R$  of the floor of the elevator on you acting upwards.

We will use two forces to consider the apparent variation in your weight (which you will feel) as the lift:

- (a) is stationary
- (b) accelerates upwards
- (c) accelerates downwards, or
- (d) falls freely under gravity, if the cable is cut.

- (a) When the elevator is stationary or moves with constant velocity (Fig.3.8a), the acceleration,  $a = 0$  and we have that

$$W = mg = R \dots \dots \dots (1)$$

If you are standing on a spring scale, it will record your true weight.



- (b) When the lift accelerates upwards with an acceleration  $a$  (Fig.3.8.b), you will also be pulled up with an acceleration  $a$  while still standing on the floor of the lift.

The unbalanced force ( $F$ ) acting on you is given by

$$F = ma = R_1 - mg$$

$$\therefore R_1 = ma + mg = m(a + g)$$

Therefore your apparent weight  $W_1$ , when the elevator moves upwards with acceleration  $a$  (and which is balanced by  $R_1$ ), is given by

$$F = ma = R_1 - mg$$

$$\therefore R_1 = ma + mg = m(a + g)$$

- (c) Therefore your apparent weight  $W$ , when the elevator moves upwards with acceleration  $a$  (and which is balanced by  $R$ ) is given by

$$W_1 = R_1 = m(a + g) \dots\dots\dots$$

(2)

It is clear from (1) and (2) that.  $W_1 > W$ .

You will therefore feel yourself pressing downwards ( $R_1$ ) on the floor with a larger force than when the lift is at rest. Your apparent weight ( $W_1$ ) as recorded by the scale will be greater than your true weight ( $W$ ).

- (c) When the elevator accelerates downwards with acceleration,  $a$  (Fig.3.8c), the unbalanced force on you is

$$F = ma = mg - R_2$$

$$\text{iii. } W_2 = R_2 = m(g - a) \dots\dots\dots (3)$$

The scale on which you stand will read this value  $W_2$ . From (2) and (3) it is clear that  $W > W_2$ , hence your apparent weight,  $W_2$  will be less than your true weight and you will feel lighter.

- (d) In this case, the lift cable is cut and it falls freely with an acceleration,  $a = g$  (Fig.3.8d). You will also fall freely with  $a = g$ . when this happens, we see from (3), that

$$W_3 = m(g - a) = 0$$



Thus you will appear to have no weight (i.e. weightless). You will exert no force on the floor and the floor will exert no force on you.

The scale reads zero.

### 3.5.4 Satellites and Weightlessness

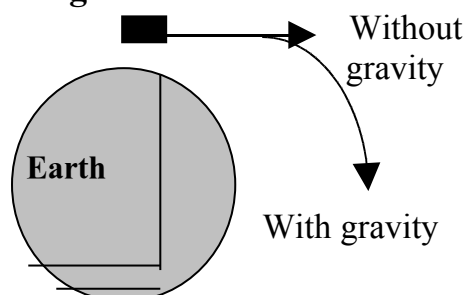


Fig.3.9: Free-fall of satellites:  
Weightlessness

Persons and objects inside a satellite orbiting close to the earth are said to experience weightlessness. This weightlessness is similar to the one you would feel in a freely falling elevator as already discussed.

When a satellite moves round in an orbit close to the earth, the force of gravity forces it to “fall” out of its natural straight line path (Fig.3.9).

While orbiting round the earth, the earth’s gravitational pull is just enough to provide the centripetal force  $\left(\frac{mv^2}{r}\right)$  required for the orbital movement, where  $m$  is mass of the satellite,  $v$  its speed and  $r$  the radius of the orbit.

$$\text{Thus } \frac{mv^2}{r} = mg$$

The satellite therefore falls all the time. It does not strike the ground, however, because its initial horizontal velocity causes it to miss the surface of the earth. As a result of this free fall situation, every person and object within the satellite is also falling freely under the gravity.

Hence such persons and objects within the satellite appear to be weightless (i.e experience seeming weightlessness)



**Worked example**

An object of mass 70 kg rests on a spring weighing machine inside a lift. The lift ascends with an acceleration of  $2.5 \text{ ms}^{-2}$ . Calculate the reading of the weighing machine. What will the reading of the machine be if the velocity of the lift is constant? ( $g = 10 \text{ ms}^{-2}$ )

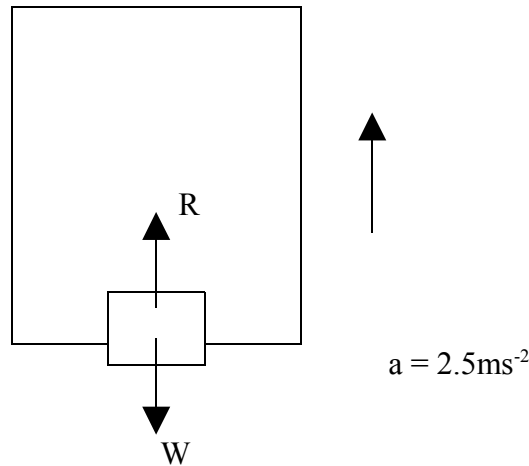


Fig. 3.10

**Solution**

- (i) The unbalanced force  $F$  is given by

$$\begin{aligned}
 F &= ma = R - mg \\
 \therefore R &= m(g + a) \\
 &= 70 \times (10 + 2.5) \\
 &= \underline{875 \text{ N}}
 \end{aligned}$$

- (ii) When the velocity is constant, acceleration is zero

$$\begin{aligned}
 R &= m(g + 0) \\
 &= mg \\
 &= 70 \times 10 \\
 &= 700 \text{ N}
 \end{aligned}$$

**SELF ASSESSMENT EXERCISE 2**

1. Distinguish between momentum and impulse and state the relationship between them. An object of mass 5 kg moving with a velocity of  $30 \text{ ms}^{-1}$  is suddenly hit by another body B moving in the same direction. If the velocity of A is changed to  $50 \text{ ms}^{-1}$  in the same direction, calculate the impulse received by it.



2. State Newton's laws of motion. Show how the second law enables us to define an absolute unit of force which remains constant under all conditions. A body of mass 0.2 kg is dropped from a height of 10 m onto a hard floor so that it bounces back to a height of 2 m. Calculate the change in momentum. If the body is in contact with the floor for 0.1 sec. find the force exerted on the body.
3. Explain what is meant by the term "inertia" of a body. A mass of 5 kg on a horizontal table surface with coefficient of friction 0.2 is connected to a second mass of 10 kg over a frictionless pulley. If  $g = 10 \text{ ms}^{-2}$ , calculate the acceleration of the system when released.
4. Why does a body in a lift appear to be weightless when the lift is A body of mass 10 kg is to be given an acceleration of  $20 \text{ ms}^{-2}$ . Find the force required when the acceleration is (i) vertically upwards, (ii) horizontal and the body moves over a surface whose coefficient of friction is 0.5. (Take  $g = 10 \text{ ms}^{-2}$ )
5. State the principle of conservation of linear momentum. A ball of mass 50 kg moving with a velocity of  $10 \text{ ms}^{-1}$  collides with another ball of mass 60 g moving with  $5 \text{ ms}^{-1}$  in the opposite direction. If the two balls stick together after collision, find the magnitude and direction of their common velocity.

#### 4.0 CONCLUSION

In this unit you learned linear momentum when we discussed the influence of unbalanced forces on bodies, the notion of objects and the laws governing such objects. You also learned the collision between bodies and the laws governing various types of collisions.

#### 5.0 SUMMARY

1. Momentum ( $p$ ) of a body is the product of its mass and its velocity  
( $p = mv$ )
2. Impulse ( $I$ ) is the product of the force and the time during which the force acts ( $I = F \cdot t$ ). Impulse acts only for a short time.
3. Newton's laws of motion are as follows:

**First law:** Every object continues in its state of rest or of uniform motion in a straight line unless acted upon by an external force. This law leads to the idea of "inertia" which is the



tendency of a body to preserve its state of rest or uniform motion.

**Second law:** The rate of change of momentum is proportional to the impressed force and takes place in the direction of the force. This law leads to the law  $F = ma$ .

**Third law:** Action and reaction are equal and opposite.

4. Principle of conservation of linear momentum states that when two or more bodies act on one another their total momentum remains constant that is conserved provided there are no external forces acting on the bodies. Such a system of bodies is called an isolated or closed system.
5. In elastic collisions, both the linear momentum and kinetic energy are conserved. In inelastic collision, linear momentum is conserved but kinetic energy decreases.
6. Mass is the quantity of matter in a body. It is also a measure of the inertia of a body. Mass is constant all over the earth.
7. “Inertial mass” is a measure of the tendency of a body to resist a change in momentum. According to Newton’s second law of motion,  $F \propto a$ , the constant of proportionality is the inertial mass.
8. Weight of an object is the force with which the earth pulls the body to the earth’s centre. Weight varies from place to place because the acceleration due to the earth’s gravity ( $g$ ) varies from place to place and weight of an object  $w=mg$ .

## 6.0 TUTOR- MARKED ASSIGNMENT

- 1
  - (a) Explain what you understand by the expression “linear momentum”
  - (b) State the principle of conservation of linear momentum and mention the condition under which the principle holds.
  - (c) A body P of mass 100 g moving with a velocity of  $10 \text{ ms}^{-1}$  collides with another body Q of mass 200 g moving with a velocity of 2 ms in the opposite direction. If P and Q stick together after the collision, calculate their common velocity  $v$  in the direction of P. Comment on your answer.



- 2 State and explain the principle on which the jet aeroplane functions. Mention two other useful applications of the same principle. A jet expels gas at the rate of  $0.5 \text{ kg s}^{-1}$ . If the force produced by the jet is  $300\text{N}$ , find the velocity with which the gas is expelled.
- 3 State Newton's laws of motion  
A man whose mass is  $80 \text{ kg}$  stands on a spring weighting machine inside an elevator. What is the reading of the weighting machine when.
- (ix) The elevator ascends with an acceleration of  $2.0 \text{ ms}^{-2}$ ?
  - (x) The elevator is moving with a uniform velocity?
  - (xi) The elevator is coming to rest with a retardation of  $4.0 \text{ ms}^{-2}$ ?

## 7.0 REFERENCES/FURTHER READINGS

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## UNIT 4     SIMPLE MACHINES

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- 3.0 Main Content
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### 1.0 INTRODUCTION

In this unit, you will learn about machines, their types and modes of action including their efficiencies and the effect of friction on them. You will also see a machine as a system which enables work to be done more easily or more quickly. You will learn about the working principles of a machine before studying some simple machines.

### 2.0 OBJECTIVES

By the end of the unit you will be able to:

- define a machine and list at least five simple machines.
- define (a) force ratio (b) velocity ratio (c) efficiency and write down the mathematical relationship between them.



- relate (a) force ratio (b) velocity ratio and efficiency of a machine
- calculate force ratio, velocity ratio and efficiency for a simple machine.
- draw (a) an inclined plane (b) wheel and axle, (c) a lever (d) a pulley (e) a screw (f) a wedge and where possible do this to achieve a specified velocity ratio.
- identify the simple machines that make up a given complicated machine such as a bicycle.
- apply simple machines to do work.

### **3.0 MAIN CONTENT**

#### **3.1 Concept, Definition and Examples of Simple Machine**

A machine enables us to overcome a large resistance or load by applying a small effort. A machine enables us to do work more easily and conveniently than could be done without it. Its purpose is to exert force on an object which is usually applied to the machine. Sometimes the force which the machine exerts is in a different direction from that of the applied force.

##### **3.1.2 Definition**

A machine is a device or tool which allows a force (or effort) applied at one point to overcome a resisting force (or load) at another point.

##### **3.1.3 Examples of Simple Machines**

You already know some of the simple machines which you come across in your everyday life. These machines are the lever, pulleys, pliers, wheelbarrows, nut-crackers, the wedge, wheel and axle, inclined plane, screw jack and many others.

#### **3.2.1 Definitions of (i) Force Ratio (ii) Velocity Ratio (iii) Efficiency of a Machine**

- Force ratio or mechanical advantage

We define effort as the force applied to a machine and load as the force or resistance overcome by the machine. The ability of a machine to overcome a large load through a small effort is seen as Force Ratio or Mechanical Advantage (MA). This is given by:



- Force ratio or mechanical advantage

$$MA = \frac{\text{LOAD}}{\text{EFFORT}}$$

The load you will take as the output force and the effort as the input force. You can therefore define mechanical advantage or force ratio by:

$$\text{Mechanical Advantage or Force Ratio} = \frac{\text{Output force}}{\text{Input force}}$$

i.e. MA or FR =  $\frac{\text{Output force}}{\text{Input force}}$

You will note that the mechanical advantage of a machine is influenced by friction in the parts. You will use part of the effort applied to overcome friction and part to overcome resistance or lift a load.

- **Velocity Ratio (VR)**

We define Velocity Ratio (VR) as the ratio of the distance moved by the effort and the load in the time interval

$$VR = \frac{\text{distance moved by effort (e)}}{\text{distance moved by load (l)}}$$

The Velocity Ratio depends on the geometry of the machine. It is independent of friction. For an ideal machine, that is, a machine that has no friction, also called a perfect machine, work done by machine = work done on machine. Therefore, Load distance moved by load = effort  $\times$  distance moved by effort.

$$\text{or } \frac{\text{load}}{\text{effort}} = \frac{\text{distance moved by effort (e)}}{\text{distance moved by load (l)}}$$

$$= \text{Velocity Ratio}$$

Hence for an ideal or perfect machine mechanical advantage = velocity ratio.

- **Efficiency ( $E_f$ )**

We define the Efficiency ( $E_f$ ) of a machine as:

$$E_f = \frac{\text{Useful work done by the machine}}{\text{Work put into the machine.}} \times 100\%$$



Usually, the efficiency of a machine is expressed as a percentage. That is,

$$EF = \frac{\text{useful work done by the machine}}{\text{Work put into the machine}} \times 100\%$$

**Note** that work (W) = force x distance

You know that a perfect or ideal machine has a 100% efficiency. This means that all the work done by the effort is wholly used to overcome the load. You also know that in practical machines; the efficiency is usually less than 100% because of friction in the moving parts of the machine. Part of the effort applied is used to overcome frictional forces which are always present. Hence, the useful work done by the machine is less than the workdone by the effort on the machine.

### 3.2.2 Relationship between Mechanical Advantage (Force Ratio), Velocity Ratio and Efficiency

For any machine, we know that:

$$\text{Efficiency (E}_f\text{)} = \frac{\text{work done on load}}{\text{Work done by effort}} \times 100\%$$

$$= \frac{\text{Load (L)}}{\text{Effort (E)}} \times \frac{\text{distance moved by load (L)}}{\text{distance moved by effort (e)}} \times 100\%$$

$$= \frac{L}{E} \times \frac{1}{e} \times 100\% = \frac{L}{E} \div \frac{e}{1} \times 100\%$$

$$\Rightarrow \frac{L}{E} \times \frac{1}{e} \times 100\% \dots(1)$$

where l and e are the distance moved by the load L and the effort E respectively in the same time interval.

For mechanical advantage (MA) and Velocity Ratio (VR), we have the following relationships:

$$\text{Mechanical Advantage, M. A.} = \frac{L}{E} \dots\dots(2)$$

$$\text{Velocity Ratio, V. R.} = \frac{e}{1} \dots\dots\dots (3)$$

You will substitute equations (2) and (3) into equation (1) to obtain



$$\begin{aligned}
 \text{Efficiency (E}_f\text{)} &= \frac{L}{E} \times 100\% \\
 &= \left( \frac{L}{E} \right) \div \left( \frac{e}{1} \right) \times 100\% \\
 &= \frac{MA}{VR} \times 100\%
 \end{aligned}$$

This implies that

$\text{Efficiency (E}_f\text{)} = \frac{MA}{VR} \times 100\%$
---

Mechanical Advantage, Velocity Ratio and Efficiency are ratios of similar quantities and so have no units.

### 3.3 Numerical Problems and Force Ratio, Velocity Ratio and Efficiency of Simple Machines

#### Worked examples

- 1 A simple machine has a VR of 5 and an efficiency of 90%. It is used to raise a load of 2.kg through a height of 10 m in 5s. Calculate:
- The force ratio (FR) of the machine
  - The work done (W) on the load, and
  - The power (P) developed. (Take  $g = 10 \text{ s}^{-2}$ )

#### Solution

$$\begin{aligned}
 \text{(a) Efficiency (E}_f\text{)} &= \frac{FR}{VR} \times 100\% \\
 \therefore FR \text{ (MA)} &= \frac{E_f \times VR}{100} \\
 &= \frac{90 \times 5}{100} = \frac{450}{100} = 4.5
 \end{aligned}$$

The Force Ratio of the machine is 4.5

$$\text{(b) Load} = 2.5 \text{ kg} \times 10 \text{ ms}^{-2} = 25\text{N}$$

$$\begin{aligned}
 \text{Work done on load} &= \text{load} \times \text{distance moved by load} \\
 &= 250\text{N} \times 10 \text{ m} = 250 \text{ J.}
 \end{aligned}$$



The work done on the load is 250 J

$$\begin{aligned} \text{(c) Power developed} &= \frac{\text{Work done}}{\text{Time taken}} \\ &= \frac{250}{5\text{s}} = 50\text{W} \end{aligned}$$

The power developed is 50 W

2. A machine whose efficiency is 80% is used to lift a load of 1000N. Calculate the effort put into the machine if it has a velocity ratio of 6.

**Solution**

$$\text{Efficiency } (E_f) = \frac{\text{MA}}{\text{VR}} \times 100\% = 80\%$$

$$\frac{\text{MA}}{6} = \frac{80}{100}$$

$$\begin{aligned} \Rightarrow \text{MA or FR} &= \frac{6 \times 80}{100} \\ &= \frac{480}{100} = 4.8 \end{aligned}$$

But

$$\text{FR} = \frac{\text{load}}{\text{Effort}} = \frac{1000}{\text{Effort}}$$

$$\begin{aligned} \therefore 4.8 &= \frac{1000}{\text{Effort}} \\ \text{Effort} &= \frac{1000}{4.8} = 208.33 \text{ N} \end{aligned}$$

the effort put into the machine is 208.33

- 3 The efficiency of a machine is 70%. Find in joules the work done by a student using this machine to raise a load of 75 kg through a vertical distance of 3 m ( $g = 10 \text{ ms}^{-2}$ )



**Solution**

$$\text{Efficient \%} = \frac{\text{Work done by the machine}}{\text{Work done on the machine}} \times 100\%$$

$$\Rightarrow \frac{70}{100} = \frac{75 \times 10 \times 3}{\text{work done by student}} \times \frac{100}{100}$$

$$\begin{aligned} \therefore \text{work done by student} &= \frac{75 \times 10 \times 3 \times 100}{70} \\ &= 3214.29 \text{ Joules} \end{aligned}$$

The work done by the student by using the machine to raise the load is 3214.29J.

**SELF ASSESSMENT EXERCISE 1**

- 1
  - (a) Define a machine? Give 5 examples.
  - (b) What do you understand by the statement that the force ratio of a machine is 7 and its velocity ratio 12?
- 2 Define the terms: Force Ratio, Velocity Ratio and Efficiency as applied to a machine. State the relationship between these terms.
3. The efficiency of a machine is 80%. Determine the work done by a person using this machine to raise a load of 250kg through a vertical distance of 5.0 m (take  $g = 10 \text{ ms}^{-2}$ ).
4. A simple machine has a VR of 5, and an efficiency of 95%. It is used to raise a load of 2 kg through a height of 15 m in 10 s. Calculate (a) the FR of the machine (b) the work done on load, and (c) the power developed.

**3.4. Force Ratio, Velocity Ratio and Efficiency of Simple Machines****3.4.1 Lever**

The lever is one of the simplest machines we know. You can overcome a large resistance by the application of a small force with it.

The lever consists of a stiff bar or rigid rod supported at the fulcrum or pivot about which it can rotate. If you apply an effort (E) at one point



of the lever, it will lift a load (L) or overcome a resistance at some other point. The diagram is shown below in Fig. 4.1

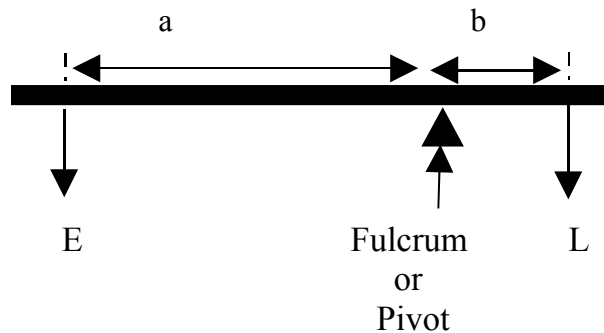


Fig. 4.1: The lever

The lever operates on the principle of moments. According to this principle, we can state from Fig. 4.1 that:

$$E \times a = L \times b \dots (4)$$

where: a is the distance of the effort from the fulcrum; b is the distance of the load from the same fulcrum.

From equation (4),

$$\frac{L}{E} = \frac{a}{b}$$

That is

$$\text{Force Ratio} = \text{Velocity Ratio}$$

where

a/b is ratio of the two arms of the lever

$$\text{That is: efficiency (Ef)} = \frac{\text{FR}}{\text{VR}} = 1$$

- **Types of Levers**

We have three classes of levers: first order, second order and third order. This classification is dependent on the relative position of the effort, load and fulcrum.

- **First Order Levers**

Here, the fulcrum (F) or pivot is between the load and the effort.



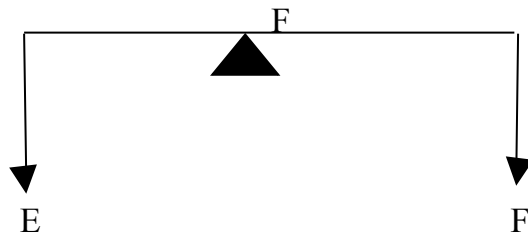


Fig. 4.2: 1st order lever

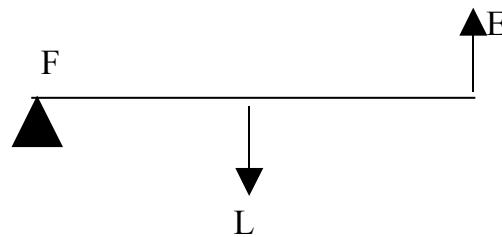
### Examples

The crowbar, a pair of scissors or pincers, claw hammer and pliers.

The velocity ratio is usually greater than 1 but could be less than or equal to 1.

- **Second order levers**

In the second order lever, the load (L) is between the effort (E) and the fulcrum or pivot.

Fig. 4.3: 2<sup>nd</sup> order lever

### Examples

Wheelbarrow and nutcrackers

The mechanical advantage and velocity ratio are always greater than 1.

- **Third order levers:**

Here, effort (E) is between load (L) and the fulcrum (F).



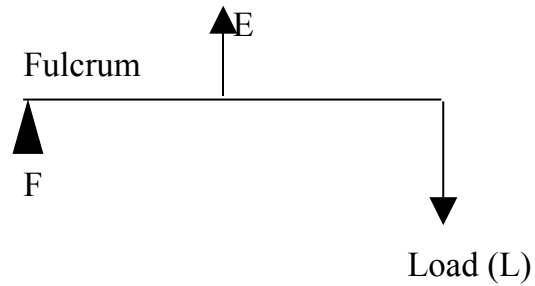


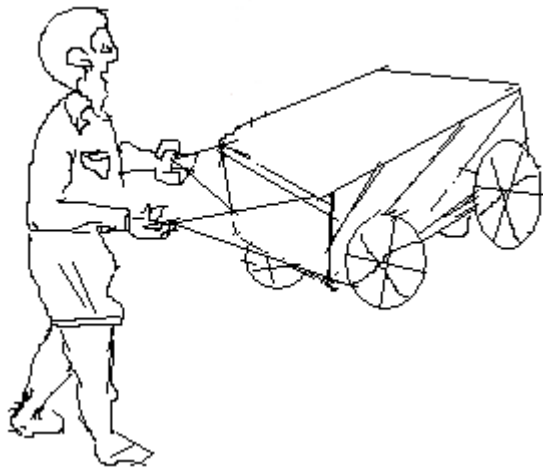
Fig. 4.4: 3<sup>rd</sup> order lever

Examples: forceps, tongs, forearm.

The force ratio and velocity ratio are less than 1.

### 3.4.2 The Inclined Plane

You have seen lorry drivers use this type of machine to raise heavy loads such as drums of oil, up a sloping plank to the high floors of L lorries. The sloping plank is an example of an inclined plane. As shown in Fig. 4.5, it is a rigid plane which is kept inclined to the horizontal at a certain angle,  $\theta$



(a)



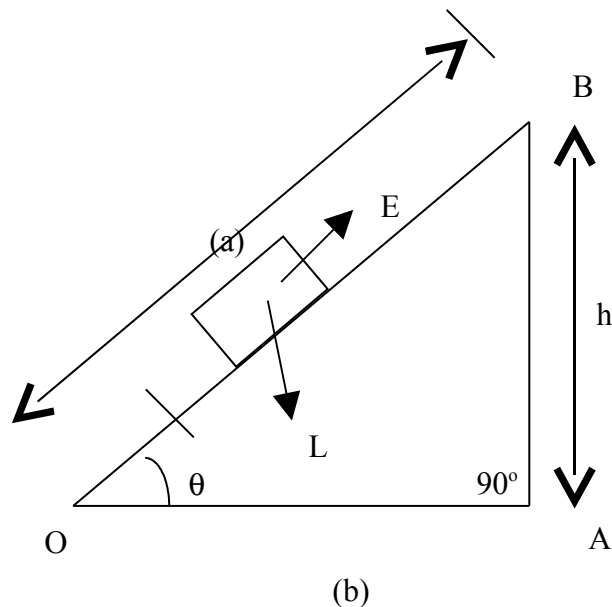


Fig. 4.5: The inclined plane (a) loading in lorry through an inclined plane (b) schematic of the inclined plane with load

As the applied force or effort E moves along OB, at an angle  $\theta$  to the horizontal, the load L is lifted up through a vertical height AB. Thus, the velocity ratio is given by:

$$VR = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$= \frac{OB}{AB} = \frac{1}{\sin \theta}$$

$VR = \frac{1}{\sin \theta}$
------------------------------

If friction forces are absent, you will have  $FR = VR = \frac{1}{\sin \theta}$ .

But friction along the plane affects the efficiency and the FR will be less than VR.

### 3.4.3 The Pulleys

The pulley system is one of the most widely used types of machine. You often see pulleys being used at construction sites to raise or lower heavy loads. As you can see, a simple pulley is a fixed wheel hung on a suitable support with a rope passing round its groove as shown in Fig 4.6.



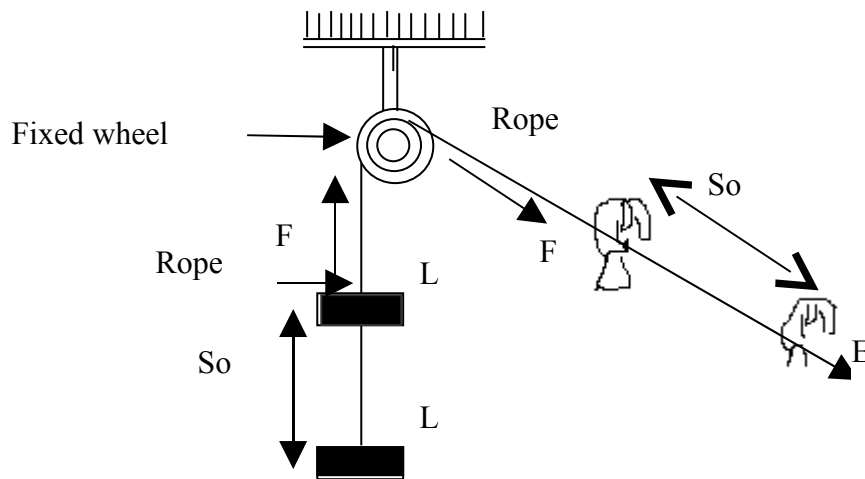


Fig. 4.6 :A simple pulley

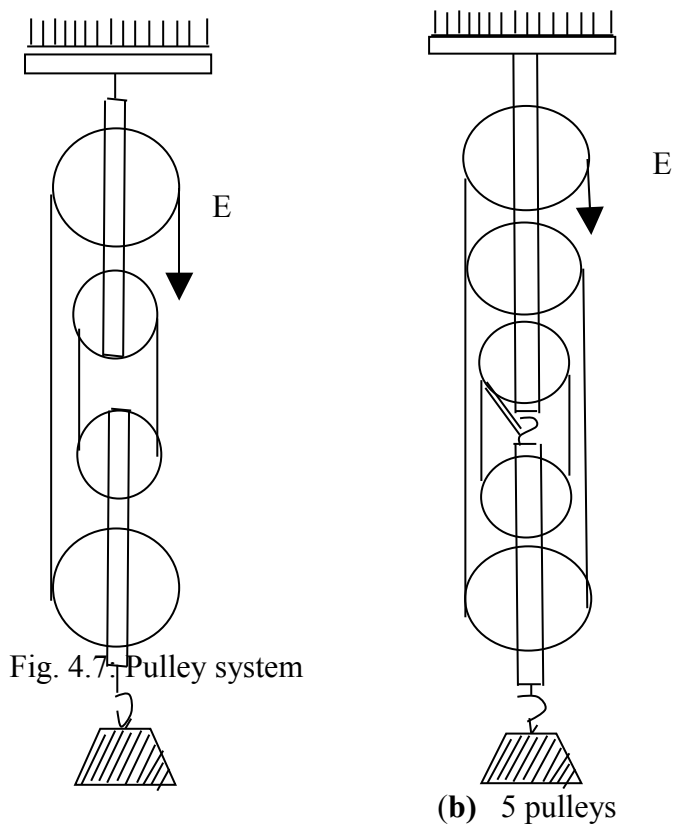


Fig. 4.7: Pulley system

(a) 4 pulleys

(b) 5 pulleys

A load  $L$  is attached at one end of the rope and an effort  $E$  is applied at the other end. If you neglect the weight of the rope and the frictional forces, the effort ( $E$ ) applied to the rope will be equal to the load lifted because the tension on the rope will be same throughout.

Hence,  $L = E$  and  $FR = 1$

Also,



The distance moved by the effort  $S_o$  = the distance moved by load  $S_o$ .

Hence,  $VR = 1$

You will realize that in practice, the effort has to lift the load (L) and also overcome the friction which hinders the pulleys from turning freely. Thus, there may be a mechanical disadvantage. However, you will still have the advantage in the use of such a pulley in that one can stand on the ground and use one's body weight in addition to his muscle to pull downwards on the rope and raise the load upwards. You know that this is easier than lifting the load directly upwards. You can also make use of such a single pulley to change the direction in which a force is to be exerted.

You can always determine the VR of a pulley system from the number of pulleys. The VR of a pulley system is to equal to the number of pulleys. For instance, in the block and tackle system of pulleys shown in figure 4.7, the VR of (a) having 4 pulleys is 4 while that of (b) having 5 pulleys is 5.

You will also note that

- The MA (FR) increases as the number of pulleys increases
- If efficiency is 100%, MA is equal to the VR, however, owing to friction and weight of pulleys, the efficiency is always less than 100%.
- Efficiency gets less as the number of pulley increases.
- With a particular pulley system, the greater the load to be lifted, the greater the efficiency. This is because the weight of the pulleys attached to one load is less in proportion to the load.



### 3.4.4 Wheel and Axle

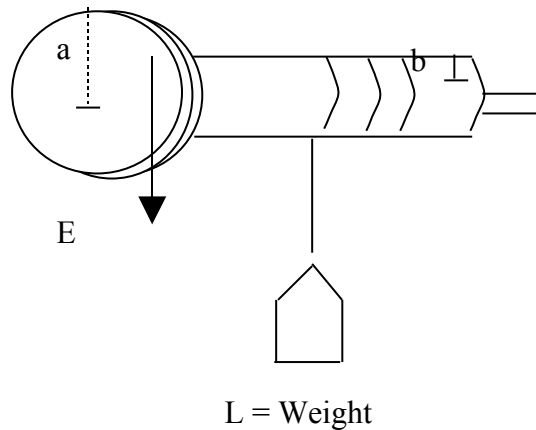


Fig. 4.8: Wheel and axle

Do you know that the wheel and axle is equivalent to a lever? The effort arm is the radius of the wheel  $a$ , and the load arm, the radius of the axle  $b$  in this case,

$$E \times 2\pi a = L \times 2\pi b$$

From which

$$\text{Mechanical advantage} = \frac{L}{E} = \frac{a}{b}$$

Here, you note that as the effort arm completes one revolution, the load is raised by the circumference of the axle.

Therefore,

$$\text{Velocity ratio} = \frac{2\pi a}{2\pi b} = \frac{a}{b}$$

Here again, in the ideal case, the efficiency is 100%. However, owing to friction, its value is less than 100% in a practical case.



### 3.4.5 The Screw Mechanisms

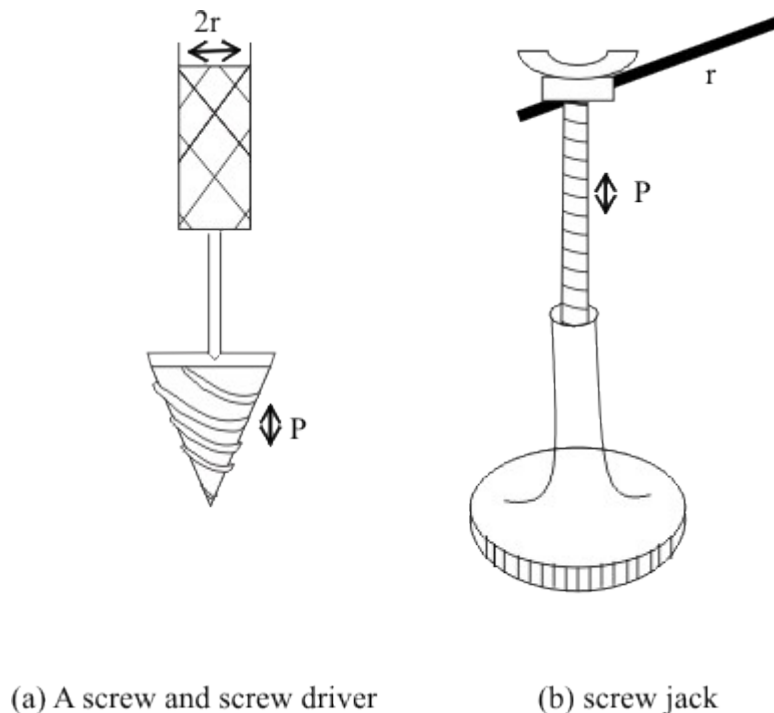


Fig. 4.9: The Screw mechanisms

You are familiar with the screw jack used to lift a car so as to change /one of its wheels. They are examples of screw mechanism (Fig. 4.9).

We call the vertical distance  $P$  between successive threads, the pitch. When you move the load through a distance  $P$ , the effort moves through a distance equal to the circumference of the handle of the screw driver in Fig. 4.9(a) or through the circumferences of the circle traced out by the lever arm.

Fig. 4.9(b)

Therefore,

$$\text{Velocity ratio} = \frac{2\pi r}{P}$$

With a high value of  $r$  compared with  $p$ , it is possible to obtain a high velocity ratio and therefore a big mechanical advantage.

In the jack, friction reduces the mechanical advantage considerable such that the efficiency is usually below 50%.



### 3.4.6 The Wedge

The wedge is a combination of two inclined planes. You may have noticed some people use it to split wood or rock into piece or to widen an opening. Examples of wedge – type of machines are axes, chisels, knives, and other cutting tools usually used by builders to separate bodies which are held together by large forces. This is shown in Fig. 4.10.

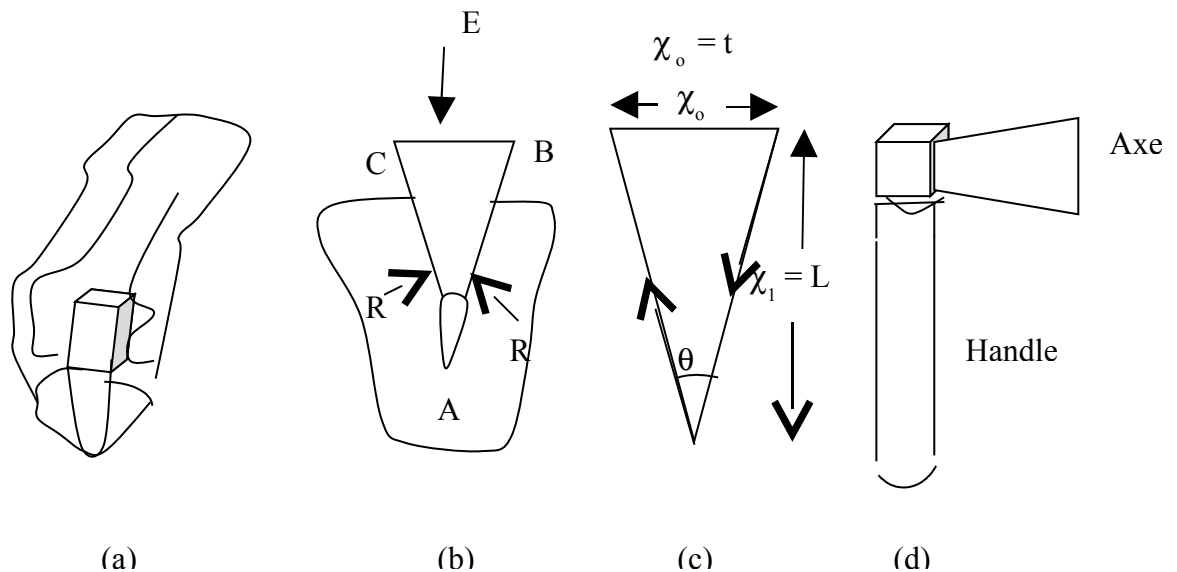


Fig. 4.10: The wedge

We noticed that when the users drive in the wedge by the effort force, a much larger force is exerted to each side by the wedge surface. Thus, as the wedge is driven in a distance  $x_1$ , the surfaces being separated are moved apart through a distance  $\chi$  as shown in Fig. 4.10.

If we take the length of the wedge to be  $L$  and its thickness to be  $t$ , then the mechanical advantage (neglecting frictional forces) is given by

$$M.A. = \frac{\text{load}}{\text{Effort}} = \frac{\chi_1}{\chi_2} = \frac{L}{t}$$

$$M. A. = \frac{\text{Slant height of wedge}}{\text{Thickness of wedge}}$$

You will note that a thin wedge has a higher mechanical advantage than a short thick one; or the smaller than angle  $\theta$  between the slant heights, the greater the M.A.



### 3.5 Simple Machines as Components of Complex Machine – The Bicycle

#### 3.5.1 The gear wheels

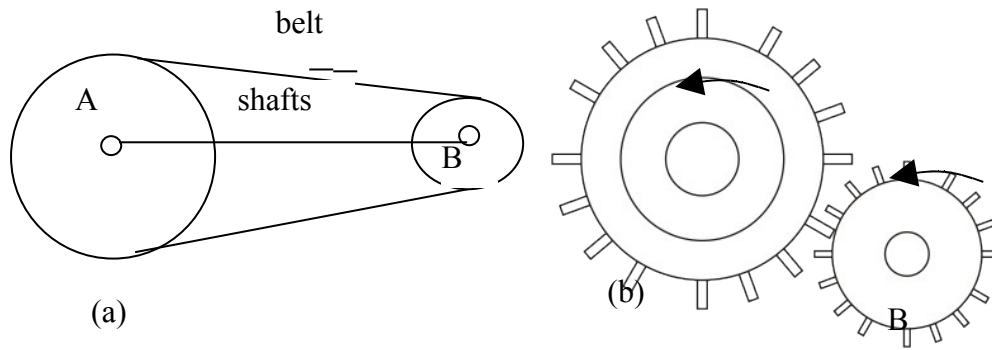


Fig. 4.11: Principle of gears

#### Principle of gears

Gear wheels are commonly used in cars, bicycles and cranes. In these machines you will apply the principle of the wheel and the axle to the two wheels (often toothed wheels) of different radii connected by a belt and rotating on separate shafts.

Here both the load and the effort are applied to the shafts and velocity ratio is given by.

$$VR = \frac{\text{number of teeth on driven wheel (A)}}{\text{Number of teeth on driving wheel (B)}}$$

#### 3.5.2 The Bicycle

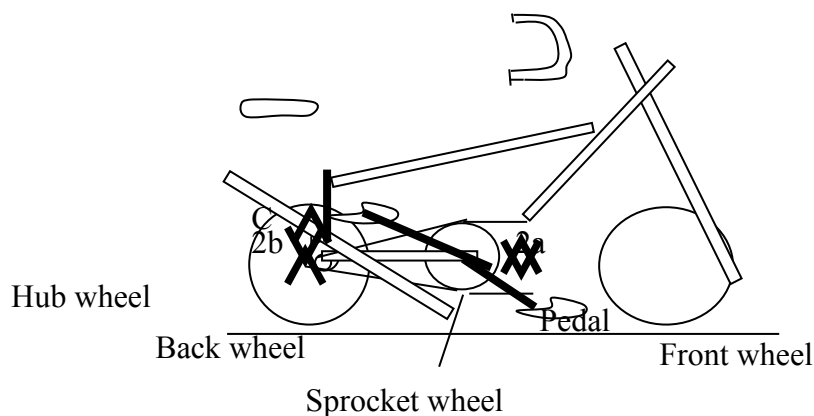


Fig. 4.12: The bicycle

You are aware that the bicycle uses two cog wheels, the sprocket wheel which is rapidly attached to the back wheel of the cycle. The cogs on the



two wheels are evenly separated. When the pedals and sprocket wheel complete a revolution, the hub wheel rotates through  $\frac{2\pi a}{2\pi b}$  revolutions.

$2\pi b$

and the back wheel does the same.

You will notice that the back wheel has moved through a distance of  $2\pi C \times a$

$b$  when the pedals complete a revolution.

We regard the friction between the tires and the ground as constituting the load while the effort is applied on the pedals.

If we take  $L$  to be length of the pedal crank, and the distance moved by effort in one revolution is  $2\pi l$ , then, velocity ratio equals

$$\frac{2\pi l}{2\pi ac} = \frac{bl}{ac}$$

The velocity ratio and the mechanical advantage (or force ratio) is considerably less than 1. But a rapid movement of the load compared with the effort is achieved.

### 3.6 Effect of Friction on Simple Machines and Methods of Reduction

In this unit, you have noted how frictional forces add to the load which has to be overcome by effort. The presence of friction between the moving parts of machines as mentioned in Module 2 unit 3.3 makes it impossible for any machine to have an efficiency of 100%. This is because part of the effort applied is used to overcome frictional forces and the remainder is utilized to do useful work. The work done in overcoming friction is 'useless' or wasted energy' and it appears as heat in the moving parts of machines.

In order to improve the efficiency of machines you will try to reduce the friction present between the moving parts of the machine by :

- 1 Lubricating which prevents metal surfaces from rubbing against each other and;
- 2 Using ball bearings or roller bearings between the two surfaces in contact.



### 3.7 Worked Examples

1. A lever of velocity ratio 25 overcomes a resistance of  $2.0 \times 10^3$  N with a force of 100N.

Calculate:

- (i) the mechanical advantage
- (ii) the efficiency of the lever

#### Solution

$$\begin{aligned}
 \text{(i) Mechanical advantage} &= \frac{\text{resistance (load)}}{\text{Effort}} \\
 &= \frac{2.0 \times 10^3 \text{ N}}{100\text{N}} = 20
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) Efficiency (E}_f\text{)} &= \frac{\text{mechanical advantage}}{\text{Velocity ratio}} \\
 &= \frac{20}{25} = 0.8 \\
 &= 80\%
 \end{aligned}$$

2. A force parallel to an inclined plane is used to raise a load of 500N through a height of 3.5m while the load moves a distance of 35m along the plane.

- (i) How much work is done on the load?
- (ii) If the efficiency is 80%, determine the effort.
- (iii) How much work is done by the effort?

#### Solution

- (i) Work done on the load = increases in P.E. of load  
( $mgh = wh$ )

$$= 500\text{N} \times 3.5 = 1750\text{J}$$

- (ii) Efficiency =  $\frac{\text{work output}}{\text{Work input}}$

$$\begin{aligned}
 \text{Work input} &= \text{Effort (E) x distance moved by effort} \\
 &= E \times 35 \text{ m}
 \end{aligned}$$



$$\text{Efficiency (E}_f\text{)} = \frac{80}{100} = \frac{1750\text{J}}{E \times 35\text{m}}$$

$$\therefore \text{Effort (E)} = \frac{1750 \text{ J} \times 100}{80 \times 35} = \frac{175000}{2800}$$

$$= \underline{62.5 \text{ N}}$$

$$\begin{aligned} \text{(iii) Work done by effort} &= \frac{\text{Work output}}{\text{Efficiency}} \\ &= \frac{1750}{0.8} = 2187.5\text{N} \\ &= \underline{2.19 \times 10^3 \text{ N}} \end{aligned}$$

## SELF ASSESSMENT EXERCISE 2

- 1
  - (a) A screw jack whose pitch is 2 mm is used to raise a motor car of mass 900 kg through a height of 20.0 cm. The length of the tommy bar of the jack is 40 cm. If the jack is 60% efficient, Calculate the:
    - (i) The velocity ratio
    - (ii) Mechanical advantage of the jack
    - (iii) Effort required
    - (iv) Work done by the effort ( $g = 10 \text{ ms}^{-2}$ )
  - (b) Explain why the efficiency of a screw jack is usually less than 100%.
2. By using a pulley system with a velocity ratio of 5, it is found that a load of 500N can be raised at a uniform speed by an effort of 125 N.
  - a What is the mechanical advantage of the machine for this load?
  - b If the efficiency of the pulley system is 80%, what effort will be required to lift a load of 10 N.

## 4.0 CONCLUSION

In this unit you learned about simple machine and its working principles. You learned the definition and meaning of terms such as effort, load, force ratio, or mechanical advantage, velocity ratio, efficiency used in connection with machines.



The simple machines you studied in the unit include the lever, the pulley, the screw, the inclined, plane, wheel and axle, the wedge.

You also learned about a simple machine as a component part of a complicated machine – the bicycle.

Finally, you solved problems arising from the unit and looked at practical effect of friction and its reduction in machines.

## 5.0 SUMMARY

- A machine is any device which allows a large weight or resistance to be overcome by a small effort.
- Mechanical Advantage or Force Ratio is defined by  $M.A = \frac{\text{Load}}{\text{Effort}}$
- Velocity Ratio V.R

$$= \frac{\text{distance moved by effort}}{\text{distance moved by load}} \quad \text{in the same time}$$

$$\begin{aligned} \text{Efficiency} &= \frac{\text{Work output}}{\text{Work input}} \times 100\% \\ &= \frac{M.A}{V.R} \times 100\% \end{aligned}$$

- M.A depends on friction but V.R is independent of friction.
- In this lever machine, V.R is the ratio of the lengths of two arms of the lever. The M.A increases with an increase in this ratio.
- In an inclined plane with  $\theta$  as the angle of inclination to the horizontal,

$$V.R = \frac{1}{\sin \theta} \quad \text{Hence V.R increases when } \theta \text{ decreases.}$$

- In the Block and Tackle pulley system, V.R is equal to the number of pulleys.
- In the screw – jack,  $V.R. = \frac{2 \pi a}{p}$

where  $a$  is the length of the tommy bar and  $p$  is the pitch.



- In the wheel and axle

$$V.R. = \frac{\text{radius of wheel}}{\text{Radius of axle}} = \frac{R}{r}$$

- In the absence of friction  $MA = VR$ .
- Friction reduces the efficiency of machines = friction is reduced by lubrication and by the use of ball or roller – bearings.

## 6.0 TUTOR-MARKED ASSIGNMENT

1. Define the terms: force ratio, velocity ratio and efficiency as applied to a machine. State the relationship between them.
2. By using a pulley system with a velocity ratio of 5, it is found that a load of 500N can be raised at a uniform speed by an effort of 125N.
  - (a) What is the mechanical advantage of the machine for this load?
  - (b) If the efficiency of the pulley system is 80% what effort will be required to lift a load of 10 newtons?
3. A wheel and axle is used to raise a mass of 50 kg by the application of an effort of 250N. If the radius of the wheel and axle is 300 mm and 100 mm respectively, determine the efficiency of the machine. (Take  $g = 10 \text{ ms}^{-2}$ ).

## 7.0 REFERENCES/ FURTHER READINGS

- Anyakoha, M.W.(2000). New School Physics for Senior Secondary Schools. Onisha: Africana FEP.
- Awe, O. and Okunola, O.O. (1992). Comprehensive Certificate Physics. Ibadan: University Press.

## MODULE 3



Unit 1	Mechanical Energy
Unit 2	Elastic Properties of Solids
Unit 3	Elastic Properties of Solids
Unit 4	Measurement of Temperature
Unit 5	Transfer of Heat and Heat Capacities
Unit 6	Latent Heat and Evaporation
Unit 7	Expansion of Gases

## **UNIT 1      MECHANICAL ENERGY**

### **CONTENTS**

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### **1.0      INTRODUCTION**



This unit introduces you to the concepts of work, energy, power and their measurements. You will learn the forms and sources of energy. Furthermore, you will learn in depth about mechanical energy, its types and transformation. Finally, you will learn to treat mechanical energy, work and power quantitatively.

## 2.0 OBJECTIVES

By the time you finish this unit, you will be able to:

- explain work, energy and power and give examples of each
- calculate
  - the work done, given a force and displacement it produces in its direction
  - the gravitational potential energy at a height  $h$  above a given reference plane
- calculate the power in watts given an applied force and the time it takes to produce a displacement
- identify the type of energy possessed by a body under given conditions
- distinguish between kinetic and potential energy
- identify energy transformation from one form into another
- state the law of conservation of energy
- calculate the kinetic energy of a body
- calculate the potential energy of a body
- show that the total energy is conserved for a given set of data on the energy of a particle in a conservative field
- verify the conservation of energy principle.

## 3.0 MAIN CONTENT

### 3.1 Forms of Energy and World Energy Sources

#### 3.1.1 Concept of Energy

Energy is the mainspring of all life and of all the activities of mankind. It may be defined as an ability or capacity to perform work. You use energy in your home in the following ways:

- Chemical energy stored in the food you eat helps you in walking, playing, doing household work and other duties.
- Chemical energy in wood or coal or kerosene is converted by combustion to heat energy for cooking your food, and heating purposes.



- Light energy that falls on your eyes enables you to see things;
- Sound energy that enters your ears makes you hear;
- Electrical energy operates most appliances such as fan, TV sets, radios, electric pressing irons, refrigerators and other appliances in your home.

### 3.1.2 Energy Sources

You know that the sun is the main source of energy in the universe. However, we can identify other sources which we have classified under:

- Natural sources which include food, sun, wood, coals, natural gas, oils and fats, tides/waves, wind, waters.
- Manufactured sources include energy from batteries, generators, electricity from the mains.
- Petroleum products include that from kerosene, gasoline and refinery gases (methane, ethane, propane and butane).

### 3.1.3 Forms of Energy

You are aware that energy can exist in different forms. Among these are:

- Chemical energy (stored in food and other fuels)
- Kinetic energy (associated with motion).
- Gravitational potential energy (associated with height above the ground).
- Elastic potential energy (stored in springs, rubbers, stretched bows, catapults).
- Nuclear energy (stored in nucleus of atoms).
- Heat energy (flows from one point to another as a result of temperature difference between the point).
- Sound energy (produced by vibrating objects – violins, guitars, piano, drums).
- Light energy (produced by light bulbs)
- Mechanical energy (associated with machines at work)
- Electrical energy (taken in by TV set and other electrical appliances).
- Magnetic energy (associated with magnetism)
- Solar energy (associated with the sun).

## 3.2 Concept and Measurement of Work

### 3.2.1 Concept of Work



In every day life, different people apply the term work to any form of activity that requires the exertion of physical or mental effort. However, in Physics (or in mechanics), we use the term in a very specific sense. We say that work is done whenever a force is applied to cause a body to move.

When you push a car a certain distance, you are said to do work. When no movement takes place after you have applied a force to a body such as a car, no work is done.

Work is said to be done whenever a force moves its point of application a distance in the direction of the force.

### 3.2.2 Measurement of Work

You will measure work by obtaining the product of force moving a body and the distance moved by the body in the direction of the force. That is, if a force  $F$  moves a body a distance  $S$  in the direction of the force, the work done is given by:

$$\begin{array}{lcl} \text{Work} & = & \text{Force} \times \text{displacement} \\ \text{i.e. } W & = & F \times S \end{array} \quad \text{-----(1)}$$

The S.I. unit of work is Joule (J)

The Joule is equivalent to Newton – metre (Nm).

In general, when a constant force  $F$  at an angle  $\theta$  = with the direction of motion causes a body to move a distance,  $S$ , we define the work done by

$$W = F \cos \theta \times S \quad \text{----- (2)}$$

where  $F \cos \theta$  is the component of  $F$  in the direction of motion.

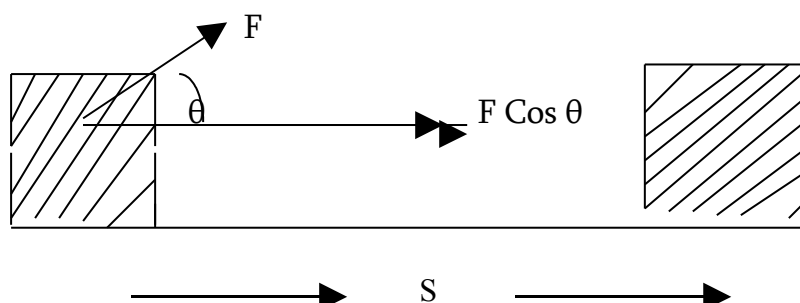


Fig .1.1:  $W = F \cos \theta \times S$

Work is a scalar quantity

You can also obtain equation (2) by resolving  $S$  in the direction of force



i.e.  $S \cos \theta$

$$\Rightarrow W = F \cos \theta \times S \text{ or } F \times S \cos \theta \quad (3)$$

When the force and the motion are in the same direction,  $\theta = 0$ ;

$$\begin{aligned} W &= F \times S \cos 0 \\ &= F \times S \end{aligned} \quad (4)$$

But, if the force is at right angles to the displacement, it has no component in the direction of the displacement. Hence,

$$F \cos 90 = 0; \text{ work done} = 0.$$

### 3.2.3 Work Done in a Force Field

- (a) Lifting a body
- (b) Falling bodies

#### (a) Lifting a body

You know that to lift a body through a height  $h$ , a pulling force must be applied to overcome the weight of the body. When an object is lifted upwards vertically, work is done against the force of gravity or against the weight of the body. You will obtain the magnitude of the work done from

$$\begin{aligned} \text{Work} &= \text{force} \times \text{distance} \\ &= mg \times h \\ &= mgh \end{aligned} \quad (5)$$

where  $m$  = mass of the body,

$g$  = acceleration due to gravity

$h$  = height

#### (b) Falling bodies



When a body falls freely under gravity, we say that the earth's gravitational force does work on the body. For instance, if a body of mass  $m$ , falls through a vertical distance  $h$ , the work done by gravity on the body is obtained from.

$$W = mgh.$$

Also, if a body of mass  $m$  rolls down a hill of height  $h$  and length  $l$ , the work done is  $mgh$ . In this case, you will still use the vertical height ( $h$ ) in the calculation and not the length of the slope  $l$

### Worked examples

1. A body of weight 200 N falls freely through a height of 10 m, what is the work done?

#### Solution

$$\text{Work done} = mgh$$

$$\text{Weight (force)} = 200\text{N, height} = 10 \text{ m}$$

$$\text{Work done} = 200\text{N} \times 10 \text{ m} = \underline{2000 \text{ J}}$$

- (2) A boy lifts a load of weight 200N through a vertical height of 1 m before putting it on his load. When he finds that the load is too heavy to carry he quickly drops the load in its original place. What is the work done by the boy ?

#### ∴ Solution

$$\text{Work done} = \text{zero.}$$

Reason: No displacement.

- (3) A load is pulled 7 m along a horizontal floor by a constant force of 30N which acts at  $40^\circ$  to the horizontal.

Calculate the work done by the force.

#### Solution



$$\text{Work done} = F \cos \theta \times S$$

$$F = 300\text{N}; \theta = 40^\circ ; S = 7 \text{ m}$$

$$\begin{aligned} \therefore \text{Work done} &= 30 \cos 40^\circ \times 7 \\ &= 30 \times 0.766 \times 7 \\ &= \underline{160.87\text{J}} \end{aligned}$$

### 3.3 Concept and Measurement of Energy

#### 3.3.1 Concept of Mechanical Energy

We have already said that anything capable of doing work has energy. This means that a person pushing a car along the road is doing work on the car. So far as the car moves some distance. A mango fruit falling from the top of the tree does work and therefore possess energy.

Can you mention other situations in which the application of force cause a body to move. All of them possess energy.

There are many forms of energy but here, you will concentrate on the study of mechanical energy.

Mechanical energy is classified into two types namely Potential Energy and Kinetic Energy.

#### 3.3.2 Potential Energy (P.E)

Potential energy is simply ‘stored energy’ or energy possessed by a body by virtue of its position or state.

The stored energy is used to do work when the body is free to move  
Examples of potential energy are:

- a magnet at rest in a magnetic field (magnetic potential energy).
- An electric charge at rest in an electric field (electric potential energy).
- a coiled spring when stretched or compressed (elastic potential energy).
- petrol, wood, and other fuel sources burn when they burn, chemical potential energy is released,
- a body at rest in a gravitational field e.g. a mango fruit on a mango tree, gravitational potential energy.



### 3.3.3 Kinetic Energy (K.E)

Kinetic energy is the energy possessed by a body by virtue of its motion.

Examples of kinetic energy are;

- a student running a race.
- an object falling freely under gravity.
- electrical charges in motion.
- any object in motion.

### 3.3.4 Measurement of Energy

- (i) Potential energy (P.E)
- (ii) Kinetic energy (K.E)

#### (i) Measurement of Potential Energy (P.E)

Gravitational potential energy as you know depends on the weight and height of an object above the surface of the earth. To lift an object at a constant speed under gravity, you must exert a force, which is equal to its weight. The work you do when you lift a body of mass  $m$  at a uniform speed through a height  $h$  is  $mgh$ . The work done in lifting the body to a height  $h$ , is stored and can be recovered in the form of kinetic energy by allowing the body to fall through a distance  $h$ . The potential energy of a body is not an absolute quantity. You will measure its magnitude relative to some reference position.

To measure the gravitational potential energy P.E of a body, you will measure the mass of the body and its height above a reference level  $h$ . This is given by

$$P.E = mgh.$$

The potential energy of an object in other force fields which act on the object is calculated in much the same way. First, you will select a reference point and then obtain the potential energy at some other point as the work done in moving from the reference point to the position of the body in the force field.

#### ii Measurement of Kinetic Energy (K.E)



The kinetic energy of a body in motion depends on both its mass  $m$  and speed  $v$ . To obtain the kinetic energy of a body in motion you will use the formula

$$K.E = \frac{1}{2} mv^2 \dots\dots\dots(6)$$

For you to obtain a value for the kinetic energy of a body you will know its mass and speed. For instance, suppose an object of mass  $m$  moves with a horizontal velocity  $u$  and is gradually brought to rest in a distance  $S$  by a constant horizontal force  $F$  acting on it. The kinetic energy of the body has been used in doing work against the opposing force  $F$ . Therefore, kinetic energy originally possessed by the body

$$= \text{work done against } F$$

$$= F \times S$$

But  $F = ma$

where  $a$  = retardation of the object.

Hence  $F$

$$F \times S = Mas$$

You will recall from the relation

$$V^2 = U^2 + 2as$$

That since  $v = 0$ ,  $a$  is negative (i.e. retardation)

Therefore,

$$V^2 = U^2 + 2as \text{ becomes } 0 + u^2 - 2as \\ \Rightarrow \frac{U^2}{2} = as$$

Also,

$$KE + F \times S = mas$$

You can now substitute  $\frac{1}{2} u^2$  for  $as$  in the above. When you do that,

K.E becomes

$$K.E = F \times S + mas = \frac{1}{2} mu^2$$

$$\text{ie } K.E = \frac{1}{2} mu^2 \dots\dots\dots(7)$$

$$\text{kinetic energy} = \frac{1}{2} \times \text{mass} \times (\text{velocity})^2$$



### Worked Examples

1. A bullet of mass 0.5 kg was fired with a speed of 300 m/s. Calculate its kinetic energy.

#### Solution

$$\begin{aligned}
 \text{K.E} &= \frac{1}{2} mv^2 & (m = 0.5 \text{ kg, } v = 300\text{ms}^{-1}) \\
 &= \frac{1}{2} \times 0.5 \times (300)^2 \\
 &= 22500\text{J} \\
 &=====
 \end{aligned}$$

2. A body of mass 50 kg is raised to a height of 15 m. What is the potential energy of the body ( $g = 9.8 \text{ m/s}^2$ ).

#### Solution

$$\begin{aligned}
 \text{P.E} &= mgh \\
 &= 50 \times 9.8 \times 15 \text{ (} m = 50 \text{ kg, } g = 9.8 \text{ ms}^{-1} \text{ } h = 15\text{m)} \\
 &= 7350\text{J} \\
 &=====
 \end{aligned}$$

### 3.4 Concept and Measurement of Power

Let us consider two boys Chika and Ali who are carrying an equal load mb each from the ground floor to the first floor of a building. The amount of work done by Chika and Ali in lifting is exactly the same. However, if Chika accomplishes the task before Ali, we say that Chika works at a faster rate than Ali. That is, Chika has more power than Ali.

Also, you do the same amount of work in lifting your weight when you climb up a staircase 10 metres high either in 15 seconds or in 25 seconds. But, you do not work at the same rate. You work faster when you climb it in 15 second ss than in 25 second. In Physics, the work done in unit time is called power.

- Power is defined as the rate of doing work or using energy.

$$\text{Power} = \frac{\text{Work done or energy expended}}{\text{Time taken}}$$



Usually, work is measured in joules and time in seconds, so, power is expressed in joules per second. One joule per second = 1 watt. Therefore, work is measured in watts (W).

A power of one watt is too small for practical use. So, we usually measure power in a larger unit called a kilowatt (KW).

### Worked Example

1. A boy of mass 25 kg walks up a flight of stairs 8 metres high in 32 seconds, calculate the power of the boy ( $g = 9.8 \text{ ms}^{-2}$ )

### Solution

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

$$\text{Work done} = mgh = 25 \text{ kg} \times 9.8 \text{ ms}^{-2} \times 8 \text{ m}$$

$$\text{Time taken} = t = 32 \text{ s.}$$

$$\therefore \text{Power} = \frac{25 \times 9.8 \times 8}{32} = \frac{1960}{32} = 61.25$$

W =====

## 3.5 Transformation and Conservation of Mechanical Energy

### 3.5.1 The Law of Conservation of Energy

The law of conservation of energy states that in an isolated or closed system, the total amount of energy is always constant, although energy may be changed from one form to another.

When we talk about an isolated or closed system, we mean a group of objects that neither receives energy from nor gives energy to objects outside the system.

For mechanical energy being considered here, the law simply shows that the sum of the P.E. and K.E is always constant for a given body, but the energy may change from P.E to K.E or from K.E to P.E.

### 3.5.2 The Principle of Conservation of Mechanical Energy in a Conservative Field



You will note that the above principle is true in a conservative field. An important aspect of a conservative force is that the work done by such a force is recoverable. For example, if you do a work against a gravitational force in raising a body to a height  $h$  above the ground, that work is recovered when the body falls a distance  $h$  back to its original position.

The earth's gravitational field is an example of field containing conservative forces. In such a conservative field, total mechanical energy is conserved.

Friction is an example of a non-conservative force. We noticed that when friction acts between the moving parts of a machine, some mechanical energy is lost but it reappears in the form of heat and the total energy which is now mechanical plus thermal energy remains constant.

The general principle of the conservation of energy states that the total energy in a given system is always constant or energy can neither be created nor destroyed although it can be transformed from one form to another.

### 3.5.3 The Conservation of Mechanical Energy of a Falling Body

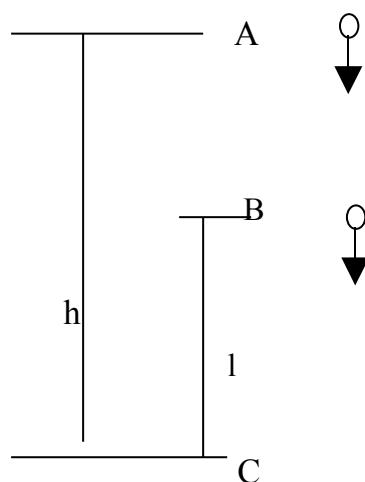


Fig.1. 2: Conservation of mechanical energy

Here, let us consider a body of mass  $m$  at rest at a point A, of height  $h$  above the ground level. As the body falls, its PE changes into KE. What it loses in PE is equal to what it gains in KE.

At A the total energy is all PE and is given by  $mgh$ . There is no KE because the body is at rest.



$$\text{Total energy at A} = mgh$$

If the ball falls to another point B at height  $l$  above the ground,

$$\text{P.E at B} = mgl$$

and

$$\text{K.E at B} = \frac{1}{2} mv_B^2$$

$$\begin{aligned} V_B^2 &= u^2 + 2gS \\ &= 0 + 2g(h-l) \end{aligned}$$

$$\therefore \text{K.E} = \frac{1}{2} mv_B^2 = \frac{1}{2} m \times 2g(h-l) = mg(h-l)$$

$$\text{Total energy, P.E} + \text{K.E} = mgl + mg(h-l)$$

$$= mgh$$

At C, the lowest level,  $h = 0$

$$\text{P.E} = mg \times 0 = 0$$

$$\text{K.E} = \frac{1}{2} mv_C^2$$

Usually  $v_C^2 = u^2 + 2bs$ , we have

$$V_C^2 = 0 + 2gh$$

$$\text{K.E} = \frac{1}{2} m \times 2gh = mgh$$

$$\text{Total energy} = \text{P.E} + \text{K.E} = mgh$$

As you have seen, the total energy of the body at each point A, B, or C is a constant ( $mgh$ ) although the energy is all potential at A, partly potential and partly kinetic at B and all kinetic at C. The total mechanical energy is therefore constant or conserved in a conservative field.



### 3.5.4 The Conservation of Mechanical Energy Involved in a Swinging Pendulum

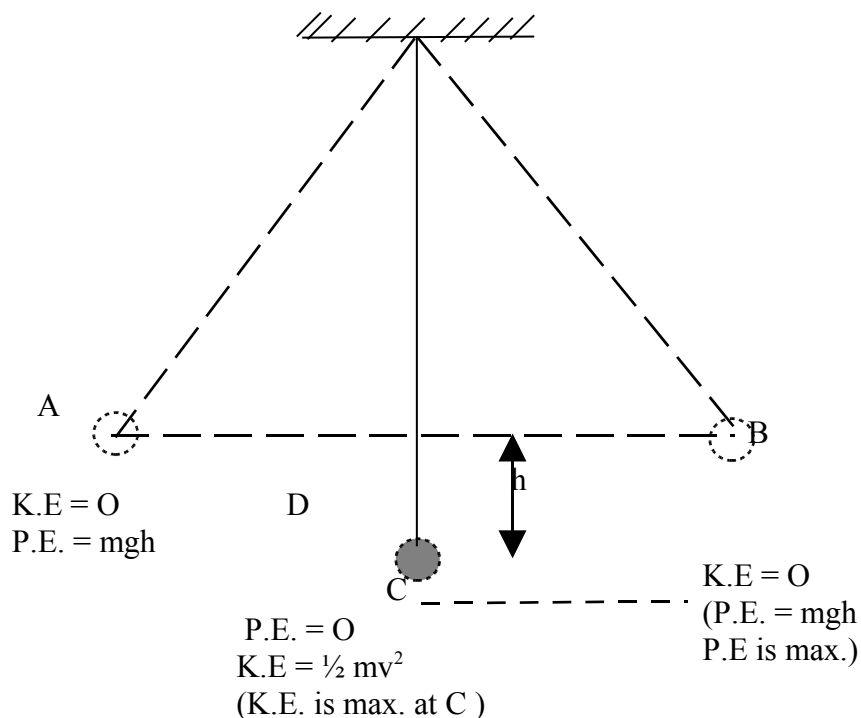


Fig. 1.3: Energy transformation in the motion of a simple pendulum

Now we will consider energy transformation in the motion of a swinging pendulum. A simple pendulum consists of a bob attached to a string the other end of which is suspended from a firm, a support. When the pendulum swings from end to end, the energy of the system changes from potential to kinetic and vice versa. At each stage of the swing, the total energy remains constant.

The pendulum swings from the highest point A through the centre of the swing C to the other highest point B,

At C, the bob is at the lowest position. Therefore, the P.E of the system is zero. Also at C, the speed of the pendulum is maximum and so the K.E is maximum at this point. That is,

At C,

$$\begin{aligned} \text{P.E} &= 0 \\ \text{K.E} &= \frac{1}{2}mv^2 \quad (\text{maximum}). \end{aligned}$$

As the bob moves from C to B, the K.E at C is gradually transformed to P.E with the P.E becoming maximum at B which is at a height h above C.



At the B, the total energy is potential energy and is equal to  $mgh$ . This is also the P.E at A which is at the same height above C as B. That is,

At B and A,

$$P.E = mgh \text{ (max)}$$

$$K.E = 0$$

You can find the maximum velocity of the swinging bob that occurs as the bob passes through C. If you take this Velocity to the  $v_{\text{max}}$ , since total energy is always conserved, you will have:

$$\frac{1}{2} mv_{\text{max}}^2 = mgh$$

$$(K.E \text{ at C}) = (P.E \text{ at A or B})$$

Therefore,

$$V_{\text{max}}^2 = 2 gh$$

$$V_{\text{max}} = \sqrt{2 gh} \dots\dots\dots (8)$$

You have seen that at A and B, the energy is all P.E. At C, energy is all K.E. At any intermediate point such as D, the energy is partly potential and partly kinetic. At each point however, the total energy (P.E + K.E) is constant and is equal to  $mgh$ .

Here also, you have seen that the total energy of the body remains constant. Although there is transformation of energy from P.E to K.E, but, there is no loss or gain in its total energy. This confirms the law of conservation of energy.

However, you will observe from experience that a pendulum bob left swinging on its own will eventually come to rest after some time. This is because its energy is gradually used up in moving the air around it. That is, in doing work against air resistance.

### Worked example

A stone of mass 500 g is thrown vertically upwards with a velocity of  $30 \text{ ms}^{-1}$

Find (a) the potential energy possessed by the stone at its greatest height; (b) the kinetic energy of the stone just before it hits the ground.

$$(g = 9.8 \text{ ms}^{-2})$$



**Solution**

(a) Using  $v^2 = u^2 + 2 gS$

where  $v = 0 \text{ ms}^{-1}$  (stone momentarily at rest).

$g = -g$  (because it acts in opposite direction to that of  $u$ )

When you substitute for  $v$  and  $g$  in the equation above, you obtain

$$0 = u^2 + 2(-g)S$$

$$0 = u^2 - 2gS$$

$$2gS = u^2$$

$$S = \frac{u^2}{2g}$$

$$U = 30 \text{ ms}^{-1}, g = 9.8 \text{ ms}^{-2}$$

and

$$S = \frac{(30 \text{ ms}^{-1})^2}{2 \times 9.8 \text{ ms}^{-2}} = 45.92 \text{ m}$$

This is the greatest height reached by the stone. Therefore, the potential energy of the stone at a height,  $h$ , of 45.92 m is given by

$$\text{Potential energy} = mgh$$

$$= 0.5 \text{ kg} \times 9.8 \text{ ms}^{-2} \times 45.92 \text{ m}$$

$$= 225 \text{ J}$$

The potential energy possessed by the stone at its greatest height is 225J

The kinetic energy of the stone just before it hits the ground equals the potential energy possessed by the stone at its greatest height i.e. 225 J. This is also the potential energy of the stone in falling through a height 45.92 m to the ground.



### 3.6 Pendulum

#### 3.6.1 Applications of Mechanical Energy

Mechanical energy is applied in the working principles of machines where forces move over distances. You learned about this extensively in Module 2 Unit 4.

#### SELF ASSESSMENT EXERCISE 1

- 1 Define the terms work, energy and power.
- 2
  - a. State the principle of the conservation of mechanical energy.
  - b. How is the law illustrated by
    - (i) the swing of a simple pendulum?
    - (ii) the energy of a falling orange?
3. A cable car is pulled up a slope by a constant force of 5000 N at a uniform speed of 6 m/s. It takes 4 minutes to complete the journey.
  - (a) How much work is done in getting the car to the top of the slope?
  - (b) How much work could be done if the speed were 12 m/s (the force remaining the same)?
  - (c) How does the power developed in (a) compare with (b)?

### 4.0 CONCLUSION

In this unit you learned about work, energy and power. You also learned in depth mechanical energy and its conservation.

### 5.0 SUMMARY

You have learned in this unit that

- Work is Force (F) x Displacement(S) in the direction of force. If F and S are inclined at an angle  $\theta$ , then work done (W) =  $Fs \cos \theta$ . It is measured in joules.
- Energy is capacity to do work. It is measured in joules.
- Mechanical energy can be classified as kinetic energy and potential energy



- Potential Energy is the energy due to position.  $P.E = mgh$ ;
- Kinetic energy is the energy due to motion.  $K.E = \frac{1}{2} mv^2$
- Power is the rate at which work is done. It is also the rate at which energy is expended. It is measured in watt.
- The total energy in the universe is constant.
- The principle of the conservation of mechanical energy states that in a closed or isolated system, the total mechanical energy is always constant, although energy may change from P.E to K.E or vice versa.

## 6.0 TUTOR-MARKED ASSIGNMENT

1. What kind of energy is possessed by:
  - (a) the wheel of a moving motor car?
  - (b) a man standing on the roof of a car?
  - (c) cooking gas?
  - (d) a ball rolling down a slope?
  - (e) Music
2.
  - (a) In every day life we say that you have great power if you have great strength or much authority such as a senate president. Is the scientific meaning of power the same as its everyday meaning? Give reasons for your answer.
  - (b) Friction is often used to slow down objects. Is friction used for this purpose doing work? Give a reason for your answer.
3. A boy weighing 200 N climbs a staircase that is 20 m long and reaches a height of 8 m. How much work has he done to raise his height. What happens to his kinetic energy at the end of the climb? (Take  $g = 9.8 \text{ ms}^{-2}$ ).
4. If you weigh 75 kg and in 5s you can run up a flight of stairs consisting of 36 steps, each 15 cm high, what is your power?  
(Take  $g = 9.8 \text{ ms}^{-2}$ )
5.
  - (a) State the Principle of Conservation of Mechanical Energy
  - (b) On what factors do the measurement of
    - (i) kinetic energy
    - (ii) Potential energy of a body depend.

## 7.0 REFERENCES/FURTHER READINGS



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## UNIT 2 ELASTIC PROPERTIES OF SOLIDS

### CONTENTS

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- 2.0 Objectives
- 3.0 Main Content
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  - 3.2 Verification and Demonstration of Hook's Law
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  - 3.4 Young's Modulus of Elasticity
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  - 3.6 Elastic Potential Energy
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### 1.0 INTRODUCTION

In Module 1 unit 2, we studied the particle nature of matter and stated that particles that make solids are closely packed together and held in relatively fixed positions by strong intermolecular forces. As a result of these forces, it is not easy to dislodge the molecule of a solid because any external force applied on the solid is stiffly resisted by the intermolecular forces. In this unit, we shall study the behaviour of those solids which change their shape or size when a sufficiently strong external force is applied to them and return to their original shape or size when the applied force is removed. Specifically we shall study Hooke's law which governs the relationship between the stretching force and the extension produced by it in an elastic material. We shall also discuss Young's modulus of elasticity and study the work done in stretching or compressing spring and elastic strings.

### 2.0 OBJECTIVES

By the end of this unit you will be able to:

- explain the elastic properties of substances
- state Hooke's law and explain the meaning of the terms such as stress, strain, elastic limit, young's modulus
- describe and carry out an experiment to verify Hooke's law



- draw a graph of load versus extension for an elastic material
  - before the elastic limit is reached, and
  - beyond the elastic limit
- solve simple problems on Hooke's law and on work done in stretching or compressing a spring and elastic string.

### 3.0 MAIN CONTENT

#### 3.1 Concept of Elasticity

Solids tend to change their shape or size when sufficiently strong external forces are applied to them and to return to their original shape or size after the forces causing the change are removed. Solids which retain their shape or size after the force causing the change has been removed are said to be 'elastic', and this property of solids is called elasticity.

##### Definitions

- (1) Elasticity is the ability of a substance to regain its original shape and size after being distorted by an external force.
- (ii) An elastic material is one that regains its original shape and size after the distorting external force has been removed.

##### 3.1.1 Hooke's Law

##### 3.1.2 Statement of Hooke's Law

Hooke's law states that, provided the elastic limit of an elastic material is not exceeded, the extension,  $e$ , of the material is directly proportional to the applied force,  $F$ .

Mathematically,

$$\begin{array}{l} F \propto e \\ \text{i.e. } F = K e \dots\dots\dots (1) \end{array}$$

where  $k$  is the constant of proportionality called elastic constant or force constant or stiffness of the material.

$$\text{From (1), } K = \frac{F}{e} \dots\dots\dots (2)$$

If  $F$  is in newtons and  $e$  in metres, then  $K$  is in newtons per metre ( $\text{Nm}^{-1}$ )



**Definition:**

Elastic constant or stiffness of an elastic material is the force required to produce unit extension of the material.

The working of the spring balance is based on Hooke's law. In this case  $F = mg$ , the weight of the body which is proportional to  $e$ , the extension of the spring.

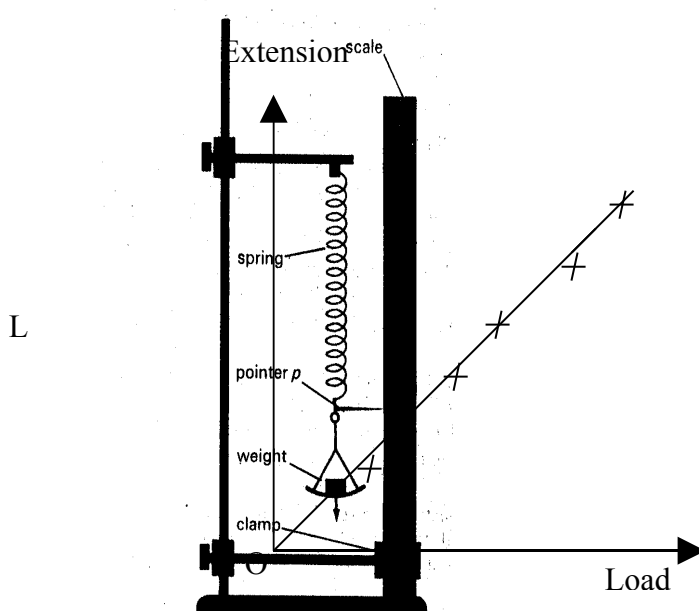
**3.2 Verification or Demonstration of Hooke's Law**

Figure 2.1a

Fig. 2.1b: Graph of extension against load.  
Hook's law**Fig.2.1:** Verification or demonstration of Hook's law

You can verify or demonstrate Hooke's law using experimental set up as shown in Fig.2.1 as follows:

- (i) Obtain a spiral spring and attach one end of it to a firm support. To the other end attach a scale pan which has been previously weighed.
- (ii) Attach a pointer, P, to the spring with plasticize and arrange vertical scale over which the pointer movement can be read. Set up the whole apparatus as shown in the Fig. 2.1.a
- (iii) Note the initial pointer reading on the scale and call it  $\ell$ .



- (iv) Add weights to the scale pan in equal steps of about 0.05 kg and note and record the pointer reading for each increasing load. Note that the total load stretching the spring in each instance is the weight in the scale pan plus the weight of the scale pan.
- (v) Remove the load from the scale pan in equal steps also and note the record the corresponding pointer readings.
- (vi) Record your readings as shown in the table below:

	Scale reading			
Load	Load increasing	Load decreasing	Average reading	Extension

- (vii) Plot a graph of extension against load. If the total maximum load has not extended the spring permanently, the graph obtained will be a straight line graph passing through the origin (Fig.2.1b). Since the graph is a straight line, it means that the extension of the spring is directly proportional to the load. Hence, Hook's law is verified.

### SELF ASSESSMENT EXERCISE 1

- 1 Explain what is meant by (a) elasticity, and (b) elastic material?
- 2 State Hooke's law and show how the law leads to the idea of stiffness of the elastic material. State the unit (S.I.) in which stiffness is measured, showing clearly how this unit may be derived.
- 3 Describe an experiment to verify Hooke's law using a graphical method.



- 4 Suppose you performed Hooke's law experiment and obtained a set of readings of loads and their corresponding extensions. Sketch the graph of;
- Extension against load,
  - Load against extension.

State two important similarities and one difference between the two graphs.

### 3.3 Graph of Extension against Load for a Permanently Strained Elastic Material

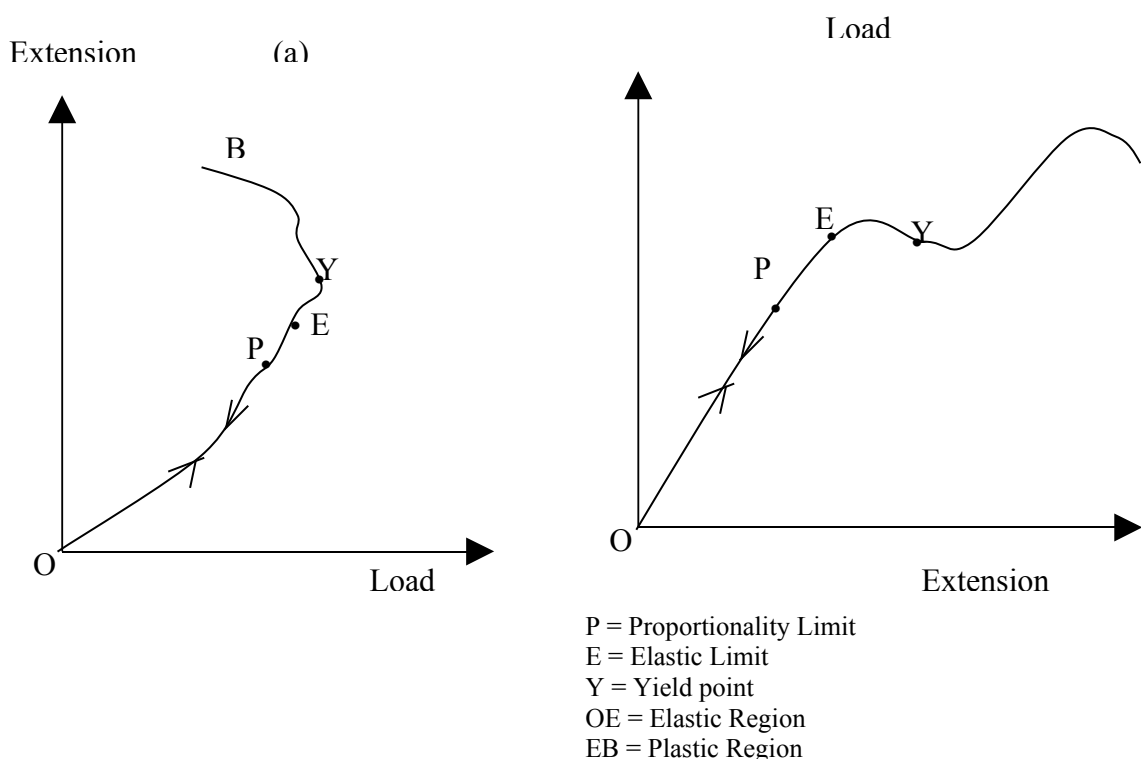


Fig. 2.2: Graphs of (a) extension against load (b) load against extension up to breaking point

If, in the Hooke's law experiment, you continue to increase the load until the spring wire snaps or breaks, the graph of extension against load will be as shown in figure 2.2 above. The graph is a straight line up to OP, followed by the curved part PEYB which rises slowly initially and then sharply.

Along OP, the straight portion of the graph, Hooke's law is obeyed and the extension is proportional to the load.



P is called the ‘proportionality limit’. However, up to the point E (slightly above P) which is called the ‘elastic limit’. The spring will return to its original length if the applied force is removed.

### Definition

The elastic limit of an elastic material is the limit of force beyond which the stretched material does not return to its original length when the stretching force is removed.

Beyond the elastic limit, E, the material loses its elasticity. The region OE from the origin to the elastic limit E is called the ‘elastic region’. The difference between the ‘proportionality limit’ P and the ‘elastic limit’ E is that, strictly speaking, Hooke’s law holds up to P, but elasticity holds up to E.

Beyond E, the wire stretches rapidly with increasing load and then enters the ‘plastic region’ which begins at E. In this ‘plastic region’ the wire does not return to its original length upon removal of the stretching force. At Y, the ‘yield point’ the material is said to have ‘yielded or lost all elasticity permanently and has become ‘plastic’.

### Definition

Yield point is the point beyond the elastic limit in which the elastic material has yielded all its elasticity permanently and has become plastic.

The difference between the “elastic region” (OE) and the “plastic region” (EB) is that in a latter, a sudden and rapid increase in the extension for any slight increase in load. B is the ‘breaking point’ where the material may finally ‘break’ or ‘snap’. It corresponds to the maximum possible extension that can be produced in the material.

### Worked examples

1. A force of 10N is required to extend the length of a spiral spring by 4 cm. Find the extension of the spring when a mass of 360 g is suspended from its end ( $g = 10\text{ms}^{-2}$ ).



**Solution**

By Hook's law,  $F = ke$

$$\begin{aligned}\therefore k &= \frac{F}{e} \\ &= \frac{10\text{N}}{0.04\text{ m}}\end{aligned}$$

when a body is suspended, the weight  $w$  should produce an extension,  $e$

$$\therefore W = mg = k \times e$$

$$\begin{aligned}\therefore e &= \frac{mg}{k} \\ &= \frac{\frac{360\text{kg}}{1000} \times 10\text{N}}{1000 \text{ kg}} \\ &= \frac{1000 \text{ N}}{4 \text{ m}} \\ &= 1.44 \times 10^{-2} \text{ m} \\ &= 1.44 \text{ cm}.\end{aligned}$$

- 2 An elastic material is stretched 45 mm by a force of 3 N. Calculate the additional force which will stretch the material 50mm, if the elastic limit is not exceeded.

**Solution**

By Hook's law,  $F = k.e$

$$\therefore k = F \frac{e}{e} = \frac{3 \text{ N}}{45 \times 10^{-3} \text{ m}}$$

Let the force stretching the material 50 mm be  $F$

$$\begin{aligned}\therefore F &= k.e \\ &= k \times 50 \times 10^{-3} \text{ m} \\ &= \frac{3\text{N}}{45 \times 10^{-3} \text{ m}} \times \frac{50 \times 10^{-3} \text{ m}}{1} \\ &= 3.33\text{N}\end{aligned}$$

$$\begin{aligned}\therefore \text{Additional force} &= (3.33 - 3.00) \text{ N} \\ &= 0.33 \text{ N}\end{aligned}$$



### 3.4 Young's Modulus of Elasticity

Suppose a wire of length  $\ell$  (m) and cross-sectional area  $A$  (m<sup>2</sup>) is extended through  $e$  (m) by a force  $F$  (N).

- (a) The ratio of the force to the area,  $F/A$  is called the stress or 'tensile' of the elastic material.

$$\text{Stress} = \frac{F}{A} \dots\dots\dots(1)$$

- (b) The ratio of the extension,  $e$  to the original length,  $\ell$  of the wire i.e  $\left(\frac{e}{\ell}\right)$  is called the tensile strain of the wire.

$$\therefore \text{Strain} = \frac{e}{\ell} \dots\dots\dots(2)$$

$$\text{From (1)} \quad F = \text{stress} \times A \dots\dots\dots(3)$$

$$\text{From (2)} \quad e = \text{Strain} \times \ell \dots\dots\dots(4)$$

By Hooke's law,  $F = k e$

$$\therefore \text{Stress} \times A = k \times \text{Strain} \times \ell$$

$$\therefore \text{Stress} = \frac{k \ell}{A} \times \text{strain}$$

$$\therefore \text{Stress} = k^1 \text{ strain where } k^1 =$$

$$\text{constant} = \frac{k \ell}{A}$$

$$\therefore \frac{\text{Stress}}{\text{Strain}} = k^1 \dots\dots\dots(5)$$

OR

$$\text{Stress} \propto \text{Strain}$$

Hence Hooke's law can also be stated as follows:

The tensile stress of the material is directly proportional to its tensile strain provided the elastic limit is not exceeded.



The constant of proportionality,  $k^1$  (see equation (5) above) is called Young's modulus of elasticity and is represented by the symbol,  $\gamma$ .

$$\therefore \text{Young's modulus } (\gamma) = \frac{\text{Stress}}{\text{Strain}}$$

$$\therefore \gamma = \frac{F/A}{e/l}$$

The unit of  $\gamma$  is  $\text{Nm}^{-2}$  (Newton per square metre) the same unit as stress, since strain has no unit.

$$\begin{aligned} \text{Dimension of } \gamma &= \frac{\text{Dimension of stress}}{\text{Dimension of strain}} \\ &= \text{Dimension of stress} \\ &= \frac{\text{Dimension of force}}{\text{Dimension of Area}} \\ &= \frac{MLT^{-2}}{L^2} \\ &= ML^{-1}T^{-2} \end{aligned}$$



### 3.5 Work Done in Springs and Elastic String

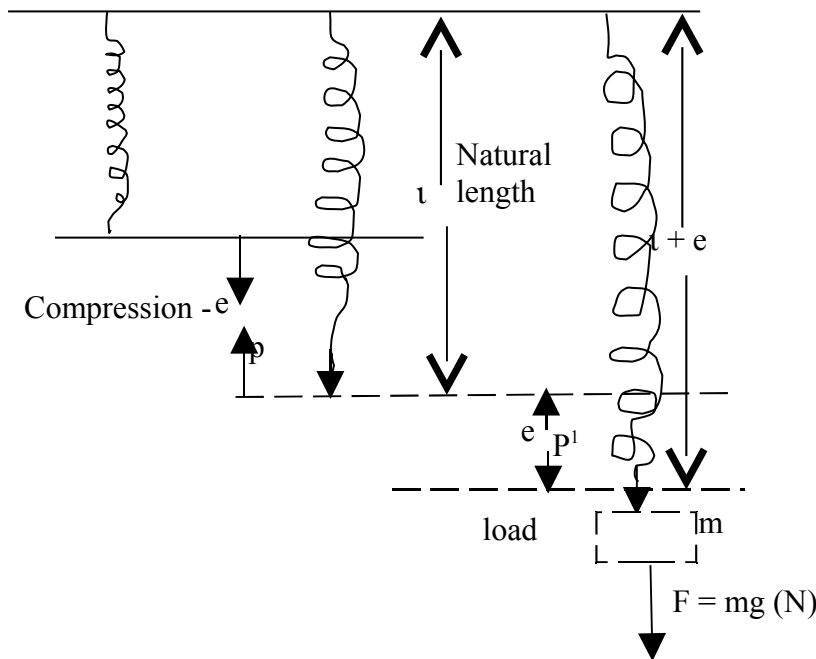


Fig. 2.3: Work done in extending or compressing a spring

Consider a spring of unstretched length,  $\ell$  (m). Suppose a force  $F$  (N) causes an extension (or compression)  $e$  (m). The force in the wire has increased from an initial value of zero at  $P$  to a value of  $F$  at 'P'. The work ( $W$ ) done in stretching the spring through the extension is given by:

$$\begin{aligned}
 W &= \text{Average force} \times \text{extension} \\
 &= \frac{\text{Initial force} + \text{final force}}{2} \times \text{extension} \\
 \therefore W &= \frac{0 + F}{2} \times e \\
 \therefore W &= \frac{1}{2} Fe \quad \dots\dots\dots (1)
 \end{aligned}$$

But  $F = ke$  (Hooke's law)

$$\therefore W = \frac{1}{2} k.e^2 \quad \dots\dots\dots (2)$$

This work done is stored as potential energy in the material which can do work when the stretching force is removed and the material regains its original length.



This type of stored energy is applied in the working of many spring clocks and toys. The energy is stored in the clock spring (or toy spring) which is compressed by winding.

### 3.6 Elastic Potential Energy

#### Definition

The elastic potential energy of a stretched or compressed material is the ability of the material to do work.

Elastic potential energy arises because work is done in stretching or compressing the material as we have seen in the previous section

$$W = \frac{1}{2} Fe = \frac{1}{2} ke^2$$

Where F is the maximum stretching (or compressing) force, e is the extension (or compression) and k is the force constant or stiffness of the material.

#### Example (or application) of elastic potential energy:

When you stretch the rubber of a catapult and project a stone, the elastic potential energy stored in the rubber is converted into the kinetic energy of the flying stone according to the law of conservation of energy.

#### Worked examples

1. An elastic material is stretch 1.2 cm by a mass of 96g suspended from one end of it. What is the elastic constant of the material?

Solution

$$\begin{aligned}
 \text{Weight of object} &= \text{stretching force} \\
 &= \frac{960 \times 10}{1000} \text{ N} \quad (W = mg) \\
 &= 9.6 \text{ N} \\
 \text{Extension} &= 1.2 \times 10^{-2} \text{ m} \\
 F &= ke \\
 &= k \times 1.2 \times 10^{-2} \text{ N}
 \end{aligned}$$



$$\begin{aligned}\therefore k &= \frac{9.6}{1.2 \times 10^{-2}} \text{ Nm}^{-1} \\ &= 800 \text{ Nm}^{-1}\end{aligned}$$

2. A spiral spring is compressed by an external force in such a way that 0.08J of energy is stored in it. If the elastic constant of the spring is  $400\text{Nm}^{-1}$ , calculate the compression of the spring.

### Solution

$$\text{Energy stored in the spring, } W = F e = \frac{1}{2} k e^2$$

$$\begin{aligned}W &= \frac{1}{2} k e^2 \\ \therefore 0.08\text{J} &= \frac{1}{2} \times 400\text{N} \times e^2\end{aligned}$$

$$\therefore e^2 = \frac{0.08 \times 2}{400} \text{ m}^2$$

$$\therefore e = 0.02\text{m}$$

### SELF ASSESSMENT EXERCISE 2

1. Explain the terms (i) elasticity, (ii) elastic limit of a material.

A mass of 300 g is suspended from the lower end of an elastic material. If the material extends by 6.0 cm, find the force needed to produce an extension of 1.0 cm.

2. (a) Define elasticity and state two reasons why this property is useful in daily life.  
 (b) Describe an experiment to illustrate the behaviour of an elastic wire which is steadily loaded and forced to extend its length beyond its elastic limit.  
 (c) Sketch a graph of the relation between the extension of the wire and the load attached to it gradually up to and beyond the elastic limit.
3. (a) States Hook's law and mention the condition under which it is obeyed.  
 (b) Describe an experiment to test its validity in the laboratory for a spiral spring.



- (c) Explain briefly how the working of a spring balance depends on Hooke's law.
  - (d) A helical spring is extended through 3.0 cm by a force of 40 N. Find (a) the force which will stretch it through 5.0 cm,  
(b) the length through which a force of 100N will extend it.
4. Defined the terms strain, stress and Young's modulus.

One end of the steel wire of length 10.0 m and uniform cross-sectional area  $1.0 \times 10^{-6} \text{ m}^2$  fixed to a rigid support. A force of 100N applied to the other end of the wire produces in it an extension of  $1.0 \times 10^{-2} \text{ m}$ . If Hooke's law is obeyed, calculate (i) the stress, (ii) the strain, (iii) the young's Modulus for the metal.

#### 4.0 CONCLUSION

In this unit you learned about elastic properties of solids. Specifically, you learned about Hooke's law, Young' modulus of elasticity and their definitions. You also learned about work done in spring/ string and elastic potential energy

#### 5.0 SUMMARY

1. 'Elasticity' is the property of a material which enables it to regain its original shape and size after the force causing the deformation has been removed.
2. 'Hook's law' states that the extension of an elastic material is directly proportional to the load or applied force causing the extension, provided the elastic limit of the material is not exceeded.
3. 'Elastic limit' is the maximum stretching force beyond which a stretched elastic material would not return to its original length when the force is removed.
4. 'Yield point' is the point at which the elastic material loses its elasticity permanently and becomes plastic.
5. Work done in stretching or compressing a material is the same as the energy stored in the material or its elastic potential energy and is given by;



$$\begin{aligned}
 W &= \frac{1}{2} Fe \\
 &= \frac{1}{2} (\text{maximum force}) \times \text{extension} \\
 \text{Also } W &= \frac{1}{2} ke^2, \text{ since } F = ke, \text{ from Hook's law}
 \end{aligned}$$

$$\text{'Young's modulus' } Y = \frac{\text{Stress}}{\text{Strain}}$$

$$= \frac{F/A}{e/l} \text{ Nm}^{-1}$$

## 6.0 TUTOR-MARKED ASSIGNMENT

- State Hook's law and explain what is meant by elastic limit.
  - A load of 2N produces an extension of 3.0 cm in a spiral spring. Find the additional force which will stretch the material by 3.5 cm.
- State the formula for the energy stored in an elastic material in terms of; the force producing an extension in the material.
  - An electric material is compressed by 2.0 cm. If the elastic constant of the material is  $400 \text{ Nm}^{-1}$ , determine the elastic potential energy stored in the material.
- A stone of mass 5 g is projected with a rubber catapult. If the catapult is stretched through a distance of 7 cm by an average force of 70 N, calculate the instantaneous velocity of the stone.

## 7.0 REFERENCES/FURTHER READINGS

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## UNIT 3 ELASTIC PROPERTIES OF SOLIDS

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### 1.0 INTRODUCTION

This unit will introduce you to the concepts of heat, temperature and molecular theory. You will learn the effects of heat and change of state and expansion, their explanation using the kinetic molecular theory. Also, you will learn expansivity and explanation of temperature changes using molecular theory. Finally, you will learn the applications of expansivity in your daily life.



## 2.0 OBJECTIVES

By the time you finish this unit, you will be able to:

- explain temperature, expansion, change of state and vaporization using the kinetic molecular theory
- solve simple problems involving linear, area and volume expansivity.

## 3.0 MAIN CONTENT

### 3.1 Concept of Heat and Temperature

When you touch a kettle of boiling water, you will feel a burning sensation. You will say that the water is hot. Also, if you hold ice block, your hand feels chilly and you conclude that the water is cold. We describe an object which has been given heat energy as hot. When we place a kettle of cold water on a heating stove, after a few minutes, the water feels hotter because it has been given heat energy. Similarly, we describe as cold a body from which heat energy has been removed. For instance, keeping a bottle of water in a refrigerator, you must have experienced that heat always flows from a hot object to a cold object.

Heat energy is the energy that is transferred from a hot object to a cooler object as a result of their difference in temperature.

Temperature is the degree of hotness or coldness of an object.

Temperature is a property of an object which decides which way heat will flow when it is placed in contact with another object. Usually heat flows from a body of higher temperature to one at lower temperature. The instrument for measuring temperature is the thermometer.

### 3.2 Effects of Heat

Here we look at some effects of heat on an object. You may have identified some of this yourself in your everyday life. These effects are summarized as follows:

Addition of heat to the body will cause:

- (a) Change in temperature of the body except during a change of state.
- (b) Change of state of the body solid to liquid, liquid to gaseous state.
- (c) Expansion of the body.



- (d) Change in the physical property of a body such as the electrical resistance, magnetic properties conductivity, elasticity, density and colour of the body.
- (e) Thermionic emission- the emission of electrons from the surface of the metal.
- (f) Change in chemical properties of the body.
- (g) Changes in pressure and volume of gases.

### 3.3 Kinetic Molecular Theory

We consider kinetic molecular theory and its assumptions. In addition, we use it to explain major concepts in this unit. The molecular theory of matter assumes the matter is made up of atoms that aggregate in molecules.

The molecule is a group of atoms of the same or different elements joined together in a simple proportion. These molecules are under the influence of two types of forces:

- (i) Attractive forces which prevent the molecule from moving apart and
- (ii) repulsive forces which prevent the molecule from moving closer. There is normally a balance between these forces in a substance.

In solid substances, the attractive forces between molecules are so strong that the molecules do not move about freely. The molecules only vibrate about their mean positions maintaining a fixed shape and volume.

In liquids, the molecules are loosely held together by weak attractive forces. The molecules are free to move about within the liquid and are always in a state of random motion. Molecules of solids and liquids are held together by intermolecular forces.

In gases, the force of attraction between the molecules is very weak. The molecules are therefore in constant motion having overcome the intermolecular force. They move very freely at a very high speed. They are always in the state of random motion and take up the shape and volume of their container.

The kinetic molecular theory assumes that:

- (i) Every substance is made up of tiny particles called molecules



- (ii) The molecules are in a constant state of random motion, colliding elastically with one another and changing their direction as a result.
- (iii) There is always an attractive force between the molecules.
- (iv) The volume of the gas molecule is negligible compared with the volumes of the gas container.

### 3.3.1 Kinetic Theory Explanation of Temperature

We say that the molecules of a substance are in constant motion, therefore, they possess kinetic energy.

The temperature of the body is a measure of the average kinetic energy of its molecules.

When you add heat to a substance, the motion of the molecules increases resulting in an increase in the average kinetic energy of the molecules of the substance, thus, an increase in its temperature. On the other hand, when you remove heat from a body, the motion of its molecule decreases resulting in reduction in their average kinetic energy and a decrease in temperature.

It follows therefore, that the temperature of a body is related to the average kinetic energy of its molecules.

### 3.3.2 Kinetic Theory Explanation of Expansion

When we heat a substance, it increases in size, we say it expands. When the substance cools, it contracts i.e reduces in size.

According to the kinetic molecular theory, when an object is heated the molecules acquire more kinetic energy which enables them to overcome their intermolecular forces resulting in increase in vibration of the molecules and their displacement about their mean positions. The average distance between the molecules of the substance becomes larger leading to an increase in the size of the substance. The increase depends on the strength of the intermolecular forces. If these forces are strong, the expansion will be small and vice versa.

The intermolecular forces are stronger in solids than in liquids and weakest in gases. Therefore, when heat is applied, gases expand more than liquids and liquids expand more than solids.



### 3.3.3 Kinetic Molecular Theory Explanation of Change of State

You are aware that substances exist in any of the three states of matter; solid, liquid and gas. With the help of heat energy, it is possible to transform a substance from one state to another. Change of state refers to the process of transforming a substance from one state of matter to another.

When a solid is heated its temperature increases until it reaches a certain maximum temperature at which the molecules acquire maximum kinetic energy. At this maximum temperature, further heating does not increase the kinetic energy of the molecules but the heat energy (latent heat) is used to break down the intermolecular forces binding the molecules of the solids in a regular pattern. The molecules then melt and move about freely as they are in the liquid state. The maximum temperature is the melting point of the solid. Also, the change from liquid to gas takes place in a similar fashion. The liquid temperature increases as it is heated and reaches its maximum temperature at which the molecule acquires maximum kinetic energy.

Further heating (latent heat) supplies the needed energy to overcome the forces of attraction between the liquid molecules and changes the liquid to vapour without temperature change. The molecules are then practically independent of each other and exist as a gas.

The energy needed in this case is much greater than from solid to liquid and the latent heat is much greater. Removal of heat energy from a substance results in a reverse process.

The heat required to break down the intermolecular forces of attraction in a solid is the latent heat of fusion.

The latent heat of vaporization is the heat needed to overcome the forces of attraction between the molecules of the liquid.

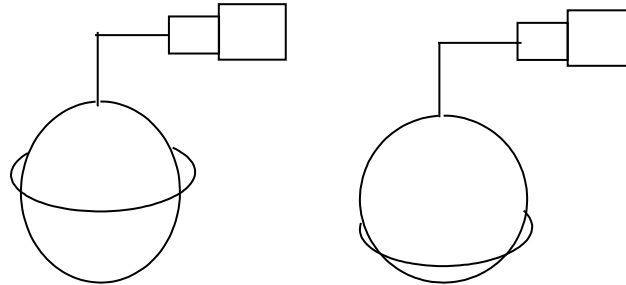
## 3.4 Expansivity

Solids expand when heated and contract when cooled. You can observe this in simple laboratory experiments such as:

- Ball and ring experiment:



When cold, a metal ball can just pass through a metal ring. On being heated strongly for some minutes, it expands and can no longer pass through the ring. When it cools, it contracts and passes through the ring again.



(a) Ball passes through the ring (before heating) (b) Ball does not pass through the ring (after heating)

Fig. 3.1

- Bar and guage experiment

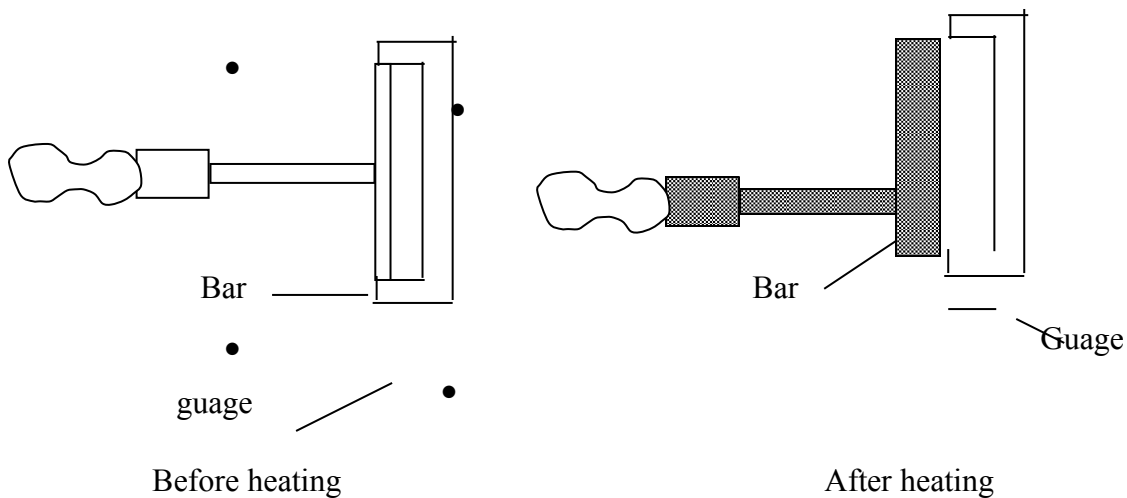


Fig. 3.2: Bar and guage

The bar in Fig. 3.2 just fits into the guage at ordinary room temperature. After heating, the heated bar expands in length when hot and can no longer fit into the guage. When cooled to the room temperature, the bar contracts and fits once more into the guage.



- Bimetallic strip;

Different substances expand by different amounts for the same temperature range. You can observe this in the heating of bimetallic strip shown in Fig. 3.3.

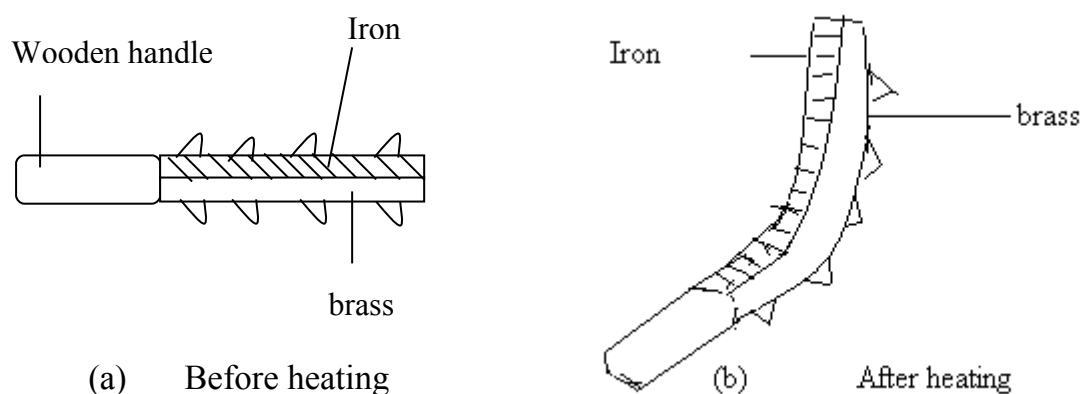


Fig.3.3: Expansion of a solid (bimetallic strip)

Before heating the bimetallic strip of brass and iron riveted together has equal length. After heating, the strip bends as shown above because brass expands more than iron for a given temperature rise.

### 3.4.1 Linear Expansivity

You can determine the extent which a unit metal substance changes in length when its temperature changes by one degree. This quantity is known as linear expansivity of the metal substance. It is represented by the symbol  $\alpha$  (pronounced alpha).

- The linear expansivity ( $\alpha$ ) of a substance is defined as the increase in length per unit length per degree rise in temperature.

In symbols, this is equivalent to:

$$\alpha = \frac{l_2 - l_1}{l_1 (\theta_2 - \theta_1)} = \frac{e}{l_1 \theta} \dots\dots\dots$$

(i)

Where:

$$\begin{aligned} \alpha &= \text{linear expansivity} \\ l_1 &= \text{length of metal at temperature } \theta_1 \\ l_2 &= \text{length of metal at temperature } \theta_2 \end{aligned}$$



$$\begin{aligned}\theta &= \text{temperature rise given by } \theta_2 - \theta_1 \\ e &= \text{expansion or increase in length } l_2 - l_1 \\ \alpha &= \frac{\text{Increase in length}}{\text{original length} \times \text{temperature rise}}\end{aligned}$$

Unit per °C or per k ( $\text{k}^{-1}$ )

- The statement that the linear expansivity of brass is  $0.000018 \text{ k}^{-1}$  or  $0.000018/^{\circ}\text{C}$  means that a unit length of brass expands by 0.00018 units when it is heated through 1k (or  $1^{\circ}\text{C}$ ) rise in temperature.

### 3.4.2 Determination of Linear Expansivity

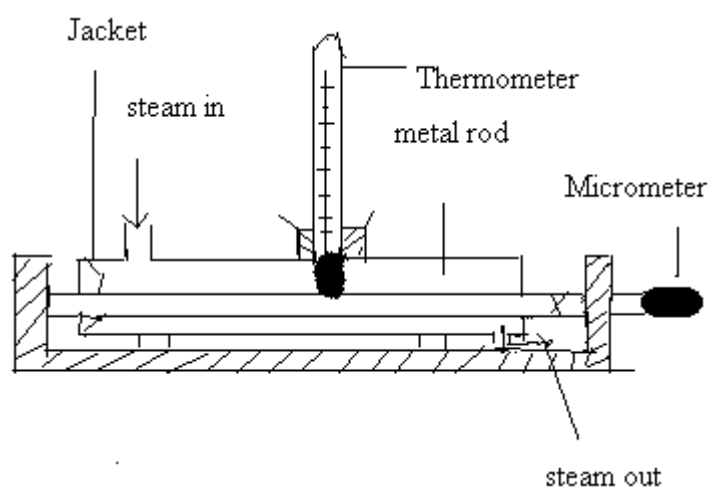


Fig. 3.4: Measurement of linear expansivity

The apparatus shown in Fig. 3.4 is used in the determination of linear expansivity of a solid.

Carefully measure the original length  $l_1$  of a metal rod (about 50 cm) with a metre rule. Enclose the rod in a fixed jacket between a fixed stop A and a micrometer guage B, so that no movement of the rod can take place at A while the other end at B is free to move. Run cold water through the jacket. Adjust the micrometer screw until it just touches the B end of the rod. Record the guage's first reading ( $e_1$ ) and the initial temperature ( $\theta_1$ ) of the rod from the thermometer placed inside the jacket.

Unscrew the micrometer screw guage from the end B of the rod to leave enough space for the rod to expand. Pass hot steam through the jacket



for several minutes to allow the rod expand as a result of this heating. Screw the guage to make contact with the rod again and take a new reading of the guage ( $e_2$ ). Repeat the process until the reading of the screw guage becomes constant. Take the final steady temperature ( $\theta_2$ ) of the rod from the thermometer inserted into the steam jacket. Calculate the linear expansivity from the equation;

$$\text{Linear expansivity } (\alpha) = \frac{\text{expansion (or change in length)}}{\text{Original length x temperature change}}$$

$$\alpha = \frac{e_2 - e_1}{l_1(\theta_2 - \theta_1)} = \frac{e_2 - e_1}{l_1 \theta}$$

### 3.4.3 Area Expansivity ( $\beta$ )

When we heat a solid, it expands in all directions that is the length, breadth and height. This results in an increase in area as well as in volume of the solid. This increase in area when the body is heated is known as area or superficial expansion. It is represented by the symbol  $\beta$  (pronounced beta).

#### Definition

The area or superficial expansivity,  $\beta$ , of a solid is the increase in area per unit area per degree Kelvin increase in temperature or the fractional increase in area per Kelvin rise in temperature.

$$\text{Area expansivity } \beta = \frac{\text{change in area}}{\text{Original area x temperature rise}}$$

$$\beta = \frac{A_2 - A_1}{A_1 (\theta_2 - \theta_1)} = \frac{a}{A_1 \theta}$$

Where:

$$\begin{aligned} a &= \text{increase in area } A_2 - A_1 \\ A_2 &= \text{area at temperature } \theta_2 \\ A_1 &= \text{area at temperature } \theta_1 \\ \theta &= \theta_2 - \theta_1 \end{aligned}$$

### 3.4.4 Volume or Cubic Expansivity ( $\gamma$ )

An increase in volume when a body is heated is known as cubic or volume expansion. We denote cubic expansivity with the symbol  $\gamma$  pronounced gamma.



### Definition

The volume or cubic expansivity  $\gamma$ , is the increase in volume of a substance per unit volume per Kelvin rise in temperature or the fractional increase in volume per Kelvin rise in temperature.

$$\text{Cubic expansivity } (\gamma) = \frac{\text{change in volume}}{\text{Original volume} \times \text{temperature rise}}$$

$$\gamma = \frac{V_2 - V_1}{V_1 (\theta_2 - \theta_1)} = \frac{v}{V_1 \theta} \quad \dots\dots\dots (3)$$

Where

$$\begin{aligned} v &= \text{increase in volume } V_2 - V_1 \\ V_2 &= \text{volume at temperature } \theta_2 \\ V_1 &= \text{volume at temperature } \theta_1 \\ \theta &= \theta_2 - \theta_1 \end{aligned}$$

### 3.4.5 The Relationship between Linear Expansivity ( $\alpha$ ), Area Expansivity ( $\beta$ ) and Cubic Expansivity ( $\gamma$ )

If  $\alpha$  is the coefficient of linear expansivity of a metal, its area expansivity ( $\beta$ ) and volume expansivity ( $\gamma$ ) are related ( $\alpha$ ) by:

$$\begin{aligned} \beta &= 2\alpha \\ \gamma &= 3\alpha \quad \dots\dots\dots (4) \end{aligned}$$

We can also write the following:

$$\begin{aligned} \ell_2 &= \ell_1 (1 + \alpha\theta) \\ A_2 &= A_1 (1 + \beta\theta) = (A_1 + 2\alpha\theta) \\ V_2 &= V_1 (1 + \gamma\theta) = (V_1 + 3\alpha\theta) \end{aligned}$$

where the symbols retain their previous meaning.



### Worked Examples on Expansivity

- 1 In an experiment to determine the linear expansivity of a solid rod, the following readings were obtained:

Initial temperature of rod ( $\theta_1$ )	=	18°C
Original length of rod ( $l_1$ )	=	50.0 cm
Final temperature of rod ( $\theta_2$ )	=	98°C
Final length of rod ( $l_2$ )	=	50.8 cm

Calculate the (a) linear, (b) area and (c) volume expansivity of the rod.

$$\text{Rise in temperature} = 98^\circ\text{C} - 18^\circ\text{C} = 80^\circ\text{C}$$

$$\text{Expansion of rod} = \text{Increase in length} = 50.8 \text{ cm} - 50.0 \text{ cm} = 0.8 \text{ cm}$$

#### Solution

$$(a) \quad \text{linear expansivity} = \frac{\text{expansion}}{\text{original length} \times \text{temperature change}}$$

$$= \frac{0.8}{50 \times 80} = 0.0002 \text{ per } ^\circ\text{C}$$

$$(b) \quad \text{area expansivity } (\beta) = 2\alpha = 0.0002 \times 2 = 0.0004 \text{ per } ^\circ\text{C}$$

$$(c) \quad \text{cubic expansivity } (\gamma) = 3\alpha = 3 \times 0.0002 = 0.0006 \text{ per } ^\circ\text{C}$$

- 2 The coefficient of linear expansivity of a metal is  $1.8 \times 10^{-5}$  cm per degree Celsius. A cube of the metal has each length as 2.5 m. Find the area of each surface and the volume of the cube if it is heated through a temperature of  $60^\circ\text{C}$

#### Solution

$$\text{Coefficient of linear expansivity } \alpha = 1.8 \times 10^{-5}$$

$$\text{Coefficient of area expansivity } \beta = 2\alpha = 2 \times 1.8 \times 10^{-5} = 3.6 \times 10^{-5}$$

$$\text{Coefficient of volume expansivity } \gamma = 3\alpha = 3 \times 1.8 \times 10^{-5} = 5.4 \times 10^{-5}$$

Area of each surface  $A_2$

$$= A_1 (1 + 2 \alpha \theta)$$

$$= 6.25 (1 + 3.6 \times 10^{-5} \times 60) \text{ m}$$



$$= 6.26\text{m}^2$$

The volume of the cube  $V_2$

$$\begin{aligned} &= V_1 (1 + \gamma \Delta T) \\ &= 15.63 (1 + 5.4 \times 10^{-5} \times 60)\text{m} \\ &= 15.69 \text{ m}^3 \end{aligned}$$

### 3.5 Some Applications of Expansion

Here are some of the applications:

- Gaps are left between rails in railway lines to allow for free expansion and contraction of the rails. Without the gaps, the rail joints will swell up on hot days, the railway line will buckle and trains would be derailed.
- The end of the steel structure of long bridges rests on rollers which allow for the expansion and contraction of the bridge without weakening the structure.
- Telegraph wires are allowed to sag when fixed in the warm, raining season, so that they do not snap when they contract in the cold harmattan.
- A bimetallic strip is used in the thermostat, a device for maintaining steady temperature. Thermostats are used in electric laundry irons, immersion heaters for hot water tanks, refrigerators and airconditioners.
- Balance wheel of watches are also made of bimetallic strips without which increase of temperature will increase the diameter of the balance wheel thereby weakening the elasticity of the hair spring and causing the watch to lose time.
- A thick glass tumbler cracks when hot water is poured into it because the inside of the tumbler expands more rapidly than the outside and causes a strain in the glass. A type of glass known as pyrex is used for making laboratory beakers and flasks to avoid the above effects. Pyrex has low thermal expansivity.
- Other applications of expansion of solids is in:



- riveting
- the fixing of metals tyres on the metal wheels in locomotives;
- in the expansion loops in steam pipes.

### 3.6 Expansion of Liquids

Liquids like solids expand on heating and contract on cooling. You can observe liquid expansion with the following activity using the apparatus shown in Fig. 3.5.

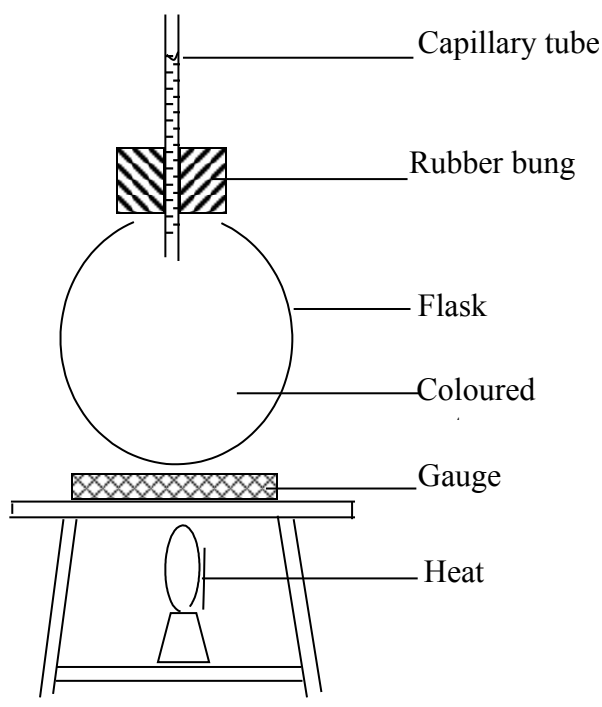


Fig. 3.5: Liquid expansion on heating

Fill the flask with coloured water to the brim. Fit the rubber bung with a good length of the capillary tube projecting outside the flask as shown in Fig. 3.5. Heat the flask gently with the bunsen burner and observe the coloured liquid in the narrow tube. Remove the liquid and observe the movement of the liquid level in the tube.

You will observe that the liquid level initially falls slightly and then rises slowly and steadily as more heat is applied. This is because on the application of the heat, the glass first expands faster than the liquid, hence, the fall of the liquid level. As more heat is applied, the expansion of the liquid increases more than that of the glass and the liquid rises in the narrow tube. On removal of heat the liquid contracts and the level of liquid in the tube is seen to fall.

By using different liquids, following the same process above but using the apparatus shown in Fig. 3.6, you will also observe that liquids



expands by different amounts when heated through the same temperature ranges.

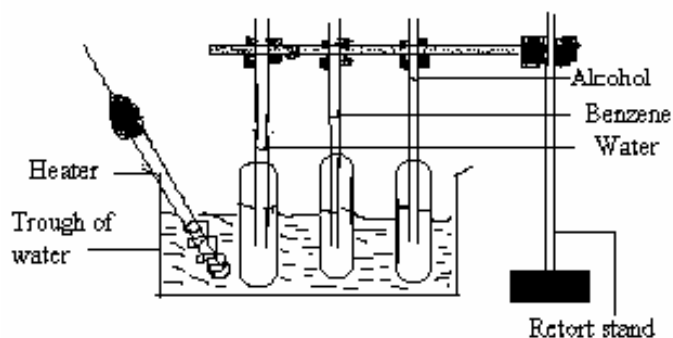


Fig. 3.6 : Different liquids expand differently

You will observe that in the three flasks, the level of liquids in the tube first falls and then rises steadily. The level of alcohol is the highest followed by benzene and least is water.

### 3.6.1 Real and Apparent Expansivity

In the last activity, we observe that when a liquid in a container is heated, both the liquid and the container expand. The expansion of the liquid as observed is the apparent expansion. This is always less than the true or real expansion of the liquid. It is therefore necessary to distinguish between the real and the apparent cubic expansion of the liquid.

Real expansion = Apparent expansion + expansion of the container

$$\gamma_{\text{real}} = \gamma_{\text{apparent}} + \gamma_{\text{container}} \quad (5)$$

- The real (or absolute) cubic expansivity ( $\gamma_r$ ) of a liquid is the increase in volume per unit volume per degree rise in temperature.
- The apparent cubic expansivity ( $\gamma_a$ ) of a liquid is the increase in volume per degree rise in temperature when the liquid is heated in an expansible vessel.

The apparent expansivity of a liquid is obtained by weighing a density bottle empty and then fills with the liquid. The bottle with the contents is then heated in a warm water bath. As the temperature increases, the glass and the liquid expand and the liquid overflows. The bottle is then



taken out of the water bath, cleaned and reweighed to determine the mass of liquid expelled. The apparent expansivity is calculated from:

$$\gamma_{\text{apparent}} = \frac{\text{mass of liquid expelled}}{\text{mass of liquid remaining} \times \text{rise in temperature}} \dots\dots\dots(6)$$

### 3.6.2 The Anomalous Expansion of Water

We know from the foregoing that most liquids expand when heated and contract when cooled. Water is an exception to this rule as it behaves in an anomalous way when cooled. It contracts on cooling from a higher temperature until the temperature falls to 4°C. At 4°C, it starts to expand if cooled further below this temperature to 0°C. That is water expands when cooled from 4°C to 0°C. As a result of this, a given mass of water has its least volume and its highest density at 4°C. This anomalous behaviour of water is illustrated in an experiment known as Hope's experiment.

### 3.6.3 The Importance of Anomalous Expansion of Water in Nature

The anomalous expansion of water is important to us because it makes ponds, lakes and rivers to freeze from the top surface rather than from the bottom. As a result, Marine lives can survive during winter since ice forms at the surface of the water while the bottom of the lake remains at 4°C a temperature warm enough for aquatic animals such as fishes.

### SELF ASSESSMENT EXERCISE 1

- 1 Which gives you greater burn, water at 100°C or steam at 100°C? Why?
- 2 Two identical cans are placed in contact, one containing hot water and the other cold water. Why is it that the temperature of the cold water rises and that of the hot water falls and not the other way round?
- 3 (a) What is meant by the statement, the linear expansivity of a solid is  $1.0 \times 10^{-5} \text{ K}^{-1}$ .
- 3 (b) i Describe an experiment to determine the linear expansivity of a steel rod.  
ii Steel bars each of length 3 cm at 29°C are to be used for constructing the rail line. If the linear expansivity of the steel is  $1.0 \times 10^{-5} \text{ K}^{-1}$ , Calculate



the safety gap that must be left between successive bars if the highest temperature expected is 41°C.

- (c) State three advantages and two disadvantages of thermal expansion of solids.
- 4 Distinguish between real and apparent cubic expansivity of a liquid.
- 5 A glass bottle full of mercury has mass 500 g. On being heated through 35°C, 2.43 g of mercury are expelled. Calculate the mass of mercury remaining in the bottle. Cubic expansivity of mercury is  $1.8 \times 10^{-4} \text{ K}^{-1}$ , linear expansivity of glass is  $8.0 \times 10^{-6} \text{ K}^{-1}$ .

#### 4.0 CONCLUSION

In this unit, you have learned a number of important concepts which relate to the effects of heat and their explanation using kinetic molecular theory. You also learned linear, area and volume expansivity of a substance and the applications of expansion. In addition, you learned about liquid expansion and the importance of anomalous expansion of water.

#### 5.0 SUMMARY

- All substances are made of molecules. When a substance is heated, its molecules begin to move faster.
- Heat is a form of energy called thermal energy. It is the energy communicated from one body at higher temperature to another at lower temperature.
- Temperature is the degree of hotness or coldness of a body.
- According to kinetic molecular theory, the temperature of a body depends on the average kinetic energy of its molecules.
- The linear expansivity  $\alpha$  of a substance is the fractional increase in its length per degree change in temperature.
- Area or superficial expansivity,  $\beta$  is the fractional increase in area per unit change in temperature.
- The volume or cubic expansivity  $\gamma$  is the fractional increase in volume per degree change in temperature.



- $\beta = 2\alpha$ ;  $\gamma = 3\alpha$
- Real volume expansivity = apparent volume expansivity + volume expansivity of container;  $\gamma_r = \gamma_a + \gamma$
- Water unlike other liquids, contract when it is heated from 0°C to 4°C. Water has a maximum density at 4°C.
- Advantages and disadvantages of expansion.
- The advantages of expansion are utilized in fitting of wheels on metal rims, fire alarm, thermostat, bimetallic thermometer, compensated pendulum.
- The disadvantages of expansion are the sagging of overhead cable, the buckling of railway lines, in the gaining or losing of time as a result of expansion of balance wheels of watches and clocks.

## 6.0 TUTOR -MARKED ASSIGNMENT

1. State three advantages and two disadvantages of thermal expansion of solids.
2. Which gives you a greater burn, water at 100°C or steam at 100°C? Why?
3. The linear expansivity of iron is  $0.000012 \text{ K}^{-1}$ . What does this statement mean?
4. The linear expansivity of a cube is  $0.000018 \text{ K}^{-1}$ . If the length of each side of the cube is 10 cm, find the area of one face of the cube and the volume of the cube when its temperature is raised by 30 K.
5. A container is completely filled with 95.1 kg of a liquid at 36°C. When the temperature of the liquid was raised to 78°C, it is found that 1.2 g of the liquid overflows. Determine the apparent cubic (volume) expansivity of the liquid.



## **7.0 REFERENCES/FURTHER READINGS**

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## UNIT 4 MEASUREMENT OF TEMPERATURE

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### 1.0 INTRODUCTION

In the previous unit (Module 3, unit 3) we discussed the concepts of heat and temperature and stated the difference between them. We noted that heat is a form of energy (thermal energy) while temperature is a measure of the degree of hotness or coldness of a body. In this unit we shall restate the difference between heat and temperature and discuss the various methods of measuring temperature. The relationship between pressure and temperature of a gas, the different types of scales used in temperature measurement and types of thermometers will be treated.

### 2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Construct a device for measuring the temperature of the body
- Use the variation of
  - Pressure of a gas with temperature
  - Expansion of solid, liquid or gas with temperature



- Electrical resistance of a material, to measure the temperature of the body.
- distinguish between heat and temperature and between temperature points and temperature intervals
- select those liquids which are suitable for use in liquid –in-glass thermometers from a given list of liquids and their properties.
- describe the absolute scale of temperature and explain the meaning of the absolute zero of temperature
- convert a given temperature in the Celsius scale to a temperature in the Kelvin scale and vice versa
- describe the kinetic molecular model of temperature.

### **3.0 MAIN CONTENT**

#### **3.1 Heat and Temperature**

Heat is a form of energy which can be transformed into other forms of energy. It is also called “thermal energy” and always flows from a body at a higher temperature to a body at a lower temperature, if the two bodies are in contact. It is a measure of the internal energy of a body. The quantity of heat contained in a body depends on the mass of the body and its temperature. The unit of heat is the joule (J).

Temperature is the degree of hotness or coldness of a body. It is related to the energy of movement (kinetic energy). It is also a measure of the average kinetic energy of the molecules of a body. The unit of temperature is degree Celsius ( $^{\circ}\text{C}$ ) or Kelvin (K).

#### **3.2 Principle Used in Temperature Measurement**

In measuring the temperature of a body we use the principle that certain physical properties of some substances vary with temperature. Any physical property of a substance which varies in a known way and can easily be measured is employed in an instrument called “thermometer” to measure temperature. The substance whose physical property that is used is called a “thermometric substance”. Table 4.1 below presents a summary of the types of thermometers and their thermometric substances.



**Table 4.1: Thermometers and Thermometric Substances**

	<b>Types of Thermometers</b>	<b>Thermometric substance</b>	<b>Physical Property</b>
1.	Liquid-in-glass thermometer	Mercury or Alcohol	Change in volume of liquid with temperature
2.	Gas thermometer	Gas	Change of gas pressure at constant volume with temperature
3.	Resistance thermometer	Resistance wire	Change in the electrical resistance of wire with temperature
4.	Thermocouple	Two dissimilar metals (e.g. copper and constantan)	Change in electric potential difference (or current) between two metal junctions at different temperature.
5.	Bimetallic thermometer	Two dissimilar metals (e.g. iron and copper)	The differential expansion of the two metals of the bimetallic strip.

### 3.3 Temperature Scales

#### 3.3.1 Fixed Temperature Points of a Thermometer

A thermometer has two reference temperatures or “fixed points”, namely:

- (a) The upper fixed temperature point
- (b) The lower fixed temperature point.

#### Definitions

- (a) The upper fixed point of a thermometer is the temperature of steam from pure water boiling at standard atmospheric pressure of 760 mm of mercury.
- (b) The lower fixed point of a thermometer is the temperature of pure melting ice at the standard atmospheric pressure of 760mm of mercury.



- (c) The fundamental interval (or temperature interval) is the difference in temperature between the upper and lower fixed points of a thermometer.

The fundamental interval of a thermometer is usually calibrated to give readings of temperature values between the upper and lower fixed points. The calibration of this interval depends on the scale of temperature chosen for the thermometer.

### 3.3.2 Types of Temperature Scales

Three types of scales that are in common use are as follows

- (i) Fahrenheit scale
- (ii) Celsius scale
- (iii) Kelvin (absolute or thermodynamic) scale.

Fig. 4.1 shows the three scales with their upper and lower fixed points and their fundamental intervals.

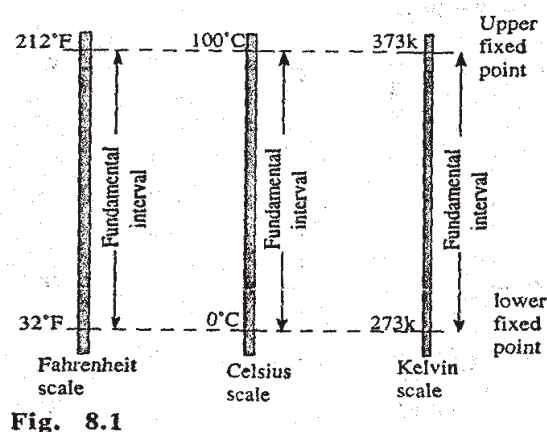


Fig. 4.1

Fig. 4.1: Fahrenheit, Celsius and Kelvin scales of temperature

#### (a) Fahrenheit Scale (°F)

- (i) Lower and upper fixed points are 32°F and 212°F respectively.
- (ii) Fundamental interval is divided into 180 equal parts (i.e.  $212 - 32 = 180$ ) each of which defines one degree Fahrenheit (1°F) in this scale.



**(b) Celsius Scale ( $^{\circ}\text{C}$ )**

- (i) Lower and upper fixed points are  $0^{\circ}\text{C}$  and  $100^{\circ}\text{C}$  respectively.
- (ii) Fundamental interval is divided into 100 equal parts or degree, each equal to  $1^{\circ}\text{C}$ .

**(c) Kelvin or Absolute Scale (K)**

- (i) Lower and Upper fixed points are 273K and 373K respectively.
- (ii) Fundamental interval is divided into 100 equal parts or units each of which is called one Kelvin (K).

You should note well that in this scale temperatures are not measured in degrees but in units called Kelvin (K). Hence the unit symbol K is written without the degree sign. The zero on the Kelvin scale is equal to  $-273^{\circ}\text{C}$  (i.e.  $0\text{K} = -273^{\circ}\text{C}$ ). It is called the “absolute zero”

**Definition**

The absolute zero temperature is the temperature at which the volume of a fixed mass of gas at constant pressure, which is steadily cooled below  $0^{\circ}\text{C}$ , would theoretically become zero.

**Conversion Formulas****(i) Conversion from Fahrenheit to Celsius:**

Scale suppose  $X^{\circ}\text{F}$  corresponds to a temperature of  $y^{\circ}\text{C}$ .

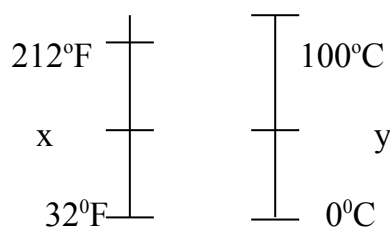


Fig. 4.2

Fahrenheit  
Scale

Celsius  
Scale



By proportions:

$$\begin{aligned}\frac{X-32}{212-32} &= \frac{Y-0}{100-0} \\ \frac{x-32}{180} &= \frac{y}{100} \\ Y &= \frac{100(x-32)}{180} \\ Y &= \frac{5}{9} (X-32)\end{aligned}$$

where X is in °F and Y is in °C.

### SELF ASSESSMENT EXERCISE 1

- (i) Give two examples of conversion from Celsius to Fahrenheit and Fahrenheit to Celsius.

Conversion from Celsius to Kelvin Absolute Scale

We have seen that

$$0\text{K} = -273^{\circ}\text{C} \text{ or } 0^{\circ}\text{C} = 273\text{ K}$$

Hence, if  $t^{\circ}\text{C}$  corresponds to T on the absolute scale, then

$$T = t + 273$$

You should note that although the fixed points on the Celsius and Kelvin scales are different, a temperature change of  $1^{\circ}\text{C}$  (on the Celsius scale) is equivalent to a temperature change of 1K (on the Kelvin scale).

### Determination of Temperature on a Celsius Scale

You can determine an unknown temperature  $\theta$  in the Celsius scale as follows:

1. Identify a physical property of a substance which varies in a known way with temperature and which can easily be measured.
2. Measure the value of the physical property at the lower fixed point,  $0^{\circ}\text{C}$ .
3. Measure the value of the physical property at the upper fixed point,  $100^{\circ}\text{C}$ .
4. Finally measure the value of the physical property at the unknown temperature  $\theta^{\circ}\text{C}$ .



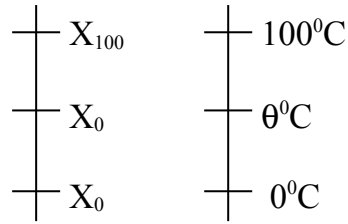


Fig. 4.3

Let value of physical property at  $0^{\circ}\text{C}$  =  $X_0$

Let value of physical property at  $100^{\circ}\text{C}$  =  $X_{100}$

Let value of physical property at  $\theta^{\circ}\text{C}$  =  $X_0$

If change in physical property is directly proportional to change in temperature, then using proportions, we have

$$\frac{X_0 - X_0}{X_{100} - X_0} = \frac{\theta - 0^{\circ}\text{C}}{100 - 0^{\circ}\text{C}}$$

$$\frac{0}{100} = \frac{X_0 - X_0}{X_{100} - X_0}$$

$$0 = \left\{ \frac{X_0 - X_0}{X_{100} - X_0} \right\} \times 100$$

This method can be used to measure an unknown temperature where a calibrated thermometer is not available.

### Worked example

The upper and lower fixed points of a certain thermometer are 30cm apart. On a certain day, the length of mercury thread in the thermometer is 9 cm above the ice point. What is the temperature recorded by the thermometer on (a) Celsius scale (b) Kelvin scale?



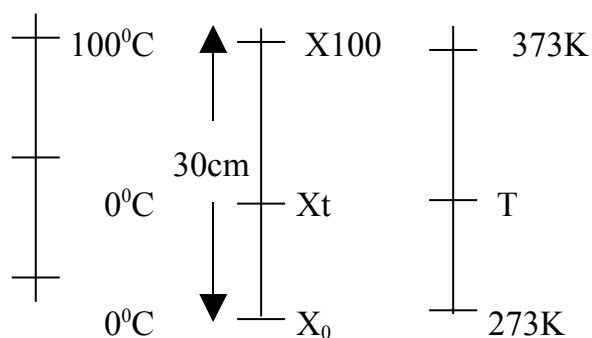
**Solution**

Fig. 4.4

(a) Using proportions, we have

$$\frac{\theta - 0}{100 - 0} = \frac{9}{30}$$

$$\frac{0}{100} = \frac{9}{30}$$

$$0 = \frac{9 \times 100}{30} = 30^\circ\text{C}$$

(b) Again using proportions, we have

$$\frac{T - 273}{373 - 273} = \frac{9}{30}$$

$$\frac{T - 273}{100} = \frac{9}{30}$$

$$T - 273 = \frac{9 \times 100}{30} = 30$$

$$T = 30 + 273 = 303\text{K}$$

Alternatively for (b), once 0 has been obtained in (a), we can apply the conversion formula as follows:

$$T = t + 273$$

$$T = \theta + 273$$

$$T = 30 + 273 = 303\text{K}$$



**(c) Constant Volume Gas Thermometer Scale**

- (i) Upper fixed point is the triple point of water.

**Definition**

The triple point of water is the temperature at which ice liquid water and water vapour coexist in equilibrium.

- (ii) Lower fixed point is the absolute zero or Ok (-273°C).

The scale is usually calibrated at these two fixed points and other temperatures are obtained by measuring the pressure of the gas at the unknown temperature and making use of the definition of temperature in the scale given below.

Determination of Temperature on the Constant volume Gas Thermometer Scale.

Suppose the unknown temperature =  $T$

Let temperature of the triple point of water =  $T_{tr} = 273.16K$

Let pressure of gas at the unknown temperature =  $P_T$

Let pressure of gas at triple point of water =  $P_{tr}$

$$\text{Using } \frac{T_1}{T_2} = \frac{P_1}{P_2}, \quad \text{we have}$$

$$\frac{T}{T_{tr}} = \frac{P_T}{P_{tr}}$$

$$\frac{T}{273.16} = \frac{P_T}{P_{tr}}$$

$$T = \frac{P_T}{P_{tr}} \times 273.16$$

For elementary work, where ice point is used instead of the triple point, we have

$$T = \frac{P_T}{P_{ice}} \times 273.16$$

**(d) Standard Thermometer Scale**

Early experiments measured temperature with mercury thermometers and often found that different instruments gave different readings for the same temperature. Other types of thermometers such as the platinum resistance or the gas thermometers which were used later had the same problem. It became clear that one type of thermometer based on one



scale of temperature would have to be taken as standard. Gas thermometers, either the constant volume or constant pressure types, showed the closest agreement over a wide range of temperatures. They were also very sensitive, accurate and highly reproduceable. It was observed that as the pressure was lowered (approaching zero), all gas thermometers indicate the same reading.

Lord kelvin suggested that the standard scale of temperature should be based on an imaginary ideal gas with properties that were those of real gases at very low i.e it obeyed Boyle's law. He proposed that the product of the pressure and volume of this ideal gas should be used as the thermometric property of the gas. Thus if  $P_1, P_2$  are the pressures and  $V_1, V_2$  the volumes of the gas at temperatures  $T_1, T_2$ , then

$$\frac{T_1}{T_2} = \frac{P_1 V_1}{P_2 V_2} \dots \dots \dots (1)$$

If the volume is kept constant, then we have

$$\frac{T_1}{T_2} = \frac{P_1}{P_2} \dots \dots \dots (2)$$

Which gives a definition of temperature on the constant – volume gas thermometer scale. If however, pressure is kept constant, we have

$$\frac{T_1}{T_2} = \frac{V_1}{V_2} \dots \dots \dots (3)$$

which gives a definition of the temperature on the constant-pressure gas thermometer scale.

For practical reasons (among which were the extreme sensitivity of the steam point to pressure changes) another upper fixed point, the triple point of water, was chosen to replace the steam point as the upper fixed point on the proposed standard temperature scale. The absolute zero (OK) was chosen as the lower fixed point.

As it has already been pointed out either the constant-volume or constant-pressure gas thermometer could be reliably used for measuring temperatures in this scale since at low pressures all gases (including the imaginary ideal gas) give the same reading. However, the constant – volume gas thermometer has been chosen and hence the standard thermometer scale “is the same as the constant-volume gas thermometer scale” It is also called “ideal gas scale “or thermodynamic temperature scale.

We have already defined unknown temperature on this scale as follows:



$$T = \frac{P_T}{P_{tr}} \times 273.16$$

where  $T$  is the unknown temperature,  $P_T$  is the pressure of the gas at the unknown temperature,  $T$ ,  $P_{tr}$  is the pressure of the gas at the triple point of water and 273.16 is the triple point of water.

#### (d) International Practical Temperature Scale (IPTS)

Measuring temperatures on the standard thermometer scale (or ideal gas scale) using the constant –volume gas thermometer is a tedious exercise. The international temperature scale (ITS) was therefore proposed and adopted as a more practical scale for general use. The scale consists of eleven primary fixed points ranging from the triple point of hydrogen (13.81K) to the freezing point of gold (1337.58K), which have been accurately determined on the ideal gas scale by a constant volume gas thermometer and also some other secondary fixed points. Particular types of thermometer are specified for measuring temperatures over a certain range using agreed formulas. At the fixed points, there is agreement between the IPTS and the Kelvin scale, the differences at the intervening temperatures being usually negligible.

### 3.4 Types of Thermometer

#### 3.4.1 Liquid – in – Glass Thermometer

This is the most common type of thermometer. Temperature is measured by measuring the volume of a fixed mass of liquid inside the thermometer owing to change in temperature. Mercury and alcohol are the common liquids used in this type of thermometer but mercury thermometers are more commonly in use. Mercury is preferred because.

- (i) It is easily seen
- (ii) It expands almost uniformly with increase in temperature,
- (iii) It does not stick to or wet glass
- (iv) Its temperature range,  $-39^{\circ}\text{C}$  to  $357^{\circ}\text{C}$ , is convenient for normal use.

Figure .4.5 shows a mercury-in-glass thermometer with fixed points indicate in the Celsius and Kelvin scale of temperature.

The thermometer is made sensitive by:

- (a) using thin glass for the bulb to enable the liquid in the bulb to assume the temperature of its surrounding very quickly,



- (b) using a narrow capillary tube of uniform bore to make it possible for small temperature changes to cause large changes in volume of mercury (or length of mercury thread).
- (c) using a liquid of high expansivity such as mercury (note the properties of mercury listed above earlier).

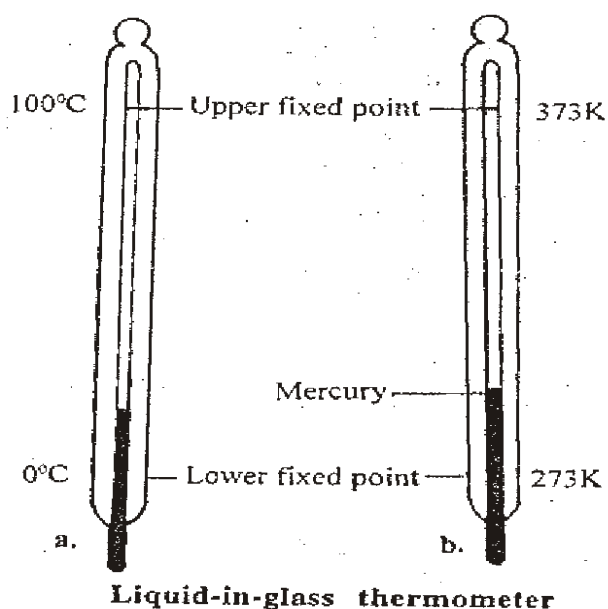


Fig. 4.5: Mercury –in – glass thermometer

### Water as a thermometric liquid

Water is unsuitable as a thermometric liquid for the following reasons:

- (i) It has a small range of expansion – it freezes at 0°C and boils at 100°C
- (ii) It does not expand uniformly and between 0°C and 4°C it contracts when heated and expands when cooled (anomalous expansion).
- (iii) It wets glass (unlike mercury it leaves traces of water as it moves on the glass).
- (iv) It is colourless, making the mercury in glass difficult to read.



### 3.4.2 Comparison of Mercury and Alcohol as Thermometric Liquids

The following (Table 4.2) shows the comparison of mercury and alcohol as thermometric liquids.

**Table 4.2: Comparison of Mercury and Alcohol as Thermometric Liquids**

	<b>Mercury</b>	<b>Alcohol</b>
1.	Has greater conductivity and expands rapidly and so indicates a temperature change quickly	Has much less conductivity and expands slowly and so does not respond quickly to temperature change
2.	Has silvery surface which makes it opaque and hence can be seen easily.	Needs to be coloured since it is colourless
3.	Does not wet glass because of its convex meniscus and thus enables the meniscus to give accurate readings of temperature	Wets glass because of its concave meniscus and so tends to cling to the walls of stem of thermometer resulting to inaccurate readings especially when the temperature is falling
4.	Does not vaporize easily	Vaporises easily even at low temperatures
5.	Can be used to measure higher temperatures since it boils at 357°C	Boils at 78°C and so cannot be used to measure temperatures above 78°C
6.	Freezes at -39°C and so not suitable for measuring temperature lower than this value	Freezes at -115°C and so is suitable for measuring much lower temperatures than mercury thermometers.
7.	Has much lower expansivity for the same	Has a much higher expansivity. Expands, six times, as much as



	rise in temperature.	mercury for some rise in temperature.
--	----------------------	---------------------------------------

Points 1-5 above represent the advantages of mercury over alcohol as a thermometric liquid while points 6 and 7 represent the advantages of alcohol over mercury (or the disadvantage of mercury over alcohol).

Other types of liquid – in – glass thermometer. Apart from the type of mercury –in-glass thermometer commonly used in schools and hospitals, we also have

### (a) Clinical Thermometer

This type of thermometer is used in hospital for measuring the temperature of the human body. It has range of  $35^{\circ}\text{C}$  to  $43^{\circ}\text{C}$  because the normal temperature of a healthy person is about  $37^{\circ}\text{C}$  but may rise to about  $41^{\circ}\text{C}$  during high fever. This is illustrated in Fig. 4.6 below:

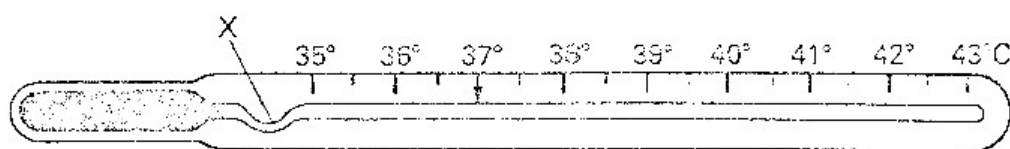


Fig. 9.3 Clinical thermometer

Fig. 4.6: Clinical thermometer

### (b) Maximum and Minimum Thermometer

This type of thermometer is used by weathermen in the meteorological department to record maximum and minimum temperature for a day in meteorological stations. The information is used to predict weather. Six's combined maximum and minimum thermometer named after the inventor, James Six, is most commonly used. Fig. 4.7 illustrates the major features of the thermometer. Two steel indices  $A_1$  and  $A_2$ , floating in the alcohol in both aims of the tube, are used to read the minimum and maximum temperatures respectively.

When temperature rises, the alcohol expands pushing the mercury round the thermometer. As a result the steel index  $A_2$  is pushed upwards. Its lower end indicates the maximum temperature, which is read from the attached scale. Decrease in temperature cause the alcohol to contract so



that mercury moves up the minimum arm, pushing the steel index  $A_1$  upwards.

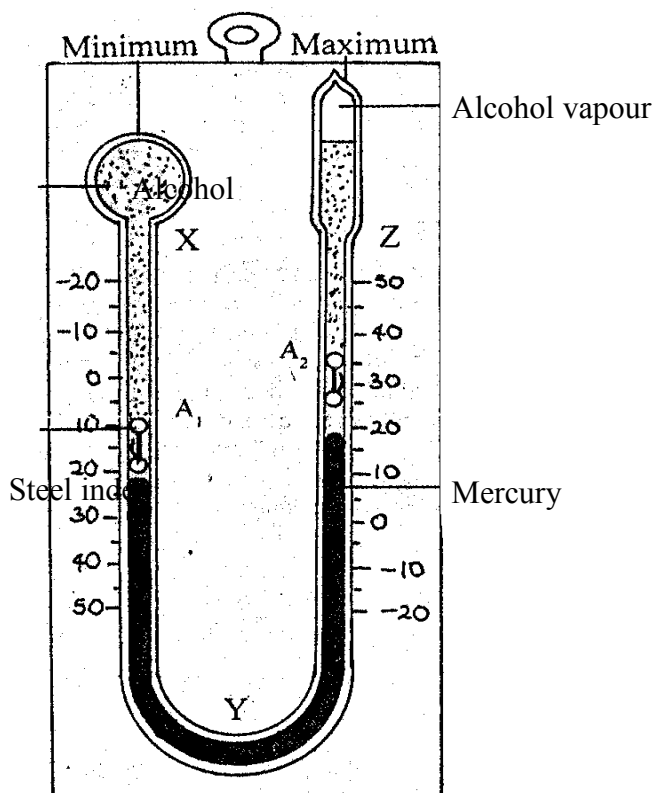


Fig. 4.7: Six's maximum and minimum thermometer

The lower end of this index ( $A_1$ ) indicates the minimum temperature reached. Tiny springs attached to  $A_1$  and  $A_2$  enable them to remain in positions to which they have been pushed. A magnet is used to reset the indices by bringing them in touch with the mercury column more.

### 3.4.3 Gas Thermometers

Liquid-in-glass thermometers are not very suitable for very accurate measurements of temperature. This is mainly because the glass containing the mercury or alcohol also expands with the liquid in an irregular manner. Again such thermometers have relatively small temperature range determined by the boiling and freezing points of the liquids used.

For accurate temperature measurements, gas thermometers are used. They are usually of two main types: constant-pressure gas thermometers in which the pressure or volume respectively of a fixed mass of gas increases linearly as temperature increases. Constant volume gas



thermometers are more commonly used than their constant – pressure counter – parts.

### Constant Volume Gas Thermometer

Fig. 4.8 shows the main features of a constant volume gas thermometer. It consists of a flask containing dry air or nitrogen connected to a manometer tube by means of a capillary tube. The mark A is the constant volume mark. In order to measure the pressure of the gas in the flask, the height of the mercury reservoir B is adjusted until the mercury level in the capillary tube reaches the level A, the

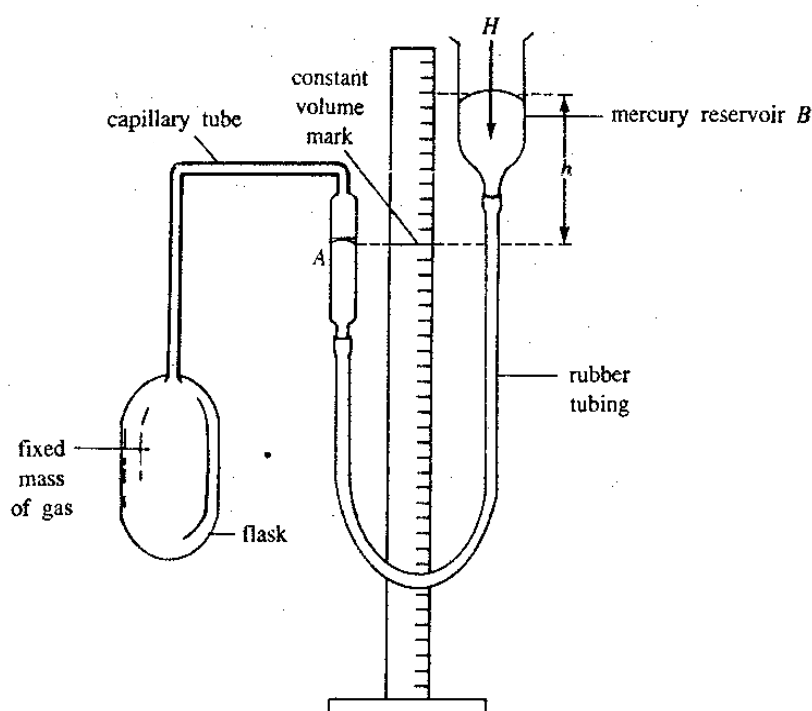


Fig. 4.8: Constant volume gas thermometer

$h$  = difference in mercury levels in the manometer at A and B (measurement in cm),

$H$  = height of mercury barometer (atmospheric pressure) then the pressure ( $p$ ) of the gas in the flask is given by the total pressure on the gas

$P$  = the total pressure on the gas ( $H + h$ ) cm mercury where  $H$  = atmospheric height.



$h_0$  = height of mercury at ice point

$h_{100}$  = height of mercury at the steam point,

$h_\theta$  = height of mercury at the temperature  $\theta$  to be measured,

$$\text{then } \theta^\circ\text{C} = \frac{\frac{P_\theta - P_0}{P_{100} - P_0} \times 100}{\frac{(H + h_\theta) - (H - h_\theta)}{(H + h_{100}) - (H - h_\theta)}}^\circ\text{C}$$

$$\theta = \frac{h_\theta - h_0}{h_{100} - h_0}$$

### 3.4.4 Platinum Resistance Thermometer

Fig. 4.9 shows the main features of a platinum resistance thermometer which makes use of the fact that the electrical resistance of a metallic conductor changes proportionally with its temperature. If the temperature increases, the resistance also increases.

Platinum Resistance Thermometer

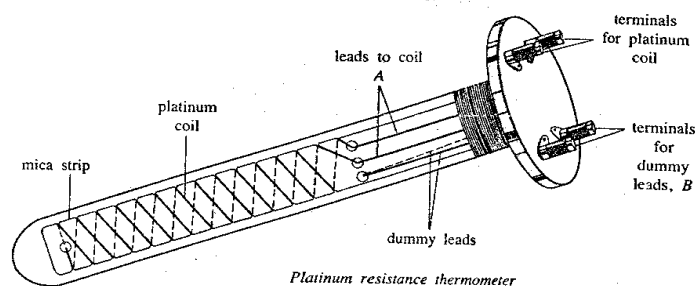


Fig. 4.9: Platinum resistance thermometer

The thermometer consists of a long thin platinum wire wound (non-conductively) round a small spool made of mica or silica. The ends of the wire are connected to a resistance measuring device such as the metre bridge. If a given platinum wire has a resistance  $R_0$  at  $0^\circ\text{C}$ ,  $R_{100}$  at  $100^\circ\text{C}$  and  $R_\theta$  at an unknown temperature  $\theta$ , then the unknown temperature is given by

$$\theta = \frac{R_\theta - R_0}{R_{100} - R_0} \times 100^\circ\text{C}$$



In most cases, the temperature scale may be calibrated to read the temperature directly.

Platinum resistance thermometers are useful for measuring very low and very high temperatures, usually of range  $-180^{\circ}$  to  $1,150^{\circ}\text{C}$ .

### 3.4.5 Thermoelectric Thermometer

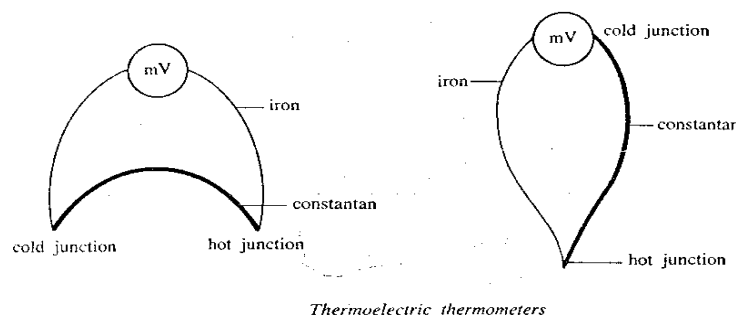


Fig 4.10: Thermoelectric thermometers

A thermoelectric thermometer is essentially a thermocouple and works on the principle of the thermometer. It consists of two different metals, e.g. copper and constantan joined at the ends. One end, the hot junction, is heated (or cooled) while the other end, the cold junction, is kept constant at the temperature of melting i.e. so that a current (usually in milliamperes) flows along the metals. The thermometer is connected in series to a potentiometer which measures the e.m.f across the ends of the metals in millivolts. The value of the e.m.f depends on the difference between the temperature of the hot and cold junction.

$$\begin{aligned} \text{Suppose } E_0 &= \text{emf at } 0^{\circ}\text{C (ice point)} \\ E_{100} &= \text{emf at } 100^{\circ}\text{C (steam point)} \\ E_{\theta} &= \text{emf at the unknown temperature } \theta, \end{aligned}$$

$$\text{Then } \theta = \frac{E_{\theta} - E_0}{E_{100} - E_0} \times 100^{\circ}\text{C}$$

Thermoelectric thermometers are particularly useful for measuring very high temperatures in industry.

Advantages of the thermometer are as follows:

- (i) It is very sensitive and can measure rapidly changing or momentary temperatures
- (ii) It can measure high temperature up to  $500^{\circ}\text{C}$ .
- (iii) It is very small in size and so can measure the temperature at a point.



### 3.4.6 Optical Pyrometers

When it is required to make accurate measurements of very high temperature, optical pyrometers are used. This is done by observing the radiation from the hot body and the process is called pyrometry and the instrument used is a radiation pyrometer or optical pyrometer.

Optical pyrometers make use of the principle that very hot substances change colour at certain known temperatures. For example the temperature of steel, when it is below red heat, can be judged by its colour. Temperatures below red heat can also be estimated by the use of paints which change colours at known temperatures.

Radiation pyrometers can be used to measure temperatures (e.g. of furnaces) only above red-heat which is about  $600^{\circ}\text{C}$ . They fall into two major classes:

- (i) Optical radiation pyrometers, which respond only to visible light and
- (ii) Total radiation pyrometers, which respond to the total radiation from the hot body, heat and light

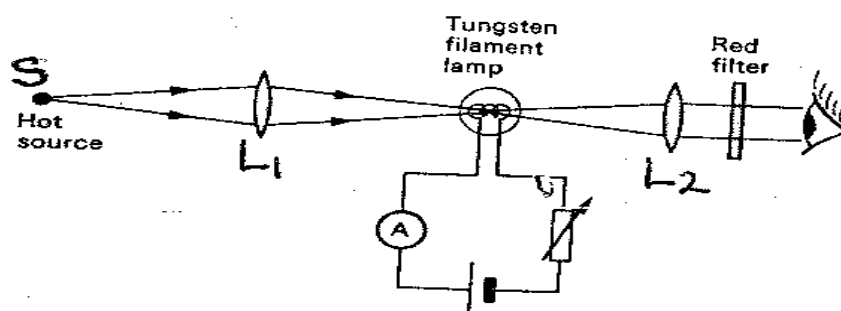


Fig. 4.11: Optical radiation pyrometer

The principle of the commonest type of optical radiation pyrometer also known as the “disappearing filament” optical pyrometer is illustrated in Fig. 4.11. Light from the hot source and the tungsten filament lamp is made to pass through a red filter of a known wavelength range before it reaches the eye. The eyepiece of the instrument (not shown in the figure but between the eye and the filter) is focused upon the filament of the tungsten light. The hot source, S whose temperature is required is then focused by the objective lens, L so that its image also lies in the plane of F. The light from both the hot source and the filament passes through



the red filter (of red glass) before reaching the eye. Both of them (source and filament) therefore appear red.

If the body is brighter than the filament, the filament appears dark on a bright background. If the filament is brighter than the source, it appears bright on a dark background. By adjusting the current through it, the temperature of the filament is adjusted until it merges as nearly as possible into its background. It is then as bright as the body or source and is said to have “disappeared”. The rheostat R which adjusts the current as well as the ammeter A which measures the current are both mounted on the body of the pyrometer.

The ammeter is usually calibrated to read the temperature directly in degrees Celsius. Radiation pyrometers of this type can measure temperatures up to  $1000^{\circ}\text{C}$  (i.e range between  $600^{\circ}\text{C}$  and  $1000^{\circ}\text{C}$  but more elaborated pyrometers can be adjusted to measure much higher temperatures. Generally the range of a radiation pyrometer can be extended by introducing a green filter between objective lense  $L_1$  and the filament lamp. This reduces the brightness of the red filter. When this is done, a second scale on the ammeter is provided for reading the temperature accurately when such a green filter is inserted

### Summary of Ranges, Advantages and Disadvantage of Thermometers

Table 4.3 presents a summary the ranges, advantages and disadvantages of the common types of thermometers discussed.

Table 4.3: A summary of types of thermometers

Types of thermometers and temperature range	Advantages	Disadvantages
Mercury-in-glass thermometer $-39^{\circ}\text{C}$ to $357^{\circ}\text{C}$	(i) Easily and quickly read – temperature read directly. (ii) Very portable. (iii) suitable for normal use.	(i) Limited range. (ii) Less accurate (iii) Easily broken.
Constant volume gas thermometer $-270^{\circ}\text{C}$ to $1\,500^{\circ}\text{C}$	(i) Sensitive. (ii) Accurate (iii) Large range (iv) Easily constructed. (v) Used for calibrating other types of thermometers.	(i) Difficult to use and time consuming. (ii) Volume of flask is large, therefore unable to measure temperature at isolated confined space.



	(vi) Used as standard thermometer for temperatures below $-183^{\circ}\text{C}$ .	
Platinum resistance thermometer $-180^{\circ}\text{C}$ to $1150^{\circ}\text{C}$	(i) Accurate. (ii) Wide range. (iii) Most suitable for measuring constant temperatures accurately. (iv) Used as standard thermometer for temperature between $-185^{\circ}\text{C}$ and $630^{\circ}\text{C}$ .	(i) Not suitable for measuring rapid changes in temperature because it takes time to attain thermal equilibrium with the surroundings.
Thermoelectric thermometer ---- $250^{\circ}\text{C}$ to $1150^{\circ}\text{C}$	(i) Sensitive. (ii) Wide range (iii) Most suitable for rapid changes of temperature. (iv) Able to measure temperature of small and isolated places. (v) Used as standard thermometer for temperatures between $630^{\circ}\text{C}$ and $063^{\circ}\text{C}$ .	(i) Less accurate if millivoltmeter is used to measure emf. A potentiometer used for measuring emf would be more accurate.
Radiation pyrometer above $600^{\circ}\text{C}$	(i) Accurate (ii) Sensitive (iii) Wide range (iv) Suitable for very high temperatures (e.g. furnaces) without risk or danger to persons involved.	(i) Not suitable for temperatures below $600^{\circ}\text{C}$

#### 4.0 CONCLUSION



In this unit you learned about heat and temperature. You also learned about the principle used in temperature measurement, temperature scales and types of thermometers.

## 5.0 SUMMARY

1. Temperature is the degree of hotness or coldness of a body. It is measured by an instrument called thermometer and it is a scalar quantity.
2. Heat is a form of energy (thermal energy) which flows from one point to another as a result of temperature difference between the points.
3. Thermometers employ any physical property of a substance which varies in a known way with temperature and is easily measurable.
4. Five major types of thermometers are in use, namely: liquid-in-glass thermometers, gas thermometer, resistance thermometers, thermoelectric thermometers and radiation pyrometers. Each type of thermometer has associated with it, a particular level of accuracy sensitivity, range, advantages and disadvantages.
5. The following temperature scale exists:
  - (i) Fahrenheit scale (no longer in common practical use)
  - (ii) Celsius Scale (also called Centigrade scale) on this scale, absolute zero is  $-273^{\circ}\text{C}$  or more accurately  $-273.15$ ; ice point is  $0.00^{\circ}\text{C}$ ; steam point is  $100.00^{\circ}\text{C}$  and triple point of water is  $0.01^{\circ}\text{C}$ .
  - (iii) Kelvin scale (also called “absolute scale”, “standard scale” or “thermodynamic scale”). The so called “ideal gas scale” or the “Constant volume gas thermometer scale” also belong to this class; On this scale absolute zero is  $0.00\text{K}$ , ice point is  $273.15\text{K}$ , steam point is  $373.15\text{K}$  and triple point of water is  $273.16\text{K}$ .
  - (iv) International practical temperature scale (IPTS) made up of eleven primary fixed points ranging from the triple point of hydrogen ( $13.81\text{K}$ ) to the freezing point of gold ( $1337.58\text{K}$ ) which have been accurately determined on the ideal gas temperature scale.



6. Kelvin scale of temperature is related to the Celsius scale by

$$T \text{ (K)} = 273 + \theta \text{ } ^\circ\text{C}$$

A more accurate relationship is

$$T \text{ (K)} = 273.15 + \theta \text{ } ^\circ\text{C}$$

Change of 1 K = change of  $1^\circ\text{C}$ .

The absolute zero of temperature = 0 (K) or  $-273^\circ\text{C}$  (or more accurately  $-273.15^\circ\text{C}$ )

7. Temperature  $\theta$  on the Celsius scale is defined by

$$\theta = \frac{X_\theta - X_0}{X_{100} - X_0} \times 100 \text{ } ^\circ\text{C}$$

where  $X$  is the physical property varying with temperature,

$X_\theta$  = value of  $X$  at  $\theta^\circ\text{C}$

$X_0$  = value of  $X$  at  $0^\circ\text{C}$ , and

$X_{100}$  = value of  $X$  at  $100^\circ\text{C}$ .

## SELF ASSESSMENT EXERCISE 2

- 1
  - (a) Distinguish between heat and temperature
  - (b) Name three physical properties used in the construction of thermometers and state how each property varies with temperature.
- 2
  - (a) Give reasons why water is not suitable as a thermometric liquid.
  - (b) State four advantages and two disadvantages of mercury as a thermometric liquid.
- (3) Explain each of the following with respect to a thermometer:
  - (i) fixed point
  - (ii) Fundamental interval.

A faulty thermometer reads  $0.5^\circ\text{C}$  at the ice point and  $99.2^\circ\text{C}$  at the steam point. What is the correct temperature when the thermometer reads  $20^\circ\text{C}$  and at what temperature will its reading be exactly correct?



- (4) (a) Name five different types of thermometer and describe the working of any two of them.
- (b) List five different scales of temperature and briefly describe each one of them.
- (5) A fixed mass of gas at constant pressure has a volume of  $200.0 \text{ cm}^3$  at the ice point,  $273.2 \text{ cm}^3$  at the steam point and  $525.1 \text{ cm}^3$  at the melting point of a solid. What is the melting point of the solid on the constant pressure gas scale of temperature?

## 6.0 TUTOR-MARKED ASSIGNMENT

1. Define an unknown temperature  $\theta$  on the Celsius temperature scale using a platinum resistance thermometer. State the principle on which such a thermometer functions. The resistance of a platinum resistance thermometer wire is  $10.40 \text{ ohms}$  at  $00^\circ\text{C}$  and  $10.71 \text{ ohms}$  at  $100^\circ\text{C}$ . Calculate the temperature when the resistance is  $9.61 \text{ ohms}$ .
2. Explain the meaning of “fixed points” and “fundamental interval” of a thermometer. The ice and steam points of a thermometer are marked  $40$  and  $120$  respectively. What is
  - (i) The temperature in  $^\circ\text{C}$  when the reading on this is  $60$ ,
  - (ii) The thermometer reading when the temperature is  $60^\circ\text{C}$ ?
3. The fixed point of a standard thermometer is the triple point of water which is given the value  $273.16\text{K}$ 
  - (i) What is meant by the fixed point of a thermometer?
  - (ii) Why is the triple point temperature given the value  $273.16\text{K}$
4. Suggest a suitable thermometer to measure the following temperatures:
  - (a) A pool of molten lead (melting point of lead is about  $600\text{K}$ )



- (b) A liquid nitrogen bath at its normal boiling point (about 80K). For each case, outline the working principle of the thermometer.

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## **UNIT 5      TRANSFER      OF      HEAT      AND      HEAT CAPACITIES**

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## 1.0 INTRODUCTION

This unit will introduce you to the different modes (conduction, convection and radiation) of transferring heat energy from one place to another. You will learn to explain the modes of heat transfer using kinetic molecular theory. In addition, you will learn the important concepts of heat energy – the heat capacities, their measurements and determination. Finally, you will learn Newton's law of cooling and cooling correction.

## 2.0 OBJECTIVES

**By the time you finish this unit, you will be able to:**

- explain conduction, convection and radiation in terms of kinetic molecular theory
- know the difference between good and bad conductors of heat using metals water and air as examples
- explain the relationship between the heat supplied to a substance and
  - its temperature change at constant mass
  - Its mass at constant temperature changes
- explain the terms specific heat capacity and thermal capacity
- explain why there is unequal rise in temperature for different substances of the same mass supplied with the same quality of heat
- state the assumptions underlying the experiments involving the measurements of heat capacities
- calculate unknown qualities using the relation  $h = mc \Delta t$  when no change of state is involved
- state Newton's law of cooling
- explain the Newton's law of cooling; and the cooling correction.

## 3.0 MAIN CONTENT

### 3.1 Conduction of Heat

It is an everyday experience that when you dip a silver spoon into a hot tea, the handle of the spoon very quickly feels hotter than it was before. The heat from the hot tea has been transferred along the metal handle to the other end of the spoon by the process of conduction.



Conduction of heat is a process by which heat energy is transferred through a material, the average position of the particles of the material remaining the same.

Heat energy is always transferred when different parts of a solid body are at different temperatures. The direction of heat transfer is always from the hotter to the cooler parts of the solid.

### 3.1.1 Good and Bad Conductors of Heat

Most metals such as copper, aluminum, silver, iron, allow heat energy to pass through them very easily. We refer to such materials as good conductors of heat. They are used where heat is required to travel quickly (eg the cooking utensil). On the other hand, we refer to those materials which do not allow heat to pass easily through them as poor conductors of heat or insulators. Examples of such materials are most non-metals such as water, air, wood, plastic, cloth, cotton wool, cork. The handle of a cooking utensil is made of wood or plastic which are insulators so that heat cannot be conducted quickly through them.

The ability of a metal to conduct heat is known as its thermal conductivity.

### 3.1.2 Experiment to compare the thermal conductivities of materials

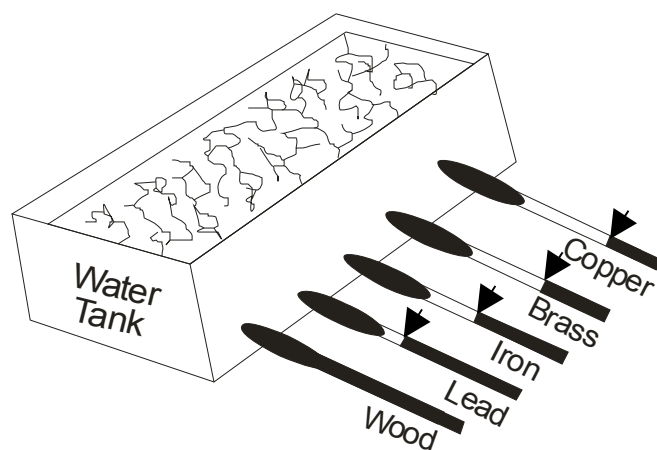


Fig. 5.1: Comparison of thermal conductivities

An experiment to compare the conductivity of different substances is illustrated Fig. 5.1.

A metal container has rods of different materials but of the same length and diameter fitted into holes in one of the sides by means of corks as



shown. The rods are first dipped into molten paraffin wax; when they are withdrawn, a coating of wax solidifies on them. Boiling water is then poured into the container so that the ends of the rods are heated to the same temperature. The wax on the rods begins to melt from the hot ends and after some time, it is found that the wax has melted to different distances along the rods, with the copper rod having the longest length, followed by aluminum, iron, lead while the wooden rod hardly melts at all. This indicated that the different substances have different thermal conductivities. Thus, in this experiment, copper has the greatest while wood has the least thermal conductivity.

### 3.1.3 Explanation of Conduction in Terms of Kinetic Molecular Theory

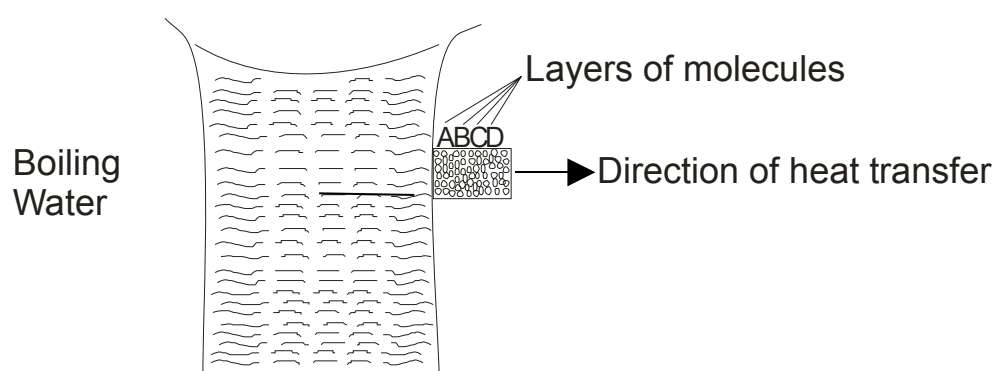


Fig. 5.2: Molecular explanation of conduction

Heat is transferred through solids mainly by conduction because of the nature of solid molecules which are very close together. When we heat a portion of solid directly as shown in Fig. 5.2, the solid molecules that are directly heated (layer A) absorb the heat and vibrate faster than before the heat was applied. They do not move from place to place because of the strong forces of attraction between them. As they vibrate, they pass on some of their heat energy to their neighbours (layer B) which in turn vibrate faster and pass on some of the heat to the next layer of molecules (C), and so on.. The heat continues to move in this way until it reaches the end of the solid. All the molecules of the solid eventually vibrate more rapidly about their fixed positions. We say that thermal energy is transferred along the solid although the average position of the molecules remains unchanged. When heat is transferred in this way, it is said to be conducted along the body.



### 3.1.4 Thermal Conductivity of Liquids and Air

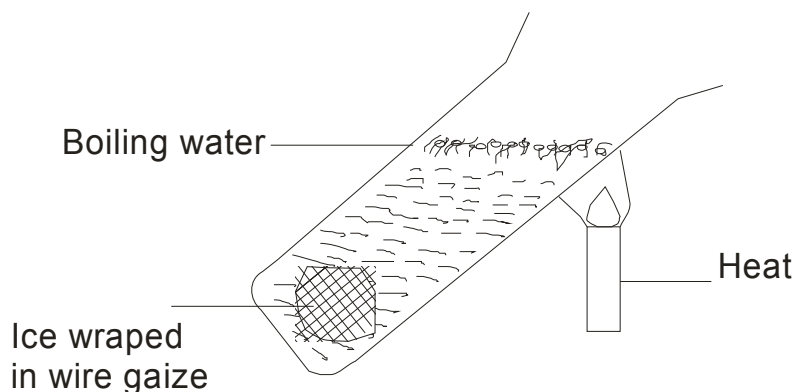


Fig. 5.3 Water, a bad conductor of heat

Water and other liquids are poor conductors of heat. The only exception is mercury which is a metal in liquid form.

You can verify that water is a poor conductor of heat by the following simple experiment.

You will wrap a piece of ice in wire gauze to make it sink to the bottom of a test tube of water. Heat the top of the water as shown in Fig. 5.3 (to prevent convection).

You will observe that the ice does not melt even when the water at the top of the tube boils. This shows that water is a bad conductor of heat.

To show that air is a poor conductor of heat, hold your hand very close but not touching the side of a kettle of boiling water. You can feel the radiation but your hand is not burned although separated from the kettle by a thin layer of air.

### 3.1.5 Practical applications of Good and Bad Conductors

Here, we will consider the following practical applications of good and bad conductors

- Cooking pots and frying pans –made of metals to ensure quick transfer of heat from the fire to the food being cooked. The handles of cooking utensils are made of insulators so that the utensils when hot can be held comfortably by the handles.
- Home cooling in the tropics – asbestos ceiling and thatched-roof houses prevent heating up of houses because their materials are insulators and do not conduct the heat from the sun into the room.



- The use of cloth to keep warm – clothes keep us warm by trapping air (poor conductors) around the body. Clothing made from woollen and fur materials are worn in the cold climates to retain heat and keep the body warm.
- Use of rugs on floors – Heat transferred from one's foot to the floor is conducted away quickly by the floor and the foot feels cold but the rug does not conduct away heat rapidly. It heats up to the temperature of the foot.
- Cotton wool and air used as insulators to reduce to the minimum the loss of heat in calorimeters. This is called lagging.

### **3.2 Convection of Heat through Liquids and Gases**

You have seen that liquids and gases are generally poor conductors of heat. However, heat is effectively transferred in them by convection.

- Convection is the process by which heat is transferred in a liquid or gas by the actual movement of the heated fluid from the hotter to the colder parts.

#### **3.2.1 Convection Current in Liquids**

You can observe convection in liquids by performing this simple activity.

You will fill about two-thirds of a large round bottomed flask with water. Drop a tiny crystal of potassium permanganate, using a long glass tube, at the bottom of the flask. Heat the bottom of the flask just below the crystal gently and observe the movement of the coloured liquid.

You will observe that the colour streaks of the potassium permanganate rises upwards from the bottom of the flask where the heat was applied. The streaks curve downwards at the top of the water as shown in figure 5.4 below.



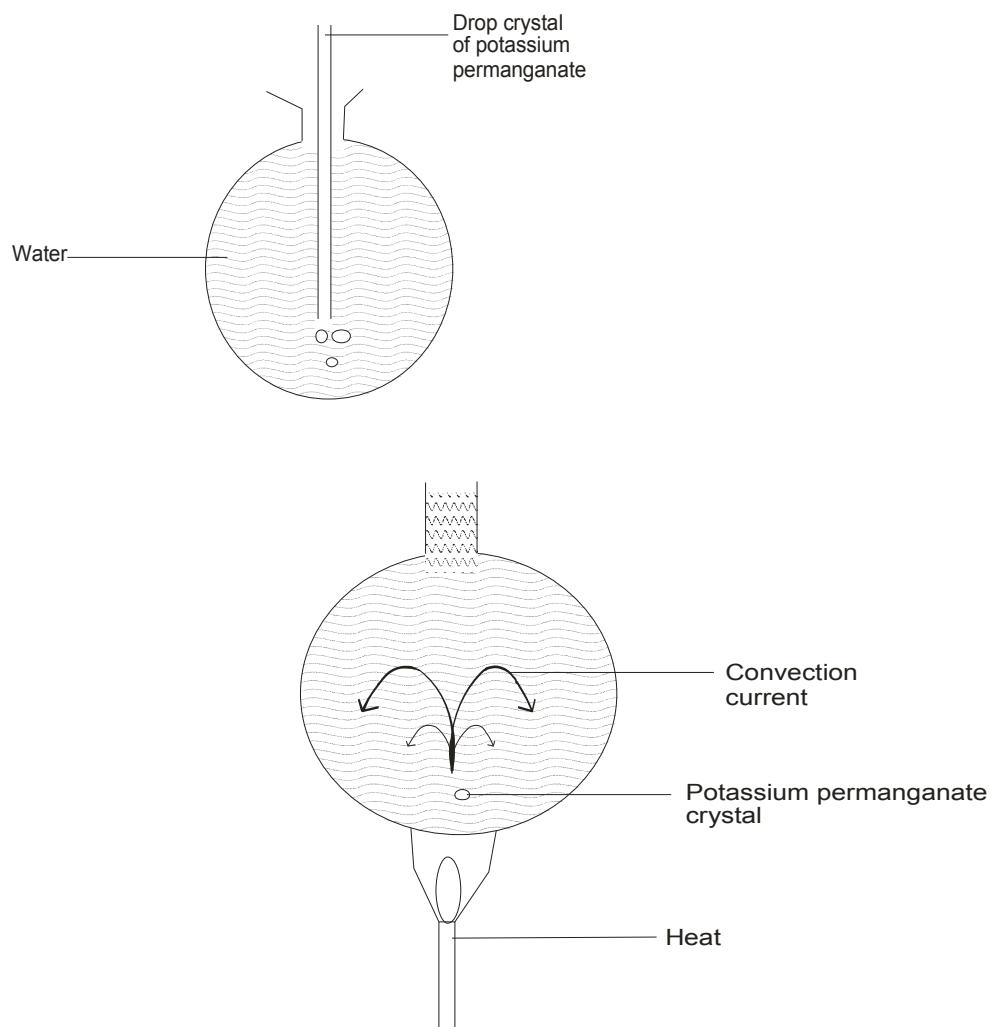


Fig. 5.4: Convection current in liquid

The upward movement of the coloured streaks is due to convection and the circulation of the liquid is called convection current. The movement of the convection current is explained as follows:

The heated liquid at the bottom of the vessel expands and rises up because warm water has a smaller density than cold water. The ascending water carries some heat with it and the colder, denser water falls to the bottom where it gets heated itself and also rises. This sets up convection current which circulates continually.

### 3.2.2 Explanation of Convection in Terms of Kinetic Molecular Theory (KMT)

We explain convection in terms of KMT as follows:

When the water is heated below, its molecules at the point of heating gain more kinetic energy, expand by vibrating farther away from their



fixed position, become less dense and therefore rise. The denser molecules from the top sink to the place of those that have risen above. These molecules in turn get heated and rise. With the heating, upward warm currents and downward cooler currents are established until the water boils.

### 3.2.3 Convection Current Artificial/Natural Current

Gases expand much more readily than liquids. As a result, convection currents are easily set up in gases when their temperature rises. Here, you will study the diagrams (Fig. 5.5) illustrating convection current in nature: Land and sea breezes, showing convection current in gases.

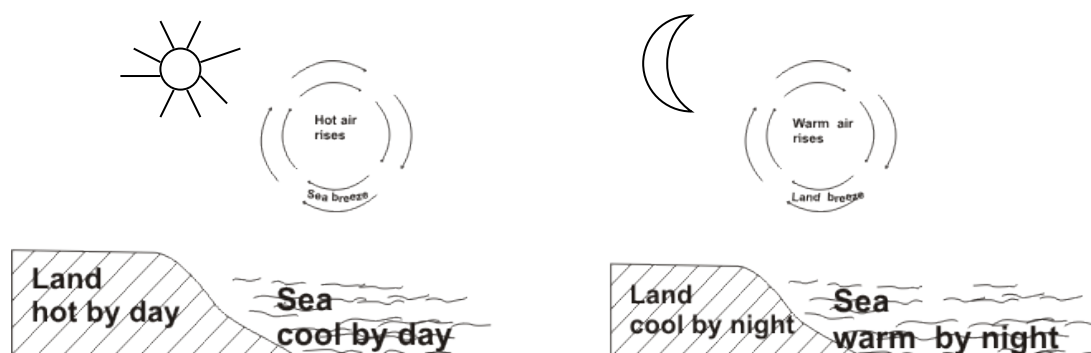


Fig. 5.5: Land and sea breezes

### 3.2.4 Practical Applications of Convection Current

Convection current is important to us in the following ways:

- **Land and sea breezes:**

In daytime, the sun warms the land to a higher temperature than the sea, because land has a lower specific heat capacity than sea-water and the sea is always in motion. The warm air above the land rises, and cooler air from the sea moves in and takes its place.

The convection current is completed by the air in the upper atmosphere moving down in the opposite direction, and we have sea breeze.

At night, the temperature of the land drops much faster than that of the sea. As a result the sea is warmer than the land. Convection current is set up in the opposite direction to the daytime. We call the breeze moving from the land to the sea land breeze. The process is illustrated in Fig. 5.5 above.



- Convection current is used in house ventilation, in motor car cooling systems in domestic hot water systems, and air conditioning of rooms.
- Glider pilots use convection current as air lift for their gliders.
- Winds are caused by convection currents in the atmosphere,. Near the equator hot air rises, and cooler and denser air from the Polar Regions flows in and takes its place.

### 3.3 Radiation of Heat

You experience radiant heat (heat energy given out by radiation) in your daily life heat from the sun and by warming yourself by the fire side. It requires no material medium.

Radiation is the process by which heat is transferred from a hotter to a cooler place without heating of the intervening medium.

We refer to radiation as electromagnetic wave of the type called infra red. It travels with the velocity of light ( $3 \times 10^8 \text{m/s}$ ). It can be detected by a radiometer and a thermopile.

#### 3.3.1 Emission and Absorption of Radiation by Different Surfaces

We know that different types of surfaces at the same temperature unit or radiate heat at different rates depending on the nature of the surface. For a given temperature, a dull black surface radiates most and a highly polished surface radiates least. You can verify this by performing this simply experiment.

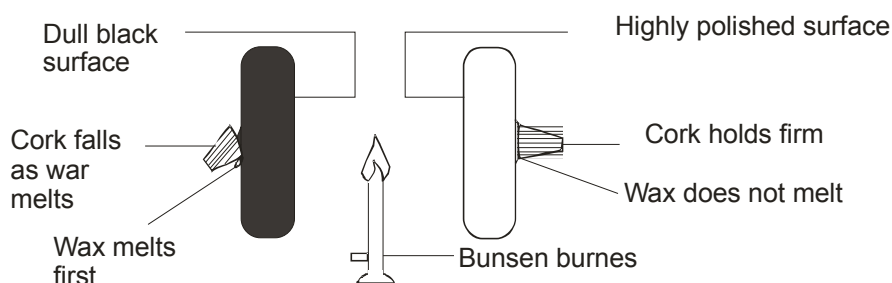


Fig. 5.6: Comparison of heat absorption by two surfaces



You will use two sheets of thin plates, one painted black and the other highly polished as shown in Fig. 5.6. Attach a cork to the reverse side of each plate using a candle wax. Place a hot object like Bunsen burner or burning candle half way between the two surfaces fixed vertically facing each other and a few centimetres apart.

After a few minutes, you will notice that the wax behind the dull black surface begins to slide off the plate. The wax behind the polished surface does not melt and the cork remains fixed.

From this activity you can conclude that the dull black surfaces are better absorbers of radiant heat than polished surface. Also, good emitters of heat are good absorbers.

Heat energy falling on a body is partly absorbed and partly, reflected. Black surfaces are good absorbers and poor reflectors; poor absorbers are good reflectors of heat.

### 3.3.2 Applications of Radiation

- Heat from the sun gets to us by the process of radiation.
- House roofs in the tropics as well as factory roofs are brightly painted in shiny aluminium colour to reduce absorption of heat during the day. It also reduces the heat loss by radiation at night.
- It is not advisable to wear a dark coloured jacket in hot sunshine because it will absorb heat and make the wearer feel hot and uncomfortable. Also, a brightly painted car is preferred to a black-painted car in Nigeria and other hot tropical countries. The black-painted car will absorb and retain heat from the sun and inside the car will be very hot.
- Petrol storage tanks are sprayed with silver paints to reflect heat rays falling on it.
- A teapot has a silvery surface and an electric iron has a silvered surface base in order to make each capable of retaining its heat for a long period.
- \* A fire-fighting suit is bright and shiny so that it does not absorb much heat to burn the fire man.



**SELF ASSESSMENT EXERCISE 1**

1. Vacuum flask is a good application of our knowledge of the three modes of heat transfer. Look carefully at the vacuum flask in Fig. 5.7 and answer the question below.

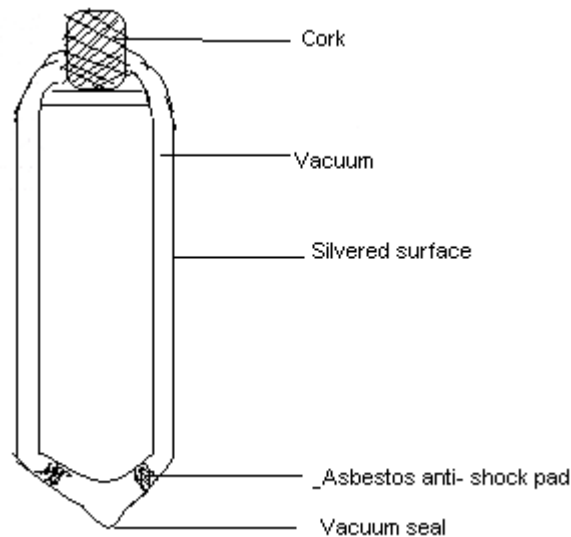


Fig. 5.7 :Vacuum flask

- (i) Why is it that a vacuum surrounds the liquid?
  - (ii) Why are the glass surfaces silvered?
  - (iii) Why is it that a cork or plastic stopper is used? (two reasons)
  - (iv) How will the heat escape if the vacuum seal is damaged.
2. An electrician is fitting an electrical heater 3.4 into a water tank. Explain whether he should put the heater at the top or at the bottom in order to heat the water in the tank.
3. Two motor cars of the same type are left in the sun. One is painted black and the other white. Which will be hotter inside? Why?
4. Explain why a tile floor feels colder to the feet than does a rug in the same room.



### 3.4 Heat Capacity and Specific Heat Capacity

#### 3.4.1 Heat capacity – definition, symbol and unit

As you know, the supply of heat to a substance increases the average kinetic energy of the molecules of the substance hence, an increase in temperature of the substance. The average kinetic energy gained by the molecules per degree Celsius rise in temperature is the heat (thermal) capacity of the substance.

##### Definition

The heat capacity of a substance is defined as the heat energy required to raise the temperature of the substance by  $1^{\circ}\text{C}$  (or  $1\text{K}$ ).

##### Symbol

C; S.I. unit: joule per Kelvin ( $\text{J/K}$ ).

#### 3.4.2 Specific Heat Capacity – Definition, Symbol and Unit

From simple experiment, we know that equal masses of different substances require different quantities of thermal (heat) energy to raise them through the same temperature range. To take care of these differences and to make it possible to calculate the amount of heat energy required to raise the temperature of a substance through a definite temperature range, the idea of specific heat capacity was introduced.

##### Definition

Specific heat capacity of a substance is the amount of heat required to raise the temperature of unit mass of the substance by  $1^{\circ}\text{C}$  (or  $1\text{K}$ ).

##### Symbol

C; S.I unit joule per kilogram per Kelvin ( $\text{Jkg}^{-1}\text{K}^{-1}$ )

You will note that

Heat capacity of a substance

C = Mass of the substance x specific heat capacity of the substance.



When the temperature of a substance changes, heat energy given out (or received).

$$Q = \text{mass} \times \text{specific heat capacity} \times \text{temperature change.}$$

In symbols:

$$C = mc \dots\dots\dots(1)$$

$$\text{and } Q = mc (\theta_2 - \theta_1) \dots\dots\dots(2)$$

where

$C$  = heat capacity of the substance

$Q$  = heat energy

$M$  = mass of substance

$c$  = specific heat capacity of the substance

$\theta_2$  = higher temperature

$\theta_1$  = lower temperature

$(\theta_2 - \theta_1)$  = temperature change)

We call  $Q$  the quantity of heat – it is sometimes denoted by  $H$ .

### 3.5 Measurement of Specific Heat Capacity of a Substance

We call the instruments designed for experiments to measure quantities of heat and specific heat capacities calorimeters. The simple calorimeter used in school laboratories is usually made of copper which is a good conductor of heat. This enables it to reach the same temperature as its contents quickly.

The calorimeter is placed inside a larger container and the two are separated by layers of lagging material- a poor conductor of heat. To ensure an even temperature, a stirrer is provided while the lid prevents evaporation of the liquid in the calorimeter. Here, we are going to consider two methods of determining the specific heat capacities of solids and liquids.

#### 3.5.1 Method of Mixtures

- (i) Specific heat capacity of solids by the method of mixtures.



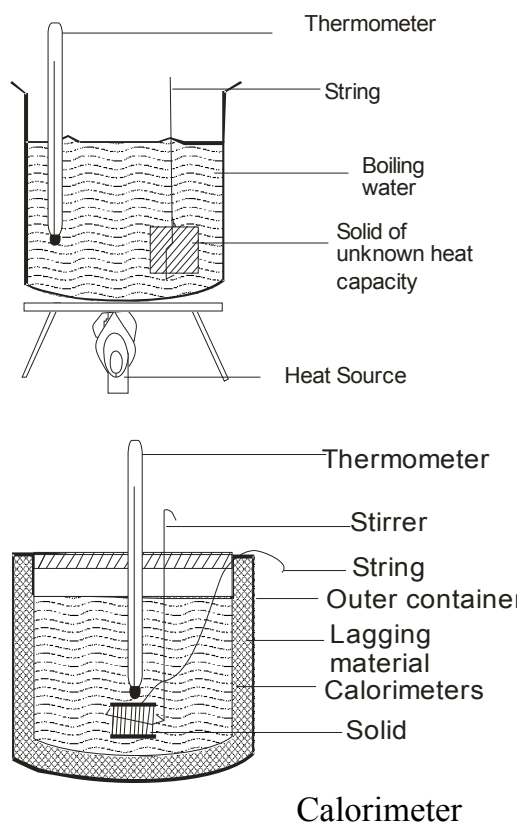


Fig. 5.8: Determination of specific heat capacity by the method of mixtures

In this experiment, the solid (eg a metal block) is first weighed and its mass recorded. It is heated in boiling water as shown in Fig. 5.8 for some minutes to allow it to assume the temperature of the boiling water. The solid is then quickly transferred to a copper calorimeter which had been previously weighed empty and when about half-full of water. The initial temperature of the water in the calorimeter is taken before the hot solid is transferred into it.

The mixture of solid and water in the calorimeter is gently stirred to attain even temperature. The highest temperature attained is recorded. The specific heat capacity is calculated as follows:

Mass of calorimeter	=	$m_1(\text{kg})$
Mass of calorimeter + water	=	$m_2(\text{kg})$
Mass of solid	=	$m_3(\text{kg})$
Initial temperature of water and calorimeter	=	$\theta_1^\circ\text{C}$
Temperature of solid in boiling water	=	$100^\circ\text{C}$
Temperature of mixture	=	$\theta_2^\circ\text{C}$
Mass of water	=	$(m_2 - m_1) \text{ kg}$
Specific heat capacity of water	=	$4200 \text{ J kg}^{-1} \text{ K}^{-1}$
Specific heat capacity of copper	=	$420 \text{ J kg}^{-1} \text{ K}^{-1}$
Let specific heat capacity of metal	=	$C \text{ J kg}^{-1} \text{ K}^{-1}$



$$\begin{aligned}
 \text{Heat lost by hot metal} &= m_3 c (100 - \theta_2) \text{ J} \\
 \text{Heat gained by water} &= (m_2 - m_1) \times 4200 \times (\theta_2 - \theta_1) \text{ J} \\
 \text{Heat gained by calorimeter} &= m_1 \times 420 \times (\theta_2 - \theta_1) \text{ J}
 \end{aligned}$$

Assuming no heat is lost to the surrounding and no heat is equally gained, we have that:

Heat lost by the solid = heat gained by the calorimeter and water.

$$M_3 C [(100 - \theta_2) = (m_2 - m_1) \times 4200 + (m_1 \times 420)] (\theta_2 - \theta_1) \dots (3)$$

You can calculate the value of  $c$  for the solid from equ (3) if the values of the other variables are known.

### Precautions

You will take the following precautions when performing the above experiment in the laboratory.

- Lag the calorimeter to prevent heat exchange with the surroundings.
- Transfer the hot metal quickly to the calorimeter without splashing
- Stir the mixture gently

(ii) Specific heat capacity of liquids by method of mixtures.

We carry out this experiment as above. But here, we use a liquid of unknown specific heat capacity instead of water in the calorimeter. We also use a solid of known specific heat capacity. The specific heat capacity is again calculated as follows:

$$\begin{aligned}
 \text{Mass of calorimeter} &= m_1 \text{ (kg)} \\
 \text{Mass of calorimeter + liquid} &= m_2 \text{ (kg)} \\
 \text{Mass of solid} &= m_3 \text{ (kg)} \\
 \text{Initial temperature of liquid + calorimeter} &= \theta_1^\circ\text{C} \\
 \text{Temperature of mixture} &= \theta_2^\circ\text{C} \\
 \text{Temperature of hot solid} &= 100^\circ\text{C} \\
 \text{Specific heat capacity of copper calorimeter} &= 420 \text{ J kg}^{-1} \text{ K}^{-1} \\
 \text{Known specific heat capacity of metal} &= C_1 \text{ J kg}^{-1} \text{ K}^{-1} \\
 \text{Let specific heat capacity of liquid} &= C_2 \text{ J kg}^{-1} \text{ K}^{-1} \\
 \text{Heat lost by hot metal} &= M_3 C_1 (100 - \theta_2) \\
 \text{Heat gained by calorimeter} &= M_1 \times 420 \times (\theta_2 - \theta_1) \\
 \text{Heat gained by liquid} &= (M_2 - M_1) \times C_2 \times (\theta_2 - \theta_1)
 \end{aligned}$$



Since,

heat gained = heat lost

We have

$$M_1 \times 420 \times (\theta_2 - \theta_1) + (m_2 - m_1) \times C_2 \times (\theta_2 - \theta_1) \\ = M_3 C_1 (100 - \theta_2) \dots\dots\dots(4)$$

The specific heat capacity of the liquid  $C_2$  would then be calculated from equ. 4 when all the other variables are known.

### 3.5.2 Electrical Method

Specific heat capacity of solid by electrical method

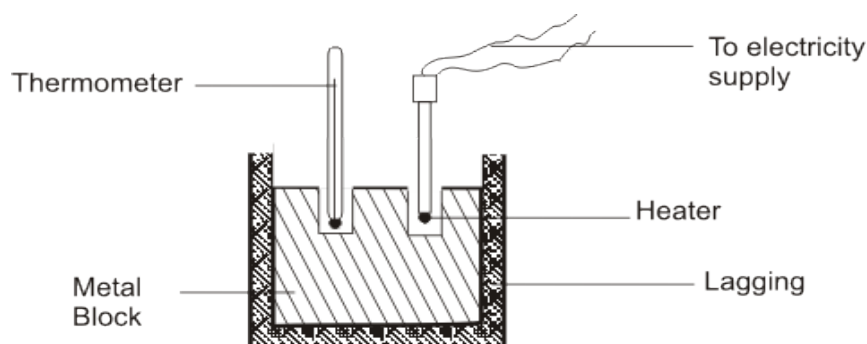


Fig. 5.9: Measurement of specific heat capacity of solid by electrical method.

Two holes are bored in the block whose specific heat capacity is to be measured as shown in Fig. 5.9. The block is weighed. A thermometer and an electrical heater are respectively inserted in each of the holes. A little oil is dropped in each hole to help establish a good thermal contact with the block. The metal block is surrounded with a lagged jacket to reduce heat losses.

The initial temperature of the block is noted and the electrical heater is switched on. The current is allowed to flow for some time until the temperature rises by about  $15^{\circ}\text{C}$ . The exact time of flow of a known current is measured with a stop-watch and the initial and final temperatures of the metal block are measured by the thermometer.

The specific heat capacity is calculated as follows:

Let: Mass of block =  $m$  (kg)  
 Value of steady current =  $I$  (A)  
 Value of p.d. across heater =  $V$  (v)



$$\begin{aligned}\text{Temperature rise of block} &= \theta^{\circ}\text{C} \\ \text{Time of flow of current} &= t \text{ (secs)} \\ \text{Specific heat capacity of block} &= C(\text{Jkg}^{-1}\text{K}^{-1})\end{aligned}$$

Heat supplied by current =  $H = 1Vt$  joules.

Heat required to raise the temperature of block

$$\text{By } \theta^{\circ}\text{C} = mc\theta$$

Assuming no heat loss to the surroundings, we have.

$$C = \frac{1Vt}{m\theta} = \frac{Mc\theta}{m\theta} \dots\dots\dots 5$$

(iii) Specific heat capacity of a liquid by the electrical method

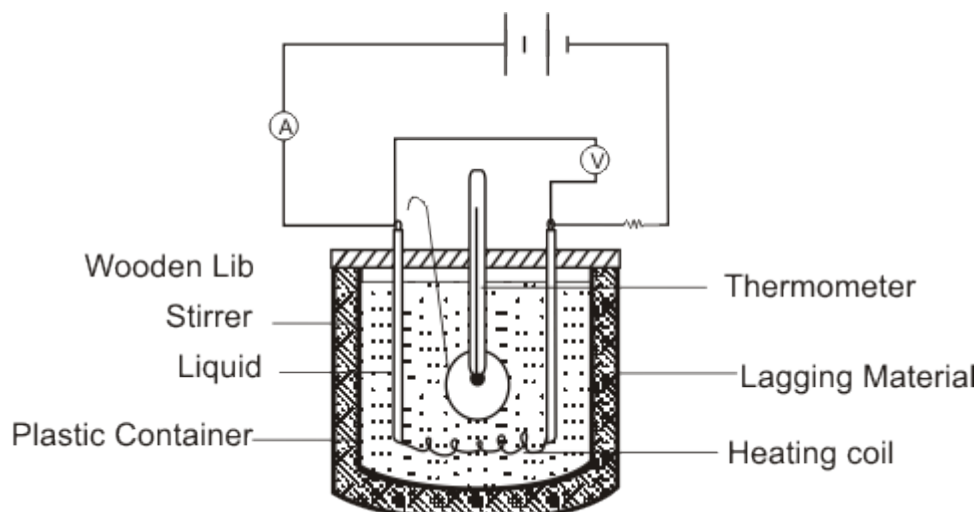


Fig. 5.10: Determination of specific heat capacity of liquid by electrical method

In this experiment, the apparatus is set up as shown in Fig. 5.10. First, we weigh a plastic container and then reweighed when about two-thirds full of the liquid (e.g. water). We then connect the ammeter, voltmeter and heating coil as shown in the circuit. A plastic stirrer and a thermometer are then fitted into the container through holes in the wooden lid. The initial temperature of the liquid is read and recorded. The current is now switched on and the rheostat adjusted to give a suitable steady current for about 3 minutes.

This is followed by stirring the liquid gently as it is being heated. The current is then switched off and the final temperature read.



We calculate the specific heat capacity of the liquid as follows:

Mass of liquid = m kg  
 Initial temperature of liquid =  $\theta_1$  °C  
 Final temperature of liquid =  $\theta_2$  °C  
 Current flowing in circuit = I A  
 Voltage across heating coil = V volts  
 Time of flow of current = t seconds  
 Let the specific heat capacity of liquid =  $C \text{ J kg}^{-1} \text{ K}^{-1}$   
 Rise in temperature

Heat gained by liquid =  $mc(\theta_2 - \theta_1)$  joules  
 Electrical energy supplied to liquid =  $1vt$  joules

Heat gained by liquid = electrical energy supplied  
 This implies that :

$$MC(\theta_2 - \theta_1) = 1Vt$$

$$C = \frac{1vt}{M(\theta_2 - \theta_1)} \dots\dots\dots 7$$

### Precautions

- Cooling the water with ice to about 10°C below room temperature to reduce heat lost to the surrounding.
- Heating water until its final temperature is about 10°C above room temperature for above reason.

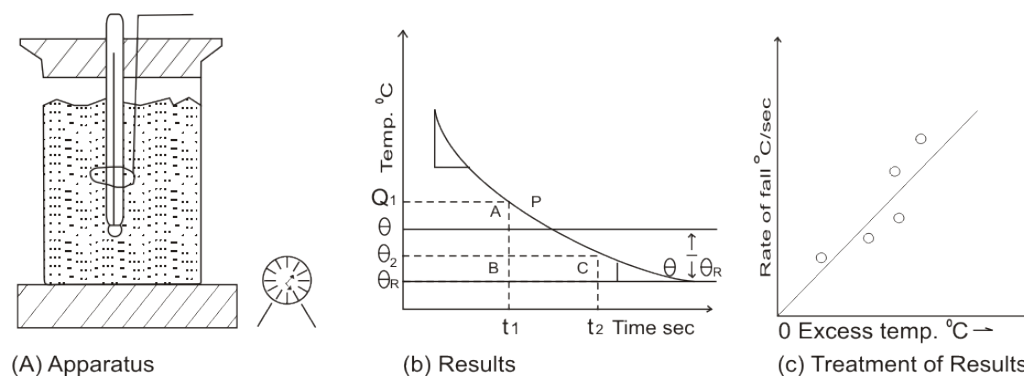
## 3.6 Newton's Law of Cooling

- Newton's law of cooling states that for a small difference of temperature between body and its surrounding, the rate at which a body loses heat is proportional to the temperature difference between the body and its surroundings.

The law is approximately true in still air only for a temperature excess of about 200°C or 300°C. It is true for all excess temperature in conditions of forced convection of the air, i.e. in a draught. At low excess temperature less than 10°C, radiation is the major contributing factor to the rate of cooling of an object.

- 3.6.1** To demonstrate Newton's law of cooling, we plot a temperature ( $\theta$ ) – time (t) cooling curve for hot water in a calorimeter placed in a draught (Fig. 5.11(a))





If  $Q_R$  is the room temperature, then the excess temperature of the water is  $(\theta - \theta_R)$ . At various temperature such as  $\theta$  in Fig. 5.11 (b), we may draw targets such as APC to the curve. The slope of the target in degrees per second, gives us the rate of fall of temperature when the water is at the temperature  $\theta$ .

If we then plot these rates against the excess temperature  $\theta - \theta_R$ , as in Fig. 5.11 (c), we will obtain a straight line passing through the origin.

The result of the experiment shows that the heat lost per second by water and calorimeter is proportional to the rate of fall of the temperature. This is in agreement with Newton's law of cooling.

### 3.6.2 Correction for Heat Losses in Calorimetry

When a hot body is transferred to a liquid in a calorimeter, the temperature of the liquid and the calorimeter rises rapidly above that of the surrounding. From Newton's law of cooling, you can deduce that an amount of heat is lost to the surrounding because allowance is not made for this when reading the final temperature of the mixture. As a result, the final temperature recorded is less than the actual final temperature and correction has to be made for this.

Newton's law of cooling enables us to estimate the heat lost in an experiment in the method of mixtures.

In doing this experiment, we take the temperature of the mixture at half minutes intervals and plot it against time as shown in Fig. 5.12.



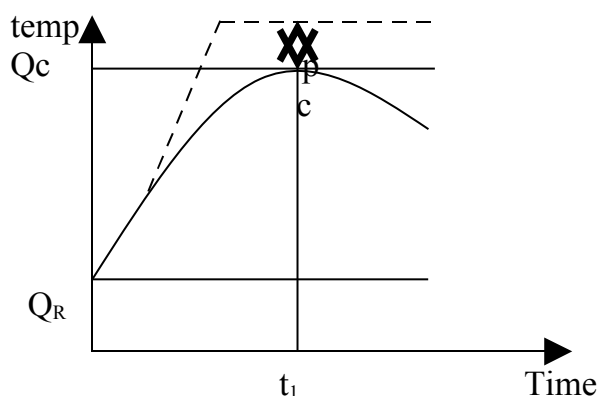


Fig. 5.12: Cooling correction

The broken line shows how we would expect the temperature to rise if no heat were lost, we have therefore to estimate the difference,  $P$ , between the plateau of this imaginary curve and the crest of the experimental curve,  $C$ .  $P$  is known as the cooling correction. That is, the temperature to be added to the observed final temperature to compensate for loss of heat (the cooling correction).

As a precaution, you may cool the liquid initially to an amount of temperature (about  $10^{\circ}\text{C}$ ) below room temperature so that the temperature rise above room temperature at steady state should be equal to the fall in temperature of the liquid below room temperature before transferring the solid into it.

### Worked examples

A lump of copper of unknown mass is heated to  $100^{\circ}\text{C}$ . If the final temperature is  $55^{\circ}\text{C}$  when dropped in 250 g of water at  $40^{\circ}\text{C}$ , calculate the mass of copper, (specific heat capacity of water is  $4200 \text{ J kg}^{-1} \text{ K}^{-1}$ , copper is  $380 \text{ J kg}^{-1} \text{ K}^{-1}$ )

### Solution

Let the mass of copper in kilogram =  $M_c$

Heat in joule lost by copper = Heat in joule gained by water

$$M_c \times C \times (\theta_2 - \theta_1) = m \times c \times (\theta_2 - \theta_1)$$

$$M_c \times 380 \times (100 - 55) = 0.25 \times 4200 \times (55 - 40)$$

$$\Rightarrow mc = \frac{0.25 \times 4200 \times 15}{380 \times 45} = 0.92 \text{ kg}$$

The mass of copper is 0.92 kg



- (2) An electric heater rated 20V 60W, fitted into a metal block supplied heat to the block of mass 2kg, and specific heat capacity of  $460 \text{ J kg}^{-1} \text{ K}^{-1}$ . Calculate the temperature rise in the block if the current flows for 15 minutes

### Solution

Power of heater =  $V \times I = 60\text{W}$

Heat produced by heater in 15 minutes = power  $\times$  time (in seconds)  
 $= 60 \times 15 \times 60 \text{ J} = 54000 \text{ J}$ .

Let temperature rise in metal block =  $\theta \text{ K}$

Heat gained by metal block =  $2 \times 460 \times \theta \text{ J}$ .

Heat produced by heater = Heat gained by metal block

$$\begin{aligned} 54000 &= 2 \times 460 \times \theta \\ \theta &= \frac{54000}{2 \times 460} = 58.7 \text{ K or } 58.7^\circ\text{C} \end{aligned}$$

The temperature rise in the block is 58.7 K or  $58.7^\circ\text{C}$ .

### SELF ASSESSMENT EXERCISE 2

1. Distinguish between heat capacity and specific heat capacity of a substance. Explain how the specific heat capacity of a substance could be determined.
2. The specific heat capacity of zinc is  $384 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ . What does the statement mean? A 60 watt heating coil is used for 3 minutes to heat a metal block of 700 g and specific heat capacity of  $1.0 \text{ J / g}^\circ\text{C}$ . What is the temperature rise?
3. The hot water of a bath delivers water of  $95^\circ\text{C}$  at a rate of  $5 \text{ kg min}^{-1}$ . The cold water tap of the bath delivers water at  $35^\circ\text{C}$  at the rate of  $15 \text{ kg min}^{-1}$ . If both baths are left on for 5 mins, calculate the final temperature of the bath water, ignoring heat losses to the surrounding.

### 4.0 CONCLUSION

In this unit you learned about the various modes of heat transfer (conduction, convection and radiation), their molecular explanations and applications. You also learned about heat capacity and specific heat capacity. The determination of specific heat capacity for solids and



liquids using method of mixtures and the electrical method. Finally, you learned about Newton's law of cooling and cooling correction.

## 5.0 SUMMARY

- Conduction is a method of transferring heat with the aid of molecules which vibrate about their mean position.
- Metals are good conductors while non metals are usually poor conductors.
- Convection is a method of transferring heat which involves actual movement of molecules from one position to another.
- Convection currents could be observed in liquids and gases.
- Radiation is a method of transferring heat which does not require any type of material medium.
- Heat transferred by radiation travels with the speed of light.
- Good absorbers of heat (eg dull black surfaces ) are also good radiations.
- Poor absorbers of heat (eg. shinny bright surfaces) are also poor radiations.
- Heat (thermal) capacity of a substance is the amount of heat required to raise the temperature of the substance by  $1^{\circ}\text{C}$ .
- Specific heat capacity of a substance is the amount of heat required to raise the temperature of 1kg of the substance by  $1^{\circ}\text{C}$ .
- In activities involving measuring of heat quantities, it is assumed that heat lost by a hotter substance is equal to heat gained by a colder substance.
- Newton's law of cooling states that for a small difference in temperature between a body and its surrounding, the rate at which a body loses heat is proportional to the temperature difference between the body and its surroundings.



## 6.0 TUTOR -MARKED ASSIGNMENT

1. Name the three modes of heat transfer. State the major difference between them.
2. What is the final temperature of a mixture if 2 kg of water at  $80^{\circ}\text{C}$  is added to 2.5 kg of cold water at  $10^{\circ}\text{C}$ . (Neglect heat absorbed by the container specific heat capacity of water is  $4200\text{J kg}^{-1}\text{ K}^{-1}$ ).
3. 4 kg of a certain metal is heated to  $86^{\circ}\text{C}$  and then dropped in 1.5 kg of water at  $26^{\circ}\text{C}$ . If the final temperature of water is  $40^{\circ}\text{C}$ , calculate the specific heat capacity of the metal (specific heat capacity of water =  $4200\text{ J kg}^{-1}\text{ K}^{-1}$ ).

## 7.0 REFERENCES/ FURTHER READINGS

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## UNIT 6 LATENT HEAT AND EVAPORATION



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## 1.0 INTRODUCTION



In unit 5 of this last Module you learnt about transfer of heat and heat capacities. In this unit, you will learn about latent heat – its concept and definitions. You will also solve numerical problems on latent heat. In addition, you will learn to determine and measure the specific latent heat of fusion of ice and of vaporization as well as applications of latent heat. Furthermore, you will be introduced to Evaporation. You will learn the concepts of evaporation, vapours, boiling, sublimation and the effect of pressure and impurities on melting and boiling points. Finally, you will learn about relative humidity, dew point, humidity and weather.

## 2.0 OBJECTIVES

By the end of this unit you will be able to:

- determine the melting point of a solid
- determine the melting point of a given liquid
- list the effects of impurities and pressure in:
  - the melting point of solid (ice)
  - the boiling point of a liquid
- explain the terms:
  - latent heat of fusion
  - latent heat of vaporization
- solve simple problems involving latent heat
- distinguish between evaporation and boiling
- explain sublimation and dew point
- explain the working principles of such common devices as
  - refrigerator
  - airconditioner
  - pressure cooker
- explain the effects of humidity on personal comfort.

## 3.0 MAIN CONTENT



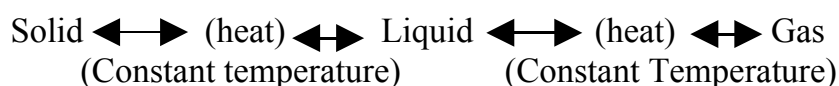
### 3.1 Latent Heat

#### 3.1.1 Concept of latent heat

You must have observed that when sufficient heat is given to a solid it melts into liquid. If heat continues to be supplied, the temperature remains constant until all the solid has changed to liquid. The heat supplied to melt the solid is called latent heat because the temperature gave no indication of the heat required to change from a solid to a liquid.

Also, if the liquid is heated further, its temperature rises until it reaches a constant temperature. Then, it begins to change into vapour or gas. Again this heat is called latent heat because of change of state from liquid to gas which gave no indication of heat required to change the substance from liquid to gas.

That is during change of state,



Temperature remains constant although heat is being supplied.

The heat supplied or removed, which causes a change of state without a change in temperature is called latent heat. It is called latent (meaning hidden) heat because it is not made apparent by a change in temperature.

#### 3.1.2 Molecular Explanation of Latent Heat

You already know that the molecules of a solid are kept in average fixed position by strong forces of attraction. Latent heat represents the energy used in breaking down the forces which hold the molecules of a solid in a regular pattern. In liquid, it is the energy required to overcome the forces of attraction between molecules of a liquid. The molecules are then practically independent of each other and exist as gas.

#### 3.1.3 Types and Definitions of Latent Heat

There are two types of latent heat namely:

- i. Latent heat of fusion (melting) (from solid to liquid)
- ii. Latent heat of vaporization (gas or vapour) (from liquid to gas)



## Definitions

- i. Latent heat of fusion of a solid is the heat energy that is used to change a solid at its melting point to liquid at the same temperature (the energy is stored in the hot liquid)
- ii. The specific latent heat of fusion of a solid is the energy required to melt 1kg of the solid at its melting point.

It is equal to the energy given out by 1 kg of the liquid in order to solidify at the freezing point.

If  $Q$  heat energy is supplied to melt mass  $M$  of solid at its melting point, then the specific latent heat  $L$  is given by

$$L = \frac{Q}{M}$$

$$Q = ML$$

where:  $L$  = specific latent heat

$Q$  = heat energy

$M$  = mass of solid at melting point

Unit  $\text{Jg}^{-1}$  or  $\text{Jkg}^{-1}$

- iii. Latent heat of vaporization of a liquid is the heat energy that is used to change a liquid in its boiling point to liquid at the same temperature (the energy is stored in the vapour)
- iv. The specific latent heat of vaporization of a substance is the heat required to change unit mass of it from liquid at its boiling point to vapour without a change in temperature.

## 3.2 Numerical Problems on Latent Heat

- i. A bucket has a mass of 1 kg and specific heat capacity of  $378 \text{ kg}^{-1} \text{ K}^{-1}$ . It contains 4 kg of water at  $40^\circ\text{C}$ . When 0.91 kg of ice at  $0^\circ\text{C}$  is added and completely melted the temperature of water is  $18.2^\circ\text{C}$ . What is the specific latent heat of fusion of ice?

(Assume no heat exchange with the surroundings)



**Solution**

Since no heat is lost to the surrounding:

Heat lost by water + heat lost by bucket  
 = Heat used for melting ice + heat gained by iced water from  $0^{\circ}\text{C}$  to  $18.2^{\circ}\text{C}$ . Eqn.....(1)

Recall that heat lost:  $Q = MC(\theta_2 - \theta_1)$   
 where  $Q$  = heat energy  
 $C$  = heat capacity of the substance  
 $M$  = mass of substance  
 $\theta_2$  = higher temperature  
 $\theta_1$  = lower temperature

Hence:

Heat lost by water =  $4 \times 4200 \times (40 - 18.2) \text{ J}$   
 Heat lost by bucket =  $1 \times 378 \times (40 - 18.2) \text{ J}$   
 Heat gained by iced water =  $0.91 \times 4200 \times (18.2 - 0)^{\circ}\text{C}$   
 Heat used to melt  $0.91 \text{ kg}$  of ice =  $0.91L$   
 where  $L$  = the specific latent heat.

To obtain the specific latent heat of the fusion of ice we will substitute the above values in 'eqn 1' and solve for  $L$ .

That is

Heat lost by water + heat lost by bucket  
 = Heat used for melting ice + heat gained by iced water from  $0^{\circ}\text{C}$  to  $18.2^{\circ}\text{C}$

$\Rightarrow MC(\theta_2 - \theta_1)$  for water +  $MC(\theta_2 - \theta_1)$  for bucket  
 =  $ML$  for ice +  $MC(\theta_2 - \theta_1)$  for iced water

$\Rightarrow [4 \times 4200 \times (40 - 18.2)\text{J}] + [1 \times 378 \times (40 - 18.2)\text{J}]$   
 =  $0.91L + [0.91 \times 4200 \times (18.2 - 0)\text{J}]$

$0.91L = [4 \times 4200 \times (40 - 18.2) \text{ J}] + [1 \times 378 \times 40 - 18.2)\text{J}]$   
 $- [0.91 \times 4200 \times (18.2 - 0)\text{J}]$   
 $= 366240 + 8240.4 - 69560.4$   
 $= 374480.4 - 69560.4$

$\therefore L = \frac{304920}{0.91}$   
 $= 335076.92 \text{ Jkg}^{-1}$



- ii. A copper calorimeter of mass 170.0 g contains 100.0 g of water at 19.5°C. Naphthalene (melting point 80.0°C) is melted in a test tube, cooled at 80°C, and then poured into the calorimeter. If the highest temperature reached by the calorimeter after stirring is 30.0°C and the final mass of the calorimeter and its content is 185g. Calculate the latent heat of fusion of naphthalene (specific heat capacity of copper 0.4 kJ kg<sup>-1</sup>, of naphthalene 1.3 kJ kg<sup>-1</sup>, water 4.2 kJ kg<sup>-1</sup>).

### Solution

Mass of naphthalene = 185.0 – 170.0 g = 15 g

Heat lost by naphthalene  
= heat gained by water and calorimeter

$$\begin{aligned}
 &\Rightarrow 15L + [15 \times 1.3 \times (80.0 - 30.0)] \\
 &= [100 \times 4.2 \times (30.0 - 19.5)] + [170 \times 0.4 \times (30.0 - 19.5)] \\
 &\Rightarrow 15L = (100 \times 4.2 \times (30.0 - 19.5)) + (170 \times 0.4 \times (30.0 - 19.5)) \\
 &\quad - (15 \times 1.3 \times (80.0 - 30.0)) \\
 &= 4410 + 714 - 975 \\
 &= 5124 - 975 = 4149 \\
 L &= \frac{4149}{15} \\
 &= 276.6 \text{ Jkg}^{-1} \quad \cong \quad 276.6 \text{ kJkg}^{-1}
 \end{aligned}$$

## 3.3 Measurement of Specific Latent Heat

### 3.3.1 Experimental Determination of Specific Latent Heat of Fusion of a Solid (Ice)

- i. By method of mixture

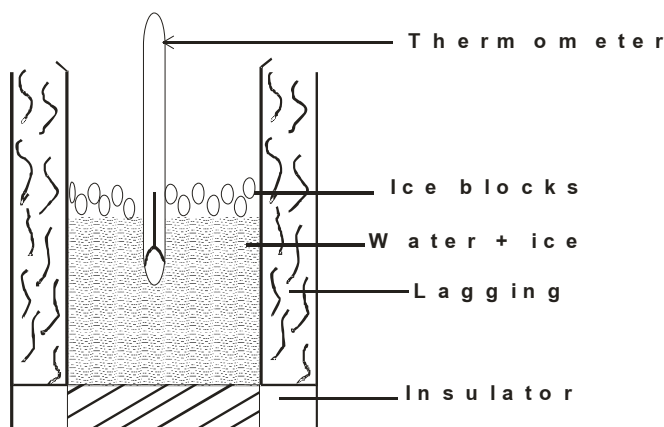


Fig. 6.1: Latent heat of fusion by method of mixture



To determine the specific latent heat of fusion of solid (ice) by method of mixture, we use the apparatus as set up in the diagram above. The calorimeter is weighed empty and reweighed when it is about half-filled with water whose temperature is about  $5^{\circ}\text{C}$  above room temperature. Some small ice blocks are dried with blotting paper and then added to the water in the calorimeter. The mixture is gently stirred and more dry ice added until the temperature of the mixture is about  $5^{\circ}\text{C}$  below room temperature. The lowest temperature of the mixture is noted after all the ice particles has melted. The calorimeter is reweighed to find the mass of ice added. The result is then calculated as follows:

Assuming

$$\begin{aligned}\text{Mass of calorimeter} &= M_1 \\ \text{Mass of calorimeter + water} &= M_2 \\ \text{Mass of calorimeter + water + ice} &= M_3\end{aligned}$$

Then:

$$\begin{aligned}\text{Mass of water} &= M_2 - M_1 \\ \text{Mass of ice} &= M_3 - M_2\end{aligned}$$

$$\begin{aligned}\text{If: Original temperature of water} &= \theta_1^{\circ}\text{C} \\ \text{Final temperature of mixture} &= \theta_2^{\circ}\text{C} \\ \text{Specific heat capacity of water} &= 4200(\text{Jkg}^{-1}) \\ \text{Specific heat capacity of calorimeter +} & \\ \text{Material} &= C(\text{Jkg}^{-1}) \\ \text{Specific latent heat of fusion of ice} &= l(\text{Jkg}^{-1}) \\ \text{Heat required to melt the ice} &= (M_3 - M_2) l(\text{joules})\end{aligned}$$

$$\begin{aligned}\text{Heat required to raise the temperature of melted ice from }^{\circ}\text{C to } \theta_2^{\circ}\text{C} \\ = (M_3 - M_2) \times 4200 \times \theta_2 \text{ (joules)}\end{aligned}$$

$$\begin{aligned}\text{Heat lost by the calorimeter and water in cooling from } \theta_1 \text{ to } \theta_2 \\ = (M_1, C + (M_2 - M_1) \times 4200) (\theta_1 - \theta_2) \text{ J}\end{aligned}$$

Since:

$$\begin{aligned}\text{Heat gained} &= \text{Heat lost} \\ (M_3 - M_2) l + ((M_3 - M_2) \times 4200) (\theta_1 - \theta_2)\end{aligned}$$

From the above equation, the latent heat of fusion  $l$  can be calculated.

Precautions

You will take the following precautions when performing the above experiment in the laboratory:



Use only dry pieces

Add small quantity of ice at a time

Stir the mixture gently to obtain even temperature.

ii By electrical method

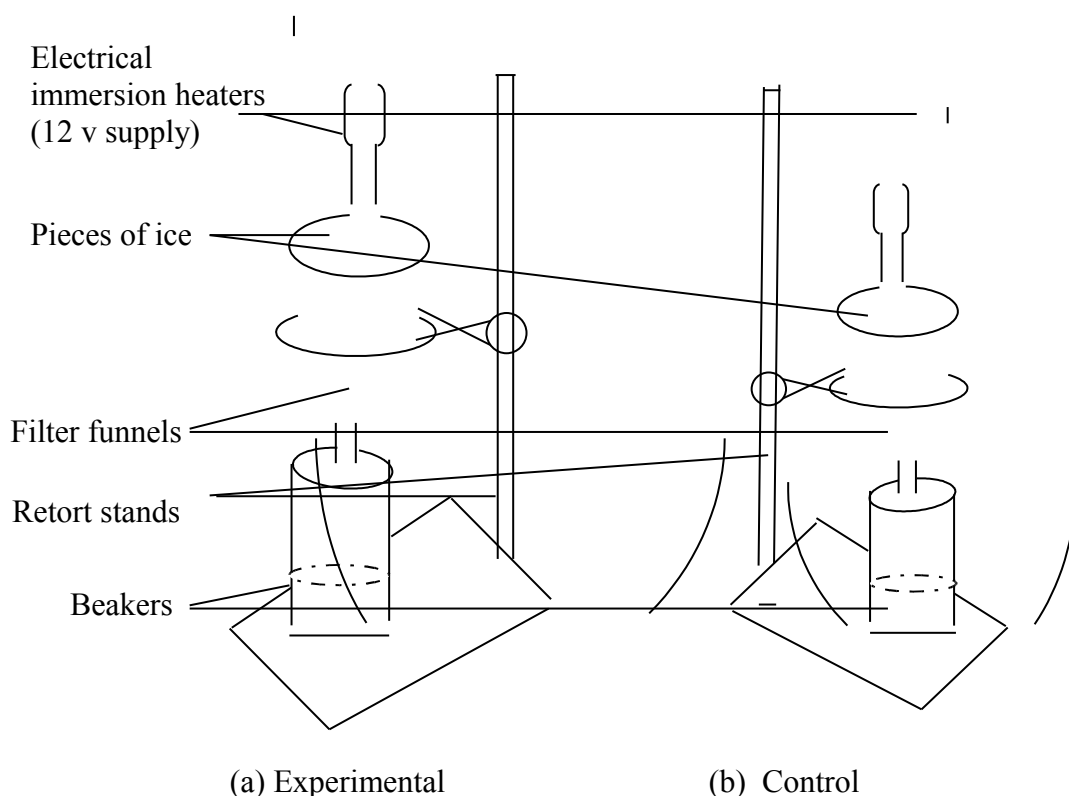


Fig. 6.2: Latent heat of fusion by electrical method

To determine the specific latent heat of fusion of solid (ice) by electrical method, we use the apparatus as set up in the diagrams above (Fig. 6.2)

A 50W immersion heater is placed in a glass filter funnel and some pieces of ice are packed round the heater. The heater is connected to a 12v supply with the circuit as shown in Fig. 6.2a. Since ice will melt at room temperature the same set-up is arranged as in figure 6.2b with the heater not connected to the 12v supply. This forms a control for the experiment. Two similar beakers of the same mass are used to collect drip water by placing one below each beaker.

Quickly empty the beakers when the drip rates from the funnels appear to be the same. Then replace them and switch on the current. Ensure that the heater is in good thermal contact with the ice throughout the



experiment. Allow current to flow for about 200 seconds and note the value of the current and the potential difference. When the drip rate appear to be constant again, remove the beakers and weigh them with their contents. Use your result to calculate the specific latent heat of fusion in the following manner:

Assuming these results are obtained:

Mass of the experimental beaker + water	=	$M_1$
Mass of the control beaker + water	=	$M_2$
Mass of the ice melted by electrical heating	=	$M_2 - M_1$
Current	=	$I$
Potential difference	=	$V$ (12v)
Time of flow of current	=	$t$
Energy supplied to ice	=	$VIt$

This implies that:

Energy supplied to melt 1 kg of ice

$$\begin{aligned}
 &= \frac{\text{Energy supplied to ice}}{\text{Mass of ice melted by electrical heating}} \\
 &= \frac{V \times I \times t}{M_2 - M_1} = \left( \frac{VIt}{M_2 - M_1} \right) \text{ Jkg}^{-1}
 \end{aligned}$$

**You should note that:**

1. If  $Q$  heat energy is supplied to melt mass  $m$  of solid at its melting point then the specific latent heat  $L$  is given by

$$L = \frac{Q}{M}$$

$$Q = ML$$

2. The immersion heater should never be switched on unless it is in a liquid or a block of metal as it will overheat in air.



### 3.3.2 Experimental Determination of Specific Latent Heat of Vapourisation (Steam)

#### i. By method of mixture

Set up the apparatus as shown in Fig. 6.3. Weigh the calorimeter empty and reweigh it when it is about half filled with water.

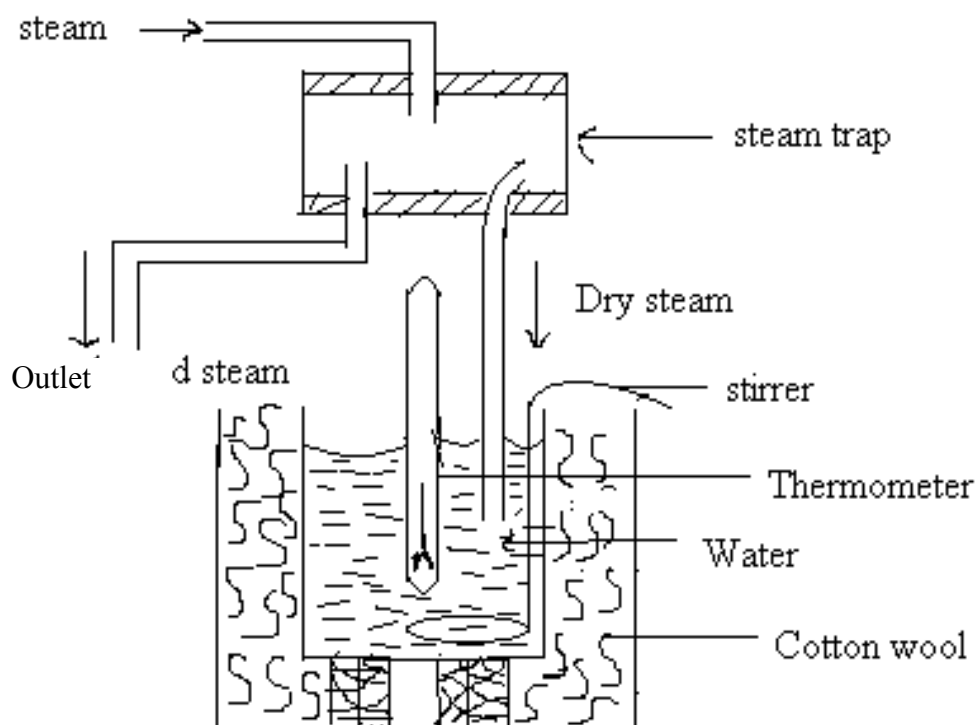


Fig. 6.3 Latent heat of vaporization by method of mixture

Then place it in an insulating jacket. Note the temperature of the water. Pass dry steam into the water until the water temperature rises by about  $25^{\circ}\text{C}$  above room temperature. Note the final temperature and find the mass of steam by reweighing the calorimeter and its content. Tabulate your readings as shown below:

Mass of calorimeter	=	$M_1$
Mass of calorimeter + water	=	$M_2$
Mass of calorimeter + water + steam	=	$M_3$
Initial temperature of water + calorimeter	=	$\theta_1^{\circ}\text{C}$
Final temperature of mixture	=	$\theta_2^{\circ}\text{C}$
Temperature of steam	=	$100^{\circ}\text{C}$



Let:

Specific latent heat of vapourisation of steam =  $L(\text{Jkg}^{-1}\text{kg}^{-1})$

Mass of water	=	$M_2 - M_1 (\text{kg})$
Mass of steam	=	$M_3 - M_1 (\text{kg})$
Specific heat capacity of water	=	$4200(\text{Jkg}^{-1}\text{K}^{-1})$
Specific heat capacity of calorimeter material	=	$c(\text{Jkg}^{-1}\text{K}^{-1})$
Heat lost by steam in condensing	=	$(M_3 - M_2)\text{joules}$

Heat lost by condensed steam in cooling from  $100^\circ\text{C}$  to  $\theta_2^\circ\text{C} = (M_3 - M_2) \times 4200 \times (100 - \theta_2)\text{J}$

Heat gained by water and calorimeter  
 $= (M_2 - M_1) \times 4200 \times (\theta_2 - \theta_1) + M_1 \times C (\theta_2 - \theta_1)$

Since

$$\begin{aligned} \text{Heat gained} &= \text{Heat lost} \\ (M_2 - M_1) \times 4200 \times (\theta_2 - \theta_1) + M_1 C (\theta_2 - \theta_1) \\ &= (M_3 - M_2)L + (M_3 - M_2) \times 4200 \times (100 - \theta_2) \end{aligned}$$

Hence

$L$ , the specific latent heat of vaporization can be calculated from the above equation.

Precautions:

While performing the above experiment you should take the following precautions:

Use only dry steam

Take care to prevent the calorimeter from receiving or losing heat from or to its surroundings.

Cool the water to about  $10^\circ\text{C}$  below room temperature before passing the steam. Continue to pass the steam until the temperature of mixture is about  $10^\circ\text{C}$  above room temperature.

Stir the mixture continuously throughout the experiment.



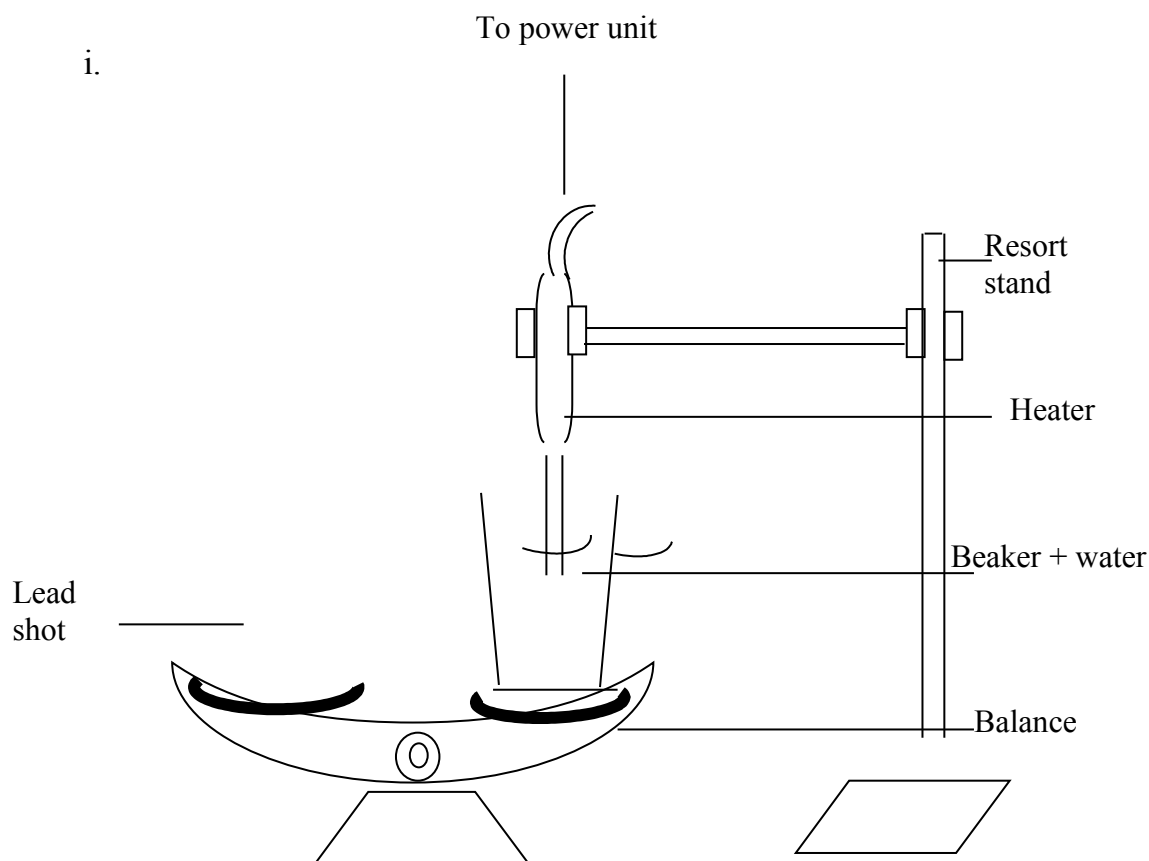


Fig. 6.4: Determination of specific latent heat of vaporization by electrical method

To determine the specific latent heat of vaporization by electrical method, you will set up the apparatus as shown in the diagram above.

Place a polystyrene (polyphenylethene) cup (beaker) on one side of a balance and partly fill it with water. Suspend an immersion heater in the water so that it is clear of the sides and bottom.

Switch on the current. When the water is boiling, pour leadshots into the other pan until balance is almost obtained. As the water boils off, the two sides will balance. At this instance start a stop-watch. Do not allow any condensed water vapour to drop back into the cup. Now, place a 20 g mass on the pan beside the cup. When 20 g of water boils off, the two sides will again balance. Note the time when this happens. State your results as shown below:

Power rating of heating coil	=	$p$
Time noted	=	$t$
Energy supplied (electrical)	=	$p \times t$
Mass of water boiled off	=	0.020 kg



Calculate the specific heat of vaporization from:

$$L = \frac{p \times t}{0.02}$$

### 3.4 Applications of Latent Heat

From the foregoing, you must have realized that whenever a liquid is converted into vapour, heat is absorbed from the liquid or any object in contact with it. The heat is the latent heat used in causing the transformation from liquid to vapour. The phenomenon is utilized in the working principles of some common household devices such as: (i) Pressure cooker (ii) The refrigerator (iii) The air conditioner.

Later in this unit, we would discuss the working principle of these devices.

#### SELF ASSESSMENT EXERCISE

1. Explain the terms:
  - (i) Latent heat of fusion
  - (ii) Specific latent heat of fusion
  - (iii) Latent heat of vaporization
  - (iv) Specific latent heat of vaporisation
2. A bucket has a mass of 100 g and specific heat capacity  $420 \text{ J kg}^{-1} \text{ K}^{-1}$ . It contains 400 g of water at  $20^\circ\text{C}$ . How many grammes of steam at  $100^\circ\text{C}$  are needed to raise the temperature of the water and bucket to  $95^\circ\text{C}$ ? Specific latent heat of steam  $L = 2.26 \times 10^6 \text{ J kg}^{-1}$ .
3. (a) Define latent heat (b) Describe the measurement of the specific latent heat of (i) fusion of ice (ii) vaporization of water under school laboratory conditions.



### 3.5 Melting and Boiling Point

#### 3.5.1 Melting point

You know that the melting point of a substance is the temperature at which the substance changes from solid to liquid state (without further change in temperature)

To find the melting point of a solid, you should follow this procedure listed below:

- i. Put a solid substance (e.g naphthalene) in a tube and melt it by heating the test tube through a beaker of boiling water. The molten naphthalene should half-fill the test tube.

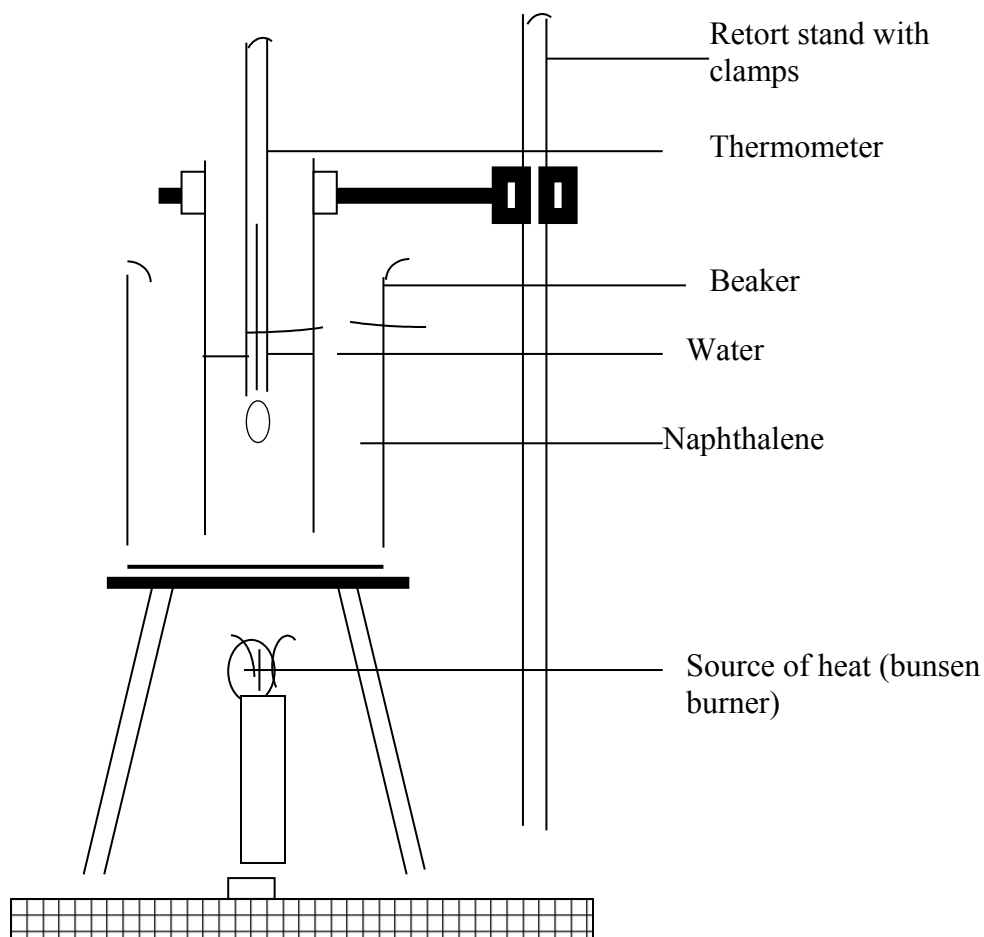


Fig. 6.5: Determination of melting point of Naphthalene

- ii. Insert thermometer so that its bulb is midway in the naphthalene.
- iii. Remove the test tube from the flame and allow the naphthalene to solidify with the thermometer inside it.



- iv. Warm the test tube in a beaker of water and record the thermometer readings at interval of 30 seconds.
- v. Stop taking readings 4 minutes after the naphthalene has completely melted.
- vi. Plot a graph of temperature against time.
- vii. Remove the test tube of molten naphthalene from the water bath and allow it to cool.
- viii. Record the readings of the thermometer at the intervals of 30 seconds.
- ix. Stop taking of readings 10 minutes after the naphthalene has completely solidified.
- x. Tabulate your readings and use it to plot a graph of temperature against time.

**a. Table of Values**

Temperature ( $^{\circ}\text{C}$ )	Time (sec)

**(b) Graph**

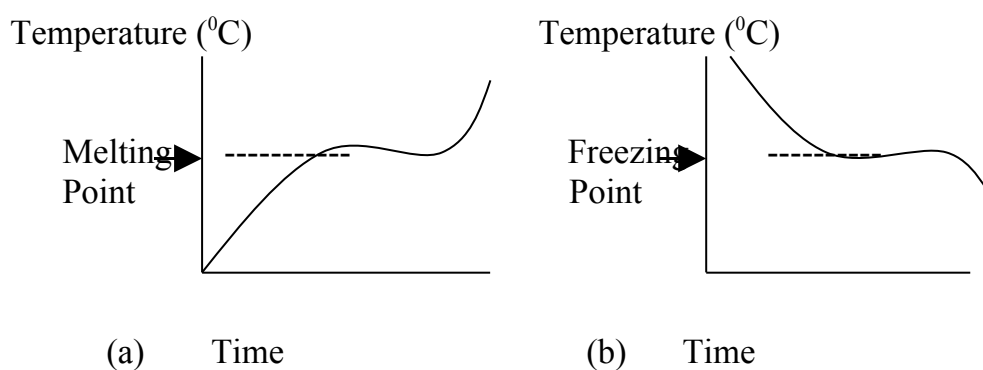


Fig. 6.6 : Heating and cooling curves of naphthalene



## Observation and Results

You would observe that as the solid is heated, the temperature rises. At a certain stage, the temperature remains constant until all the solid has changed to liquid. This is shown by the flat portion (plateau) of the graph (Fig. 6.6a) after which the temperature starts to rise again.

This temperature at which the naphthalene melts is constant and is called the melting point of the solid naphthalene.

When the naphthalene is cooling, energy is abstracted from it. At a stage, the liquid naphthalene starts to solidify. This is shown in the flat (plateau) portion of the cooling curve (Fig. 6.6b) after which the temperature starts to fall.

This temperature at which a liquid changes to a solid is called the freezing point of the liquid. You should note that the melting point and the freezing point of a substance are the same.

### 3.5.2 Boiling point

The boiling point of a substance as you know is the temperature at which the substance begins to boil and changes from liquid to gaseous state (without further change in temperature).

To find the boiling point of a substance (Water).

For you to determine the boiling point of a liquid, you should follow those steps below:



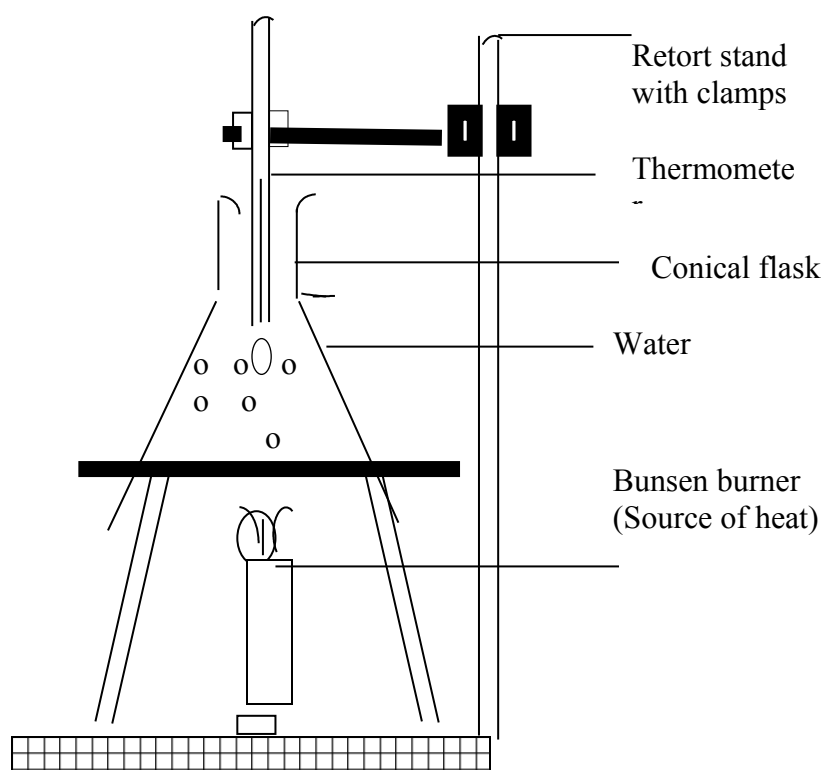


Fig. 6.7: Determination of the boiling point of liquid (water)

- i. Put the liquid substance (eg water) in a conical flask containing a thermometer.
- ii. Use a bunsen burner to heat the water
- iii. Take the water temperature at 1 minute intervals until the water has boiled for seven minutes.
- iv. Plot a graph of temperature against time to obtain a graph of the type shown overleaf using your tabulated table of values.

**a. Table of Values**

Temperature ( $^{\circ}\text{C}$ )	Time (sec)



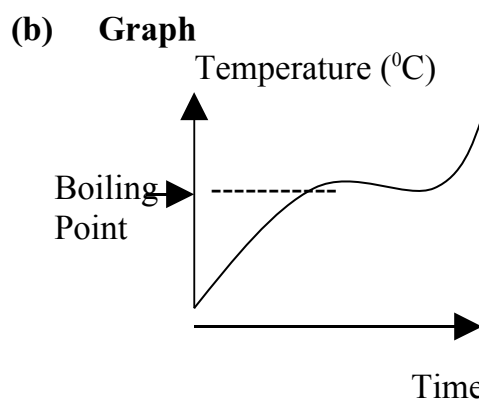


Fig. 6.8: The boiling point of a liquid (water)

#### Observation and result

You must have observed that as you heat the water in the conical flask, its temperature rises and the liquid evaporates. As the heating is continued, the temperature is found to rise until the water begins to boil. During boiling bubbles of air formed inside the liquid rise to the surface. The temperature of the liquid remains steady during boiling. This is indicated in the flat portion of the graph (plateau) Fig. 6.8 this constant temperature is called the boiling point of the liquid.

### 3.5.3 Effects of Impurities and Pressure on the Melting and Boiling Points of a Substance

The melting and boiling points of a substance are affected by pressure and impurities. You can verify these by investigating these scientific observations.

#### i. Effect of pressure on boiling point

Increase in pressure increases the boiling point of a liquid. The pressure cooker is a practical application of the effect of pressure on boiling point. Increased pressure of the trapped air (gas) above the liquid in a pressure cooker raises the boiling point of the liquid. Thus a high cooking temperature is reached very fast. Hence, food is cooked faster in the pressure cooker.



**ii. Effect of pressure on the melting point of ice (or freezing point of water)**

Increased pressure lowers the melting point of ice (or the freezing point of water). This can be shown by passing a piece of wire with heavy weights attached to both ends through a block of ice as shown in Fig. 6.9 below:

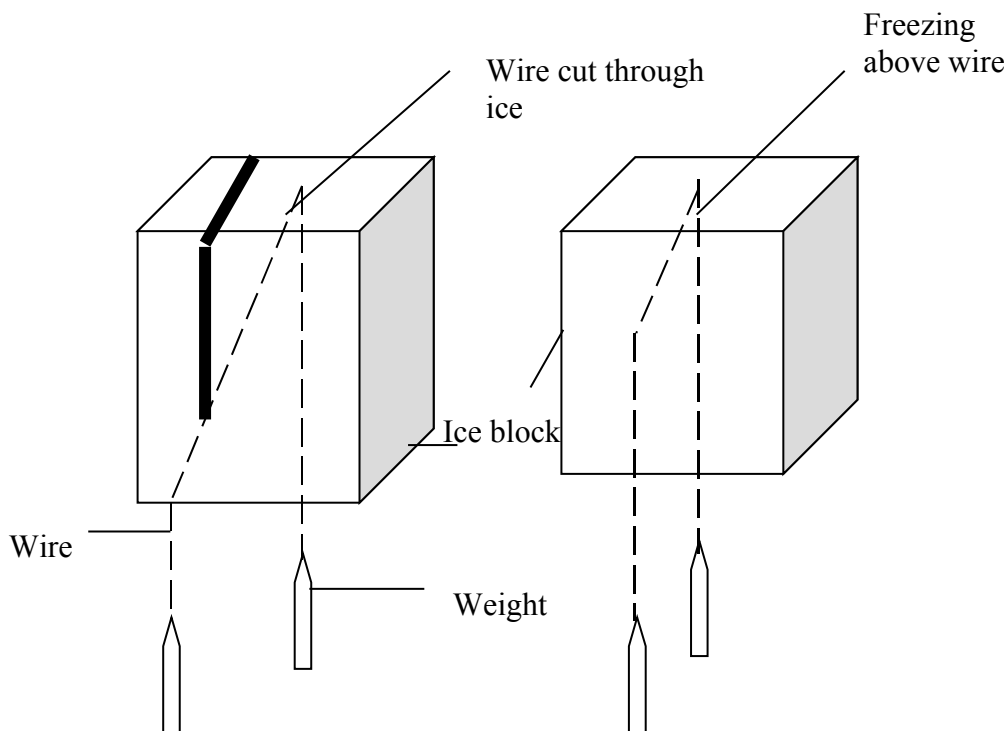


Fig. 6.9: Effect of pressure on melting point

The piece of wire cuts through the block of ice but the block remains solid behind the wire. The ice directly below the wire melts because the increased pressure lowers the freezing point. The wire passes through the ice. Above the wire, decreased pressure raises the freezing point of the melted ice and the water freezes again. The above process is known as regelation.

**iii. Effect of impurities on the boiling point of a substance**

Impurities raise the boiling point of a pure solvent. Thus, the boiling point of a solution of salt in water is higher than that of pure water.

**iv. Effect of impurities on the melting point of a substance**

The melting point of a pure solid is lowered by the presence of impurities. For instance, a freezing mixture of ice and salt has a lower



temperature than  $0^{\circ}\text{C}$ , the freezing point of pure water (or melting point of ice)

### 3.6 Evaporation and Boiling

#### 3.6.1 Concept of evaporation

You are familiar with the concept of evaporation. You know that evaporation is the change of liquid to vapour without the application of heat to the liquid. It is a slow and quiet process which takes place at all temperatures. This phenomenon can often be seen in our daily lives. For instance;

- i. when a piece of wet cloth is spread out in the open during harmattan, it dries up quickly;
- ii. liquids left in shallow open containers soon disappear;
- iii. when some liquid perfume is poured into a cup and left uncovered, the cup is found to be empty after few hours.

#### 3.6.2 Factors that affect the rate of evaporation

Can you guess the factors that affect the rate of evaporation? Compare your answers with these factors that affect the rate of evaporation. They are:

- i. **Temperature** – Evaporation is more rapid if the surrounding temperature is higher. This is because the process of evaporation involves absorbing the latent heat of vaporization from the surroundings. The warmer the surroundings, the more the amount of heat supplied by it to provide the latent heat of vaporization.
- ii. **Pressure** – The greater the pressure, the slower the evaporation and vice versa. If an open container of water is placed inside a bell-jar from which air has been pumped out (reduced pressure), evaporation takes place at faster rate.
- iii. **Area of liquid surface exposed** – The greater the surface area of liquid exposed, the more rapid will be the evaporation.
- iv. **Dryness of the air** – The drier the air (i.e the less water vapour it contains) the quicker the evaporation. Water droplets on a watch-glass evaporate faster if left in the open than when covered with a bell jar. As the water evaporates, the air in the bell jar gets filled with water vapour and this slows down the evaporation. In the



case of the watch glass in the open, the water vapour quickly escapes into the open air and more evaporation takes place. This is the reason why wet clothes dry faster in dry weather than in wet weather.

- v. **Motion of the Air** – Wet clothes placed in the draft from a fan will dry up sooner than if placed in still air. The air stream or wind removes the vapour and brings fresh and drier air into contact with the liquid. Hence, evaporation is more rapid.
- vi. **Nature of the Liquid** – Different liquids evaporate at different rates. The lower the boiling point of a liquid, the greater will be the rate of evaporation. For example, mercury with a boiling point of  $357^{\circ}\text{C}$  hardly evaporates while ether (boiling point  $35^{\circ}\text{C}$ ) evaporates very rapidly. Liquids like ether which evaporates rapidly are called volatile liquids.

### 3.6.3 Concept of boiling

Boiling occurs when a substance in the liquid state is thrown into violent agitation as a result of continuous supply of heat to the liquid molecules.

### 3.6.4 Evaporation and boiling

Whether a liquid changes rapidly into its gaseous state by boiling or by slow evaporation, it requires the same amount of latent heat to do so. In the case of boiling, this heat is supplied by the heat source. In the case of evaporation where there is no particular heat source, the liquid obtains its latent heat of vaporization from the surroundings.



	<b>Evaporation</b>	<b>Boiling</b>
i	This is the change from liquid to vapour state at temperature below normal boiling point	This is the change from liquid to vapour state at the boiling point of the liquid.
ii	Evaporation takes place at all temperatures	Boiling takes place at a fixed temperature for a particular liquid.
iii.	It occurs when molecules escape from the surface of a liquid	It takes place throughout the volume of the liquid
iv.	It is influenced by pressure, temperature, dryness of the atmosphere, wind and amount of exposed area of the liquid	It is influenced by pressure and the amount of impurities present in the liquid
v.	It takes place slowly and proceeds more rapidly as the temperature increases.	It occurs at a constant rate for a particular temperature
vi.	It is a silent phenomenon	It is a noisy phenomenon
vii.	It causes cooling	It does not cause cooling

### 3.7 Vapour and Vapour Pressure

#### 3.7.1 Concept of Vapour

As you know a liquid changes into vapour if sufficient heat energy is supplied to it. The change from liquid to vapour takes place throughout the entire liquid at a particular temperature.

#### 3.7.2 Concept of Vapour pressure

If a liquid is allowed to evaporate in a closed container, the vapour formed will exert a certain pressure. This is known as vapour pressure.

#### 3.7.3 Saturated and unsaturated vapour

- i. **Saturated vapour** – A saturated vapour is one that is in contact with its own liquid in a closed space. The pressure exerted by such a vapour is the saturated vapour pressure (S.V.P).



The saturated vapour pressure varies with temperature as shown in Fig. 6.10.

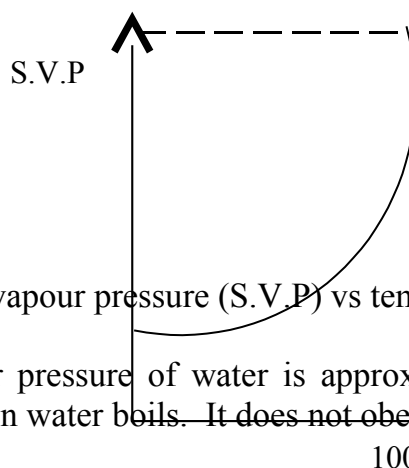


Fig. 6.10: Saturated vapour pressure (S.V.P) vs temperature

The saturated vapour pressure of water is approximately equal to the external pressure when water boils. ~~It does not obey the gas laws.~~

- ii. **Unsaturated vapour** – An unsaturated vapour is one that is not in contact with its own liquid in a closed space. The gas laws are approximately obeyed in such a vapour.

According to the kinetic theory, the number of molecules leaving the liquid during evaporation is greater than those returning to the liquid in an unsaturated vapour. On the other hand, there is equal number of molecules leaving and returning to the liquid in a saturated vapour.

At higher temperatures, the energy of the molecules increases and more molecules are then able to leave the liquid and the saturated vapour pressure will thus increase.

### 3.8 Concept of Sublimation

This is a process whereby some solid substances when heated, change directly into gaseous state without the intermediate liquid state. For example, solid iodine and solid carbon dioxide changes from solid state to gaseous state without changing to the intermediate liquid state.



### 3.9 The Working Principle of Some Common Devices

You have seen that whenever liquid is converted into vapour, heat is absorbed from the liquid or any object in contact with it. This heat is the latent heat used in causing the transformation from liquid to vapour. The absorption of the latent heat from the liquid brings about a fall in its temperature. The faster the evaporation, the greater is the fall in temperature. This phenomenon is utilized in the working principle of these common household devices.

#### i. The Refrigerator

The cooling effect of evaporation is used in a refrigerator. The volatile liquid used is usually liquid ammonia or Freon. Ammonia evaporates inside coiled copper tubes surrounding the freezing compartment, assisted by a pump which reduces the pressure. As it evaporates in these coils, the ammonia absorbs heat from the surrounding air, thus cooling the inside of the refrigerator and its contents. The vapour produced is pumped away and expressed in a condenser where it condenses to liquid ammonia. The heat released during this condensation is quickly dissipated by an arrangement of cooling fans at the back of the refrigerator.

The liquid is recycled through the evaporator coil (Fig. 6.11). The process is repeated. This sets up a continuous circulation of liquid and vapour.



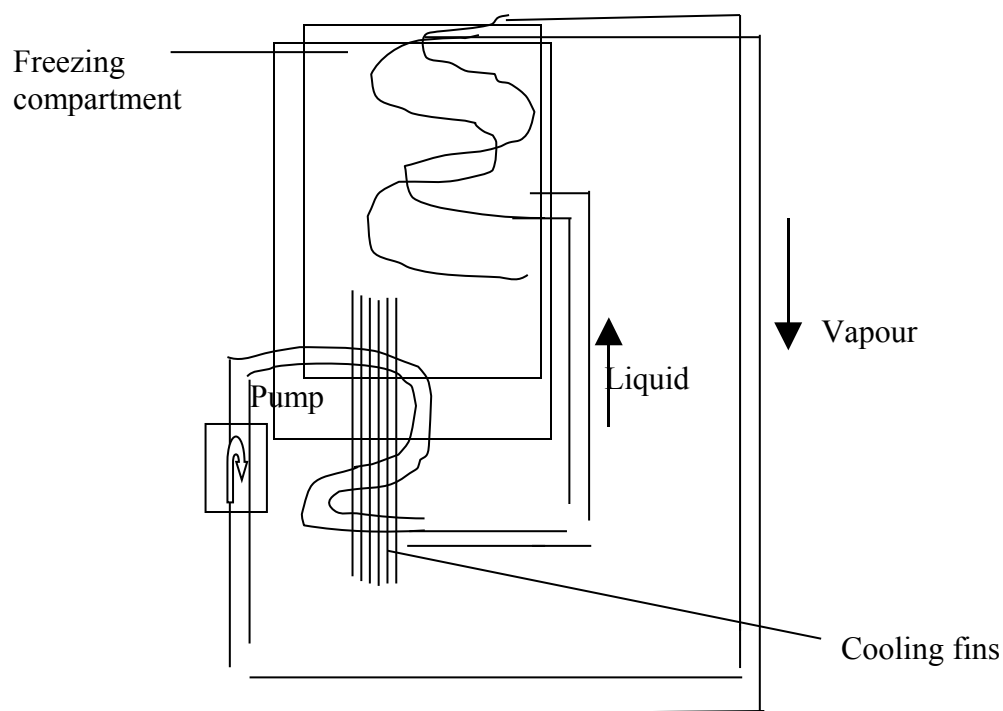


Fig. 6.11: Household electric refrigerator

## ii. Control of humidity (Air conditioning)

The humidity of an enclosure could be controlled to suit our personal comfort. The basic principle of the working of an air conditioner is illustrated in Fig. 6.12 below.

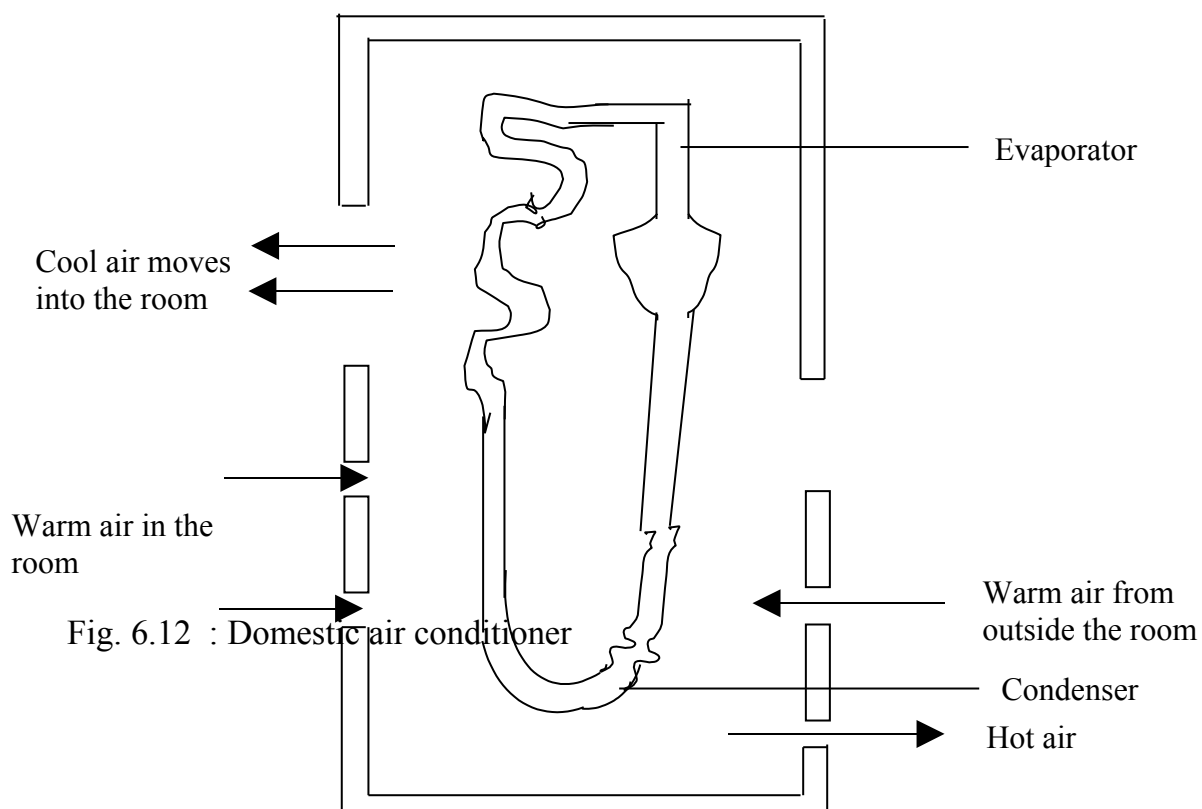


Fig. 6.12 : Domestic air conditioner



The air conditioner contains a powerful fan which draws air from outside to inside the enclosure (e.g a room, a car). It first cleans the air by removing dust particles from it. It further humidifies the air to a specified value (to make it cold) before circulating it within the enclosure. The humidity value of the circulating air is usually controlled by setting the air conditioner to certain convenient levels. Air conditioners are generally found in cars, public buildings, offices, ships and so on.

### iii. Pressure Cooker

The basic principle of the working of the pressure cooker is based on the fact that increased pressure raises the boiling point of water. Substances inside a pressure cooker are heated at about a pressure of 2 atmosphere. Such a high pressure can raise the boiling point of water to about  $120^{\circ}\text{C}$ . Food items will cook more quickly when subjected to such a high temperature condition inside a pressure cooker (Fig. 6.13)

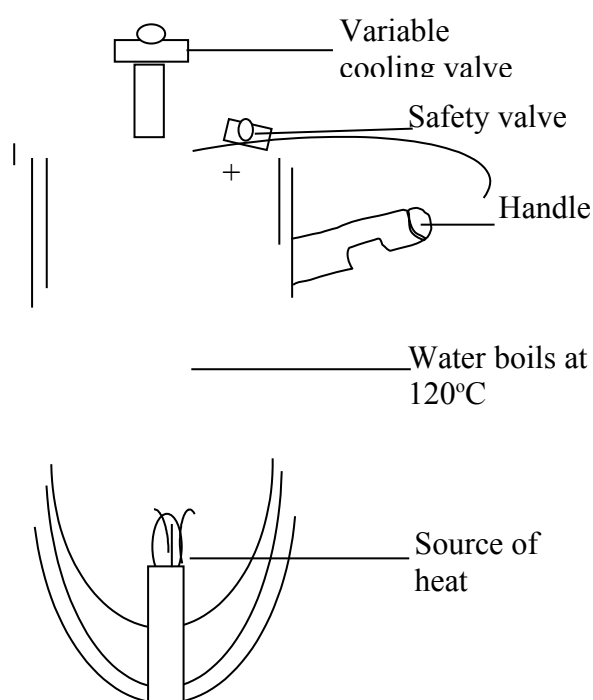


Fig. 6.13: A pressure cooker

Pressure cookers are found very useful by people and explorers in mountainous areas of the world where atmospheric pressure is generally lower than normal, where water can boil at  $91^{\circ}\text{C}$ . This makes cooking to be expensive in terms of time and fuel. Pressure cookers are usually used to remedy the situation.



### 3.10 Humidity

The atmosphere contains varying amounts of water vapour. Humidity is the amount of water vapour in the atmosphere.

#### i Relative humidity

If the air is saturated or nearly saturated with water vapour, it is described as very humid. If the air is highly unsaturated it is said to be dry. Humid weather makes one feel uncomfortable as perspiration does not evaporate easily from the skin into the already unsaturated air. Also, very dry air evaporates the moisture from the skin too rapidly. This is harmful to the skin. A comparison between the amount of moisture actually present in the air with the amount of moisture it can hold when saturated is termed relative humidity. This is expressed as a ratio which indicates the extent to which air is saturated with water vapour.

$$\frac{\text{Mass of water vapour in a given volume of air}}{\text{Relative Humidity}} =$$

Mass of water vapour required to saturate the  
Same volume of air at the same temperature

When the atmosphere is fully saturated with water vapour, the relative humidity reaches 1. also, since the pressure exerted by the water vapour is proportional to its mass per unit volume, the relative humidity can also be expressed thus:

$$\frac{\text{Pressure of water vapour present in the air}}{\text{Relative Humidity}} =$$

Saturated vapour pressure at the same air pressure

#### ii Dew point

This is defined as the temperature at which water vapour present in the air is just sufficient to saturate it. It is dependent upon prevalent atmospheric conditions like temperature, wind and the amount of water vapour in the atmosphere. For instance, when ice blocks are kept inside glass tumbler, moisture would form on the outside surface of the tumbler after sometime. This is because the cold glass surface cools the air around it to temperature at which the water vapour contained in the air is sufficient to saturate the air.

$$\frac{\text{Saturated vapour pressure at dew point}}{\text{Relative Humidity}} = \frac{\text{Saturated vapour pressure at original Temperature of air}}{\text{Saturated vapour pressure at original Temperature of air}}$$



### 3.11 Humidity and Weather

The need for effective weather forecasting is becoming very important in all countries of the world. However, to achieve a good level of accuracy in such weather forecasting, it requires accurate determination of the humidity level of the air. This job in most cases is left for the meteorologist (specialists in aspects of atmospheric physics). Their reports concerning humidity and weather are found very useful in scheduling or flights in airports, navigation, radio transmission, factories, food storage, farming etc.

Also, the construction of certain household equipment such as refrigerators, air conditioners and the like require knowledge of how to keep the relative humidity of an enclosure at a specified level. The formation of clouds, rain, fog, hailstone and snow depends among other things on the relative humidity of the atmosphere.

### 3.12 Formation of Dew, Mist, Fog and Rain

#### i Formation of dew

It is a common experience that in the evening, the ground and the air in contact with it begins to cool as they lose heat through radiation. A clear sky and still air may cause temperature air to fall even lower so that the air becomes nearly saturated with water vapour. Furthermore, the surfaces of some objects particularly grass and other plants, give off water vapour, This adds to the amount of vapour in the air. When compared with the surrounding air, these surfaces are good conductors and lose heat sooner than the air. As the temperature of the ground and the grass falls below the dew point, the moisture in the surrounding air will condense and be deposited in their surfaces as dew.

If the dew point is below  $0^{\circ}\text{C}$ , the vapour changes into the solid form of ice crystals called frost.

#### ii Formation of mist and fog

If the air near the surfaces of the land cools until it reaches dew point, the water vapour begins to condense around tiny dust particles in the atmosphere. These particles have surfaces on which the condensed vapour collects. Such a collected of water particles is known as mist.

A thick mist is called a fog. The water particles of the mist or fog are small enough to float in air. When they become larger, they fall as rain.



### iii Clouds and rains

A thick mist formed high up in the atmosphere is referred to as a cloud. During the day, the land mass is warm and the warm air containing water vapour rises above the land. As the air moved upwards, it expands (due to reduced pressure at high altitudes) and becomes cooler. This cooling process, if continued long enough, causes the air to become saturated and its water vapour eventually condenses to form tiny droplets of water. When this happens, a cloud begins to form. The tiny droplets of water combine to form larger drops which fall as rain.

You should note that not all clouds are formed as water droplets. Some clouds like the fleecy cirrus clouds which form at great heights are composed of ice crystals because the water vapour in the air condenses at a height where the temperature is below freezing point.

Raindrops that are carried by strong convection currents to greater altitudes becomes frozen as they are cooled below  $0^{\circ}\text{C}$  by the cold air at such heights. These frozen raindrops fall as hail.

### SELF ASSESSMENT EXERCISE 2

1. Explain why yam cannot be easily cooked in an open pot on top of a high mountain.
2. When two ice blocks are pressed together, they stick together. Explain this observation.
3. Distinguish between evaporation and boiling.
4. List and discuss the factors that influence the rate of evaporation.
5. What do you understand by vapour, vapour pressure and saturated vapour pressure?
6. Define the terms:
  - i. Relative humidity
  - ii. Dew point
  - iii. Sublimation
7. Write short notes on the formation of the following:
  - i. Dew
  - ii. Clouds
  - iii. Fog
  - iv. Mist
  - v. Rain



## 4.0 CONCLUSION

In this unit you learned about latent heat – its concept and definitions. You also learned how to solve numerical problems involving latent heat. In addition, you learned how to determine and measure the specific latent heat of fusion of ice and of vaporization as well as applications of latent heat. You also learned about evaporation vapours, boiling sublimation and the effects of pressure and impurities on melting and boiling points. Finally, you learned about relative humidity, dew point and humidity and weather.

## 5.0 SUMMARY

The heat supplied or removed from a body which causes a change of state without a change in temperature is called latent heat.

Latent heat of fusion of solid is the heat energy that is used to change a solid at its melting point to liquid at the same temperature.

The specific latent heat of fusion of a solid is the energy required to melt 1 kg of the solid at its melting point.

Latent heat of vaporization of a liquid is the heat energy that is used to change a liquid in its boiling point to liquid at the same temperature.

The specific latent heat of vaporization of a substance is the heat required to change unit mass of its liquid at its boiling point to vapour without a change in temperature.

Increased pressure lowers the freezing point and raises the boiling point of water.

Dissolved substances in a liquid lower the freezing point and raise the boiling point of the liquid.

The factors that affect the rate of evaporation are:

- i. Temperature
- ii. Pressure
- iii. Area of liquid surface exposed
- iv Dryness of the air
- v Motion of the liquid
- vi Nature of the liquid

The vapour contained in an enclosed space above its liquid is said to be saturated when the number of molecules escaping from the liquid per



unit time is equal to the number returning to the liquid per unit time. The pressure it exerts is called the saturated vapour pressure of the liquid.

The boiling point of a liquid is the temperature at which its saturated vapour pressure is equal to the atmospheric pressure.

$$\frac{\text{Mass of water vapour in a given volume of air}}{\text{Relative Humidity}} =$$

Mass of water vapour required to saturate the  
Same volume of air at the same temperature

## 6.0 TUTOR-MARKED ASSIGNMENT

1. Define latent heat?
2. Explain why a pressure cooker boils food more quickly than on ordinary pot.
3. What are the differences between evaporation and boiling?

## 7.0 REFERENCES/ FURTHER READINGS

Anyakoha, M.W. (2000). New School Physics for Senior Secondary Schools Onisha: Africana FEP.

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## UNIT 7      EXPANSION OF GASES

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- 4.0 Conclusion
- 5.0 Summary
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### 1.0 INTRODUCTION

In this unit you will learn how the three properties of a gas-the temperature, the pressure and the volume that are necessary to describe the state of the gas are related for a given mass of gas in the gas laws. Also, you will learn how to state, verify and derive the general gas equation for the ideal gas based on the laws. Finally, you will learn about other laws governing the real gases.



## 2.0 OBJECTIVES

By the time you finish this unit, you will be able to:

- Explain, using the ideas of the kinetic theory of gases
  - The variation of volume with temperature of a gas when the pressure is kept constant
  - The variation of pressure with volume of a gas when the temperature is kept constant
- Explain Charles' and Boyle's laws of gases
- Deduce the general gas law from a given mass of gas which obeys Charles' law
- Identify the limitations of the gas laws
- Solve simple problems involving the gas laws
- Explain
  - Molar gas constant
  - Intermolecular energy and forces
- State
  - Assumptions of the elementary kinetic theory of gases
  - Graham's law of diffusion
  - Avogadro's law
  - Dalton's law of partial pressures
  - Maxwell's distribution law.

## 3.0 MAIN CONTENT

### 3.1 Expansion of Gases

We classify substances in the gaseous state as gases or vapours. You know that a gas has no volume of its own but takes the volume of its containers. The volume of a gas is changed by varying the pressure of the gas or by changing the temperatures of the gas.

Thus, the pressure, volume and temperature of a gas may change when any one of the quantities change. Hence, the thermal expansion of a gas is complicated by the fact that the volume can be altered by a change of temperature and of pressure. For us to study the behaviour of a gas with respect to the three variables: volume, temperature and pressure, we investigate the relation between two of the variables while the third one is kept constant.



### 3.1.1 Kinetic Molecular Theory of Gas and its Assumptions

We use the kinetic molecular theory of gases to explain the effect of temperature on the volume and pressure of a gas. The theory makes the following assumptions.

- A gas is considered to be made up of a large number of molecules acting like elastic spheres.
- The molecules are in a state of constant motion knocking against each other and the walls of the containing vessel.
- The particles exert attractive forces on one another.
- The particles possess kinetic energy due to their motion.
- The volume of the molecules of a gas is negligible compared with the volume of the container.

## 3.2 Boyle's Law

Here, we will consider the relationship between the pressure and the volume of a fixed mass of a gas given by Boyle's law in 1660.

### 3.2.1 Statement of Boyle's law

Boyle's law states that the pressure of a fixed mass of gas is inversely proportional to its volume, provided that the temperature is kept constant.

In symbols, Boyle's law is written as

$$P = \frac{1}{V} \quad \text{or} \quad PV = \text{constant}$$

Where  $P$  = pressure  
 $V$  = Volume  
 of a fixed mass of gas.

### 3.2.2 Verification of Boyle's law

We can verify Boyle's law in the laboratory using simple apparatus shown in Fig. 7.1.



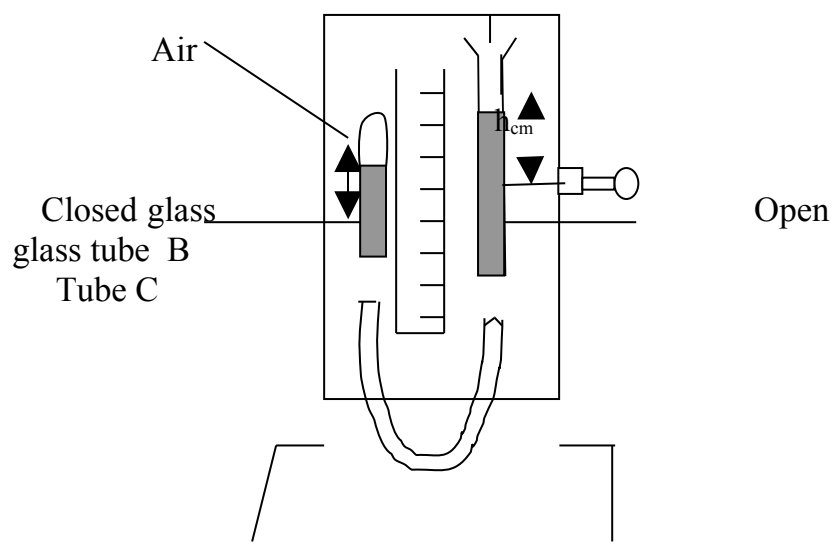


Fig. 7.1: Verification of Boyle's law

To do this, we entrap a fixed mass of dry air with mercury in a closed glass tube of uniform cross-section attached by rubber tubing to a mercury reservoir. The volume  $V$  of the dry air is proportional to the length  $L$  of the air column because the tube is of uniform cross-sectional area. The pressure and volume of the dry air varies by the open limb vertically up or down. The length which we read from the metre scale is taken as the volume of the gas while the atmospheric pressure ( $P_0$ ) is read from a Fortin barometer.

When the mercury surface in B is lower than that in C, the pressure ( $P$ ) of gas is given by:

$$P = (P_0 + h) \text{ cm Hg}$$

But, when the mercury surface in B is higher than that in C, the gas pressure ( $P$ ) is given by.

$$P = (P_0 - h) \text{ cm Hg}.$$

By lowering or raising the tube B we would obtain a set of readings of  $L$  and  $h$ . A table of these readings is then made as shown below in Table 7.1.

$l \text{ cm}$	$h_{\text{cm}}$	$(P_0 \pm h)\text{cm}$	$\frac{1}{l} \text{ cm}^{-1}$
----------------	-----------------	------------------------	-------------------------------



The values obtained in an earlier experiment when used in plotting graphs give the following result as shown in Fig. 7.2

- Graph of  $P$  against  $V$  (ie  $P_0 + h$  against  $L$ ) gives a curve as shown in Fig. 7.2a.
- Graph of  $P$  against  $\frac{1}{V}$  (ie  $P_0 + h$  against  $\frac{1}{L}$ )

is a straight line graph passing through the origin as shown in Fig. 7.2b. The straight line graph indicates that the pressure is inversely proportional to the volume of air or any gas at constant temperature.

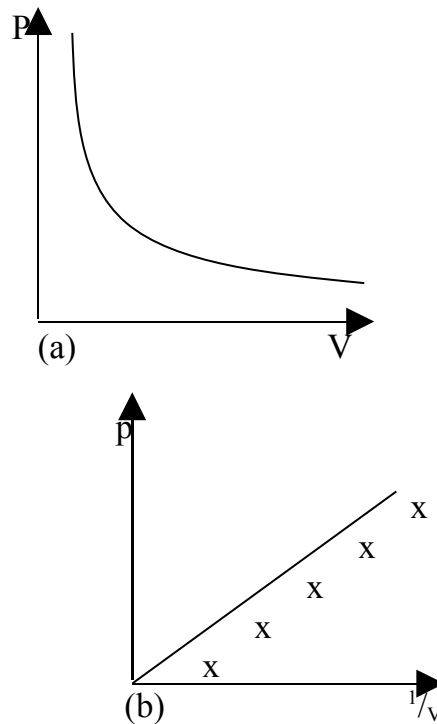


Fig. 7.2: Graph of Boyle's law

You will take the following precautions when performing this experiment.

- Ensure that the entrapped air is dry by adding a small quantity of concentrated sulphuric acid into the tube before adding mercury.
- Ensure that there is no air bubbles in the mercury.



- Ensure constant temperature by allowing the system to attained the temperature of its surrounding. You will do this by waiting for a few minutes after each variation of  $h$  and  $L$ .
- Take the readings when the mercury levels are steady and avoid parallax error of the metre rule while reading it.

### Worked example

The pressure of a fixed mass of gas is 96 cm Hg when the volume of the gas is  $5.0 \text{ m}^3$ . Find the pressure of the gas when its volume becomes  $7.0 \text{ m}^3$ . Assume the temperature is constant.

### Solution

From Boyle's law

$PV = \text{constant at constant temperature}$

$$\begin{aligned}
 &= P_1 V_1 = P_2 V_2 \\
 &96 \times 5 = P_2 \times 7 \\
 &P_2 = \frac{96 \times 5}{7} \\
 &= 68.57 \text{ cm Hg}
 \end{aligned}$$

### 3.2.3 Use of Kinetic Theory to Explain Boyle's Law

Now, let us consider a given mass of gas at constant temperature. At this constant temperature, the average velocity of the gas molecules is constant and the number of collisions they make per unit area of the wall of the containing vessel is also constant. Hence, the gas exerts a certain constant pressure,  $P$ , on the walls of the vessel.

If we now double the original volume of the gas, the gas molecules will be spread out and it will take longer time to bombard the walls and hence, fewer impact per second. As a result, the pressure of the gas will be halved. Thus, at constant temperature, volume of a gas increases as pressure decreases.

On the other hand, if we halve the original volume of the gas, the gas molecules will be more closely pack and then it will take less time to bombard the walls of the containing vessel. The gas pressure increases (double) as more impact per second are made. Hence, at constant temperature, decrease in volume of a gas leads to increase in pressure.



This means that the pressure of a given mass of gas varies inversely with its volume at constant temperature in accordance with Boyle's law.

This is illustrated diagrammatically in Fig. 7.3

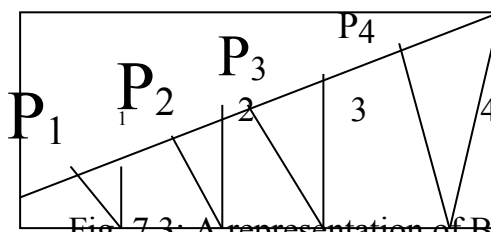


Fig. 7.3: A representation of Boyle's law

### 3.3 Charles' Law

Here we will consider the connection between the change in volume with temperature change of a fixed mass of gas kept at constant pressure published by Charles in 1787 and independently by Gay-Lussac in 1802.

#### 3.3.1 Statement of Charles' Law

Charles' law states that the volume of a fixed mass of gas increases by  $1/273$  of its volume at  $0^\circ\text{C}$  per degree Celsius rise in temperature provided its pressure remains constant.

We can also state Charles law using Kelvin temperature scale. You are aware that a temperature  $t^\circ\text{C}$  on the Celsius scale is related to an absolute or Kelvin temperature  $T$  by the equation

$$T \text{ (K)} = 273 + t^\circ\text{C}$$

Using the Kelvin scale therefore, Charles' law can be stated as

The volume of a fixed mass of gas is directly proportional to its absolute temperature if its pressure remains constant.

In symbols, we express Charles' law as

$$\begin{aligned} V &\propto T \quad ; \text{ if pressure is kept constant} \\ V &= KT \\ \text{Or } \frac{V}{T} &= K \\ \Rightarrow \frac{V_1}{T_1} &= \frac{V_2}{T_2} \end{aligned}$$

where



$V$  = volume;  $T$  = Kelvin temperature;  $K$  = a mathematical constant;  
 $V_1$  = volume of gas at temperature  $T_1$ ;  $V_2$  = volume of gas at temperature  $T_2$  of gas at constant pressure.

### 3.3.2 Verification of Charles' Law

We can verify Charles' law in the laboratory using the apparatus shown in Fig. 7.4 below.

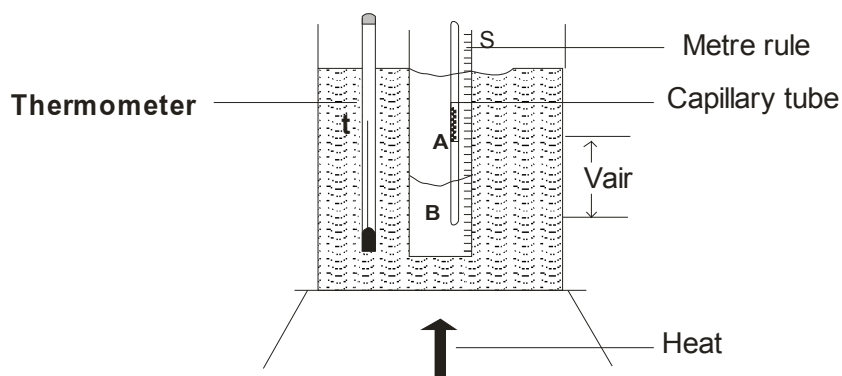


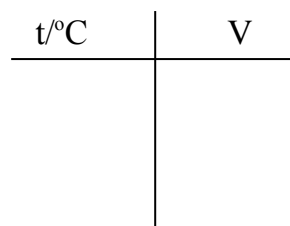
Fig. 7.4: Investigation of volume - temperature change (constant pressure)

To do this verification, we trap a column  $AB$  of air by a length of sulphuric acid which dries the air or by a length of mercury in a capillary tube  $T$  of uniform diameter. The capillary tube is sealed at the lower end and is attached to a half-metre rule  $S$  by rubber bands.

The whole arrangement is now placed inside a deep vessel filled with enough water to cover most of the tube  $T$  as shown. The water is now warmed with a burner, stirred gently with a thermometer suspended in the vessel of water. After a suitable temperature rise the temperature  $t$  is recorded as well as the length  $AB$  of the air column from the reading as the rule  $S$ . The length  $AB$  of the air column is proportional to the volume  $V$  of the air. We can obtain more readings of  $t$  and  $V$  by heating the water further to higher temperatures. The readings are as shown in Table 7.2.

Table 7.2 : Readings of temperature and the corresponding volume





When values obtained in an earlier experiment were used to plot a graph of volume  $V$  against temperature  $t^{\circ}\text{C}$ , using zero ( $0^{\circ}\text{C}$ ) on the temperature axis, the resulting graph was as shown in Fig. 7.5 below.

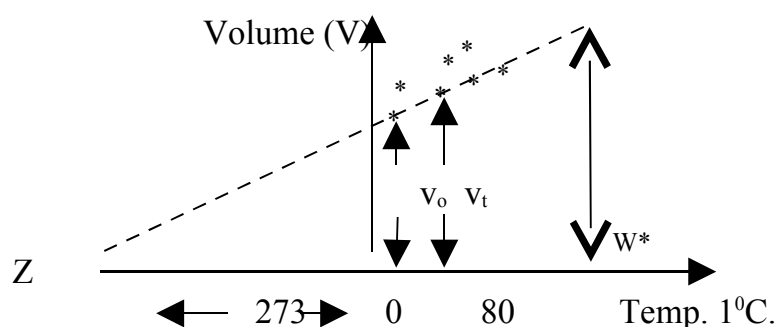


Fig. 7.5 : Volume – temperature change graph (constant pressure)

You can see from Fig. 7.5 above that we obtained a straight line graph. Because we have performed this experiment at constant pressure equal to the atmospheric pressure plus that due to the height of the liquid column, we conclude that the volume of a fixed mass of gas increases linearly as the temperature rises when the pressure is kept constant.

If you are to perform this experiment, you have to take these precautions.

- Take temperature when the thermometer reading is steady
- Stir the water continuously to ensure even temperature.
- Ensure that the bore of the capillary tube is clean and dry.
- Use only dry air to gas.
- Carry out the experiment slowly to allow heat time to pass through the thick walls of the capillary tube and bring the gas to the same temperature as the water in the vessel.

### 3.3.3 Cubic Expansivity of a Gas



You can also use Charles' law experiment to obtain the value of the cubic expansivity of a gas at constant pressure. This is defined as:

Increase in volume per unit volume at  $0^{\circ}\text{C}$  per degree Celsius rise in temperature.

That is, if

$\gamma$  = cubic expansivity of a gas

$V_0$  = volume at temperature  $0^{\circ}\text{C}$

$V_t$  = volume at temperature  $t^{\circ}\text{C}$

Then

$$\gamma = \frac{V_t - V_0}{V_0 \times t}$$

We have from careful experiments obtained the fraction  $\gamma$  in the equation above to be same for all gases and is equal to  $1/273$ .

You know the cubic expansivity of a gas unlike that of solids and liquids is the same irrespective of the nature of the gas. Also, the cubic expansivity of a gas is calculated from the fractional increase of its volume at  $0^{\circ}\text{C}$ ?

### Worked example

1. A fixed mass of gas is heated at constant pressure from  $25^{\circ}\text{C}$  to  $80^{\circ}\text{C}$ . If the volume at  $25^{\circ}\text{C}$  is  $150\text{ cm}^3$ , what volume will it occupy at  $80^{\circ}\text{C}$ .

### Solution

From Charles' law

$$\begin{aligned}\frac{V_1}{T_1} &= \frac{V_2}{T_2} \\ V_1 &= 150\text{ cm}^3 ; V_2 = ? \\ T_1 &= (273 + 25)\text{ K} = 298\text{ K} \\ T_2 &= (273 + 80)\text{ K} = 353\text{ K}\end{aligned}$$

Substituting in the above equation gives us

$$\frac{150}{298} = \frac{V_2}{353}$$



$$\begin{aligned}
 & \frac{298}{298} = \frac{V_2}{150} = \frac{150 \times 353}{298} \text{ cm}^3 \\
 & = 177.68 \text{ cm}^3
 \end{aligned}$$

The volume to be occupied by the gas at constant pressure and at 80°C is 177.68 cm<sup>3</sup>.

2. The volumes of a fixed mass of gas at constant pressure are 150 cm<sup>3</sup> at 0°C and 178 cm<sup>3</sup> at 80°C. Calculate the cubic expansivity of the gas.

### Solution

You will recall that the cubic expansivity ( $\gamma$ ) of a gas is given by the equation

$$\gamma = \frac{\text{Increase in volume from } 0^\circ\text{C}}{\text{Volume at } 0^\circ\text{C} \times \text{temperature rise}}$$

That is,

$$\begin{aligned}
 \gamma &= \frac{V_1 - V_0}{V_0 \times t} \\
 &= \frac{178 - 150}{150 \times 80} \\
 &= \frac{28}{12000} \\
 &= 2.33 \times 10^{-3} / ^\circ\text{C} \text{ or } 2.33 \times 10^{-3} / \text{K}
 \end{aligned}$$

### 3.3.4 Use of Kinetic Molecular Theory to Explain Charles' Law

To do this, let us consider a fixed mass of a gas confined in a vessel at constant pressure. Now, if we heat this gas, the molecules acquire more K. E., move faster and collide more often with the walls of the vessel. As a result, increases the pressure they exert. For us to maintain a constant pressure, we then increase the volume of the containing vessel to allow the same mass of gas to travel a longer distance before striking the walls of the container. Thus an increase in temperature leads to an increase in the gas volume. This is in agreement with Charles's law.



### 3.4 Pressure Law or Gay- Lussac's Law

Here, we will consider the connection between the pressure change with temperature of a fixed mass of gas maintained at constant volume. This connection was given by Gay-Lussac in 1802 and these are two alternatives of stating the relation (law).

#### 3.4.1 Statement of Pressure Law

- We can now state the law in the two alternative ways thus:

The law states that the pressure of a fixed mass of gas at constant volume increase by  $\frac{1}{273}$  of its pressure at  $0^{\circ}\text{C}$  for every degree Celsius (or Kelvin) rise in temperature.

We write the above in symbols as

$$P_t = P_0 \left( 1 + \frac{t}{273} \right)$$

where  $P_0$  = pressure at temperature  $0^{\circ}\text{C}$

$P_t$  = Pressure at temperature  $t^{\circ}\text{C}$ .

The alternative way of stating the pressure law is:

- The pressure of a fixed mass of gas at constant volume is proportional to the absolute temperature of the gas.

In symbols, we state the law as  $P \propto T$

$$\text{Or } \frac{P}{T} = \text{constant}$$

$$\text{That is, } \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

#### 3.4.2 Verification of Pressure Law

Fig. 7.6 below shows one of the forms of apparatus we use to verify Pressure law. It consists of a large bulb F filled with dry air; a mercury manometer with one side BC open to the atmosphere. This is connected to F by a very narrow tube D.

We place the bulbs F inside a large vessel and cover it completely with water as shown in Fig. 7.6.



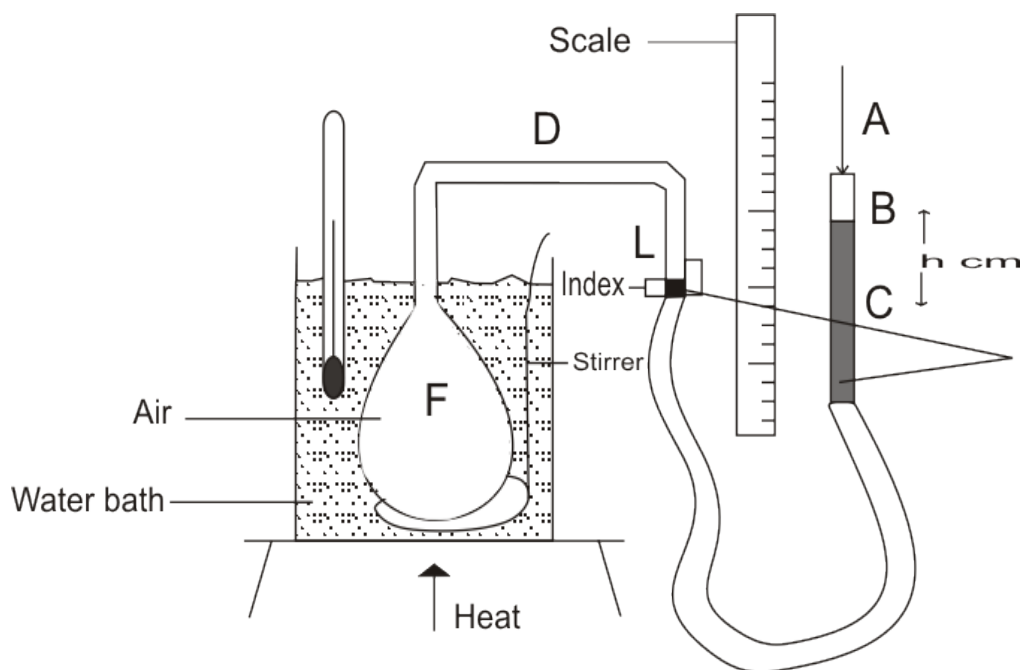


Fig. 7.6: Pressure – temperature change apparatus (constant volume)

First, we record the initial temperature  $t$  of the gas. We then raise or lower the open side BC of the manometer until the top of the mercury on the other side reaches the level L. We then read and record the height  $h$  of the mercury column above L and the atmospheric pressure A.

We warm the air in the bulb F by heating the water round the bulb and stirring it gently. At a suitable temperature rise of between  $15$  to  $20^{\circ}\text{C}$ , we take away the burner and raise BC until the mercury level on the other side again reaches L. We then read and record the new difference in levels,  $h$ , between B and L; and the temperature  $t$ .

We repeat these experimental procedures with higher temperature for the air. In each case, BC is raised until the mercury level on the other side is at L. Thus, maintaining the volume of the air constant. We read and record the corresponding values of  $h$  and  $t$ . These readings are as shown in Table 7.3 below.

Table 7.3: Readings of temperature and the corresponding values of  $h$  and  $p$

$t/^{\circ}\text{C}$	$h / \text{mm Hg}$	$P (= A + h \text{ or } A - h)$

Using the values from the table, we plot a graph of the pressure  $P$  against temperature  $t$  at constant value. If B is above L,  $p = A + h$  but if B is below L,  $P = A - L$ .



In an earlier experiment, we obtained a straight line AB which showed that at constant volume, the pressure of a gas varies uniformly or linearly with temperature. A typical graph of an earlier experiment is shown in Fig. 7.7 below.

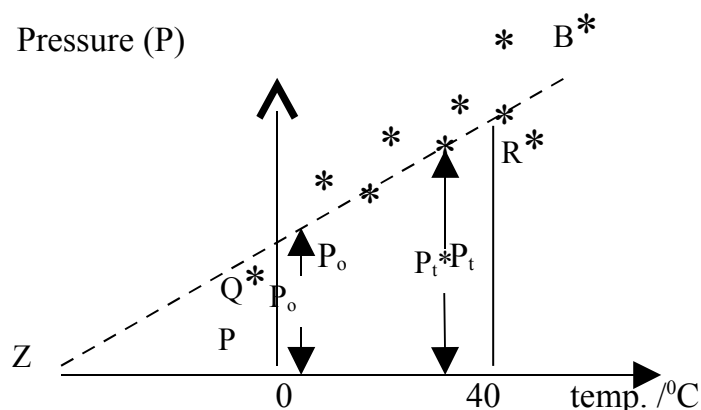


Fig. 7.7: Graph of pressure against temperature (volume constant)

From the result of the above experiment, we can deduce that for all permanent gases and dry air, the pressure increases by about  $1/273$  of its pressure at  $0^\circ\text{C}$  for each degree Celsius rise in temperature. Hence, the pressure expansivity of gases is the same as the volume expansivity. Therefore, we define the pressure expansivity of the gas as

$$\gamma = \frac{\text{Increase in pressure from } 0^\circ\text{C}}{\text{Pressure at } 0^\circ\text{C} \times \text{temperature rise}}$$

Symbolically,

$$\gamma = \frac{p_t - p_0}{P_0 \times t}$$

$$\text{or } P_t = P_0 \left( 1 + \frac{t}{273} \right)$$

### Worked example

The pressure of a car tyre at  $10^\circ\text{C}$  is  $0.20\text{N/mm}^2$ . What would be the pressure if the temperature rises to  $25^\circ\text{C}$ .

### Solution

Recall that from Pressure law,



$P \propto T$  at constant volume

$$\text{Therefore, } \frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$= \frac{0.20}{P_2} = \frac{273 + 10}{273 + 25} = \frac{283}{298}$$

$$P_2 = \frac{0.20 \times 298}{283} = 0.21 \text{ N/mm}^2$$

### 3.4.3 Use of Kinetic Molecular Theory to Explain Pressure Law

Do you remember that heat is a form of energy and that when a gas is heated the molecules gain kinetic energy and move about faster. As a result, the momentum changes produced by the molecules at the walls per impact is greater. This also leads to increase in the number of impacts of each molecule per second because of the increase in speed.

Thus, the pressure increases with rise in temperature at constant volume. This is statement of Gay-Lussac's law or Pressure law.

### SELF ASSESSMENT EXERCISE 1

- 1 (a) State and describe experiments to verify:
  - (i) Boyle's law
  - (ii) Charles's law
  - (iii) Pressure law.
- (b) Use kinetic molecule theory to explain the above laws.
2. Describe as fully as you can the kinetic picture of a gas and show how it accounts for the change in pressure when the temperature of a gas is increased at constant volume.
3. The pressure of a fixed mass of gas at constant volume increase from 60 cm of mercury at 10°C to 95 cm of mercury at 80°C. What would be the pressure of the air at 0°C?
4. A fixed mass of air occupying 500 cm<sup>3</sup> at 15°C was heated to 85°C. What was its new volume?

## 3.5 Ideal or the General Gas Law

### 3.5.1 The General Gas Law or Equation



From the gas laws we know that the volume of a gas depends on both its temperature and its pressure. We can sum up the relationship between these three variables – volume, temperature and pressure to obtain the general gas law. We have that.

From Boyle's law,

$$PV = \text{constant if } T \text{ is constant from,}$$

From Charles' law,

$$\frac{V}{T} = \text{constant if } P \text{ is constant}$$

From the Gay – Lussac's or pressure law,

$$\frac{P}{T} = \text{constant at constant volume}$$

We can combine any two of these three equations to obtain the general gas equation.

That is,

$$\frac{PV}{T} = K$$

where K is a constant for a fixed mass of gas.

$$\Rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Where

$P_1$   $V_1$  are the gas pressure and volume at temperature  $T_1$  and  $P_2$   $V_2$  are the gas pressure and volume at temperature  $T_2$ .

We can use the general gas equation to find any of the variables in the equation when the other two are known. For instance, if we know the initial volume of a gas, we can determine the final volume when both its pressure and temperature change.

That is,

$$V_2 = \frac{P_1 V_1 T_2}{P_2 T_1}$$

**You have to note that**



- The law does not hold good when a gas is near its liquefying temperature.
- The relationship ' $PV/T = \text{constant}$ ' is a law for an ideal gas. It assumes that the attraction between the gas molecules and the volume they occupy are both negligible. But, this is not the case for a real gas.

### 3.5.2 The Ideal Gas Equation

The ideal gas equation relates generally pressure, volume and absolute temperature of a fixed mass of gas as we have already discussed in the general gas equation of section 3.5.1 above.

That is,

$$\frac{PV}{T} = \text{constant}$$

However, for an ideal gas, at standard temperature and pressure, for 1 mole of gas we have

$$PV = RT$$

where  $R$  is a constant known as the molar gas constant and its value is  $8.31 \text{ J mol}^{-1} \text{ K}^{-1}$

The above equation is referred to as the Ideal gas equation. It holds true for all gases.

For  $n$  moles, however, we have

$$\text{Or } \frac{PV}{T} = \frac{nRT}{nR}$$

The above equation is the general form and is called the equation of states of an ideal gas.

### 3.6 Conversion to Standard Temperature and Pressure (S.T.P)

You have seen that the volumes of gases change remarkably with changes in temperature and pressure. For this reason, it is not easy to compare results of experiments between countries or towns across the world having temperate and tropical climates. This observation made scientists choose  $0^\circ\text{C}$  or  $273\text{K}$  and  $760 \text{ mm Hg}$  or  $1.01 \times 10^5 \text{ Nm}^{-2}$  to be the standard temperature and pressure (s. t. p. ) at which gas volumes are



given. This makes it easy and possible for us to compare volumes of different masses of a gas.

### 3.7 Intermolecular Energy and Forces

We know from common experience that forces exist between molecules. These intermolecular forces arise from two main causes: the potential energy (interactions with surrounding molecules and is electrical in nature) and thermal energy (the kinetic energy which depends on the temperature of the substance concerned) of the molecules. The intermolecular forces between molecules decrease rapidly as the distance between molecules increases.

Thus for small distances between molecules (solid or liquid) the intermolecular forces are repulsive and increases rapidly as the separation is reduced.

The forces between molecules vary in the form shown in Fig. 7.8 below. Fig. 7.8 shows the variation of the potential energy  $V$  between two molecules at a distance  $r$  apart.

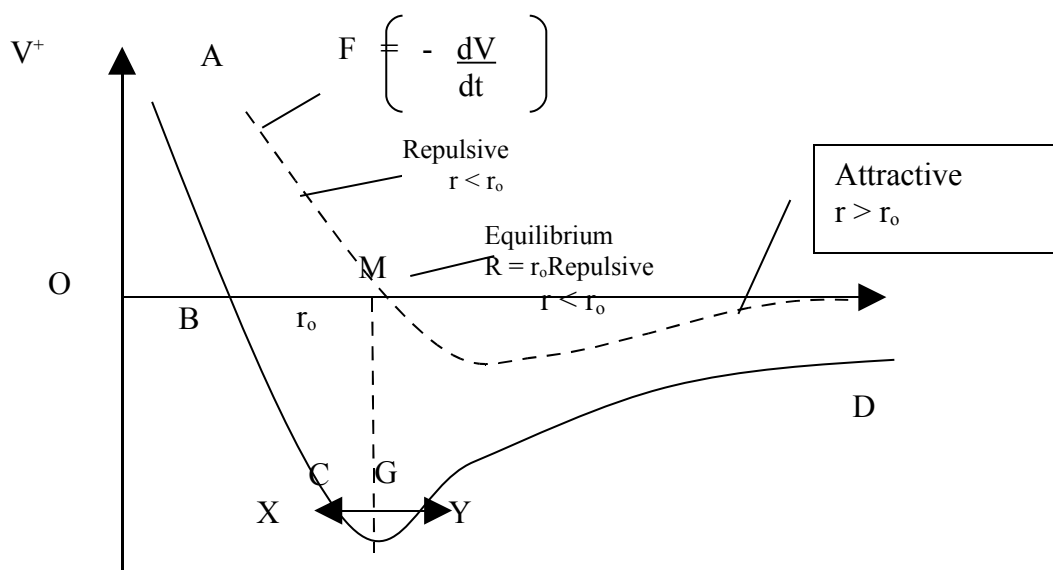


Fig. 7.8 : Molecular potential energy and force

At the point  $B$ , the intermolecular energy is zero. For distances shorter than  $OB$ , the force is negative. Along the part  $AB$ , the P.E is positive. The force between the molecules is always given by  $F = -\frac{dV}{dr} = -$  potential gradient. At distances greater than  $OB$ , the force is attractive. It increases with distance, reaches a maximum and then decrease rapidly. Along  $CD$ , the force is attractive and it decreases with distance  $r$  according to an inverse – power of  $r$ . Along  $ABC$ , the force is repulsive. This variation of  $F$  with  $r$  is illustrated in figure 7.8. At  $C$ , the



minimum P.E point of the curve, the molecules are at their normal distance apart in the absence of thermal energy. At this equilibrium distance, the attractive and repulsive forces balance each other. If the molecules are closer ( $r < r_0$ ), they repel each other. If they are further apart ( $r > r_0$ ), they attract each other.

For gases molecules, the molecules are far apart that the intermolecular attractions are small. As the molecules are brought closer by compressing the gas, the force of attraction increases, passes through the maximum and then decreases to zero at B where it exists in liquid phase.

Small cohesive force exist in gases. The intermolecular attraction between molecules of liquids is larger than that of gases because they are closer together. This makes liquid to have definite volume and higher cohesion than gases.

The intermolecular forces between solid molecules are very strong and are much more than that of liquids and gases. This makes them to have definite shape their atoms and molecules have definite pattern or lattice. Thus, solids are less compressible than liquids.

### 3.8 Pressure Calculations

Here, suppose a cubical box of side  $l$  contains  $N$  molecules of gas each of mass  $M$ . A typical molecule of this gas will have a velocity  $C$  at any instant with components of  $U$ ,  $V$  and  $w$  respectively in the direction of the three perpendicular axes  $0_x$ ,  $0_y$ ,  $0_z$  as shown in Fig. 7.9.

$$\text{Hence } C^2 = U^2 + V^2 + W^2.$$

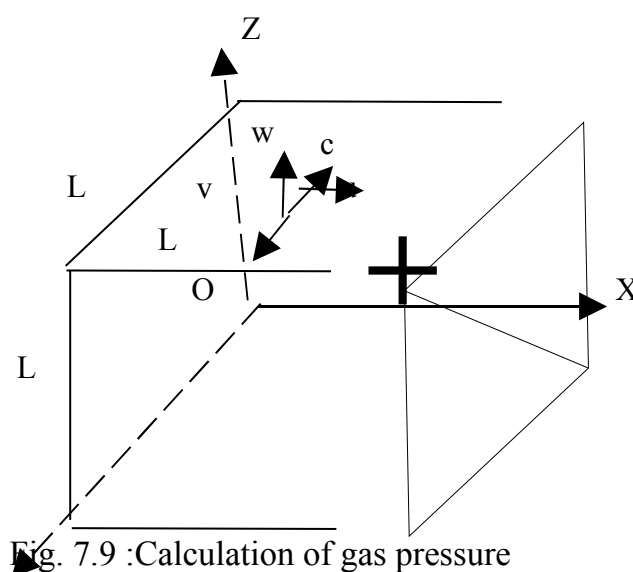


Fig. 7.9 :Calculation of gas pressure

Suppose a molecule of the gas in this box moves and collide with the box on the face  $x$  due to the component of the velocity  $u$ .



Momentum before impact =  $\mu u$

Momentum after impact =  $-\mu u$

Therefore, momentum change on impact

$$\begin{aligned} &= \mu u - (-\mu u) \\ &= -2 \mu u \end{aligned}$$

Time taken for the molecule to move across the cube to the opposite face and back to x is  $2l/u$

Therefore

Momentum change per second

$$\begin{aligned} &= \frac{\text{momentum change}}{\text{time}} \\ &= \frac{2\mu u}{2l/u} = \frac{\mu u^2}{l} \end{aligned}$$

$$\text{force on } x = \frac{\mu u^2}{l}$$

You know that

$$\text{Pressure on } x = \frac{\text{force}}{\text{area}}$$

This implies that pressure on x

$$= \frac{\text{force}}{\text{area}} = \frac{\mu u^2}{l \times l^2} \dots\dots(i) \quad l^3$$

For N molecules in the cube, in the x direction we have

$$P = \frac{\mu u_1^2}{l^3} + \frac{\mu u_2^2}{l^3} + \frac{\mu u_3^2}{l^3} + \dots\dots\dots + \frac{\mu u_N^2}{l^3}$$

$$P = \frac{m}{l^3} (u_1^2 + u_2^2 + u_3^2 + \dots\dots\dots + u_N^2) \dots\dots(ii)$$

where

$U_1^2, U_2^2, u_3^2 \dots\dots\dots U_N^2$  are components of velocity of various magnitudes in the OX direction.

If  $\bar{U}^2$  represents the average or mean value of all the squares of the velocity components in the OX direction, that is,

$$\bar{U}^2 = \frac{u_1^2 + u_2^2 + u_3^2 + \dots\dots\dots + u_N^2}{N}$$

Which implies that



$$\frac{N\bar{u}^2}{N} = \frac{u_1^2 + u_2^2 + u_3^2 + \dots + u_N^2}{N}$$

Then equation (ii) becomes

$$P = \frac{N m \bar{u}^2}{l^3} \dots\dots\dots(iii)$$

Now, if we consider a larger number of molecules of varying speed in random motion, the mean square of the component speed in any of the three axes is the same.

That is

$$\bar{u}^2 = \bar{v}^2 = \bar{w}^2$$

For each molecule, however

$$C^2 = u^2 + v^2 + w^2$$

Thus, the mean square  $C^2$  is given by

$$\bar{C}^2 = \bar{u}^2 + \bar{v}^2 + \bar{w}^2$$

Therefore

$$\bar{u}^2 = \frac{1}{3} \bar{C}^2$$

From equation (iii)

$$P = \frac{1}{3} \frac{N m \bar{C}^2}{l^3}$$

The number of molecules per unit volume

$$n = N/l^3$$

Hence,  $P = \frac{1}{3} n m \bar{C}^2 \dots\dots\dots(i)$

where  $n$  is in molecules per metre<sup>3</sup>,  $m$  in kilogram and  $c$  in metre per second. Unit of pressure: newton per metre<sup>2</sup> (Nm<sup>-2</sup>)

### 3.9 Introduction of Temperature

We can move a step further by considering the volume of gas containing  $N$  molecules. The number of molecules per unit volume is

$$n = \frac{N}{V} \quad \text{or} \quad n = \frac{N}{l^3}$$

Thus, the pressure of a gas from equ (1) becomes

$$P = \frac{1}{3} n m \bar{C}^2 = \frac{1}{3} \frac{N m \bar{C}^2}{V}$$

Therefore

$$PV = \frac{1}{3} N m \bar{C}^2 \dots\dots\dots(2)$$

Equation (2) is similar to the equation combining Boyle's law and Charles' law  $PV = RT$



We can by making kinetic theory consistent with the observed behaviour of a gas, write

$$\frac{1}{3} N m \bar{c}^2 = RT \dots\dots\dots(3)$$

Here, we assume that the mean square speed of the molecules  $\bar{c}^2$  is – proportional to the absolute temperature of the gas.

The kinetic energy of a molecule moving with a speed  $c$  is  $\frac{1}{2} M c^2$ .

Thus, the kinetic energy of translation of the random motion of the molecule of a gas is

$$\frac{1}{2} m \bar{c}^2.$$

Relating  $\frac{1}{2} m \bar{c}^2$  to the temperature enables us to put eqn (3) in the form

$$RT = \frac{1}{3} N m \bar{c}^2 = \frac{2}{3} N \left( \frac{1}{2} m \bar{c}^2 \right)$$

where

$$\frac{1}{2} m \bar{c}^2 = \frac{3}{2} \frac{R}{N} T \dots\dots\dots(4)$$

Thus, the average kinetic energy of translation of a molecule is proportional to the absolute temperature of the gas.

We note that the ratio  $R/N$  is a universal constant .  $R$  is the gas constant .while  $N$  is the number of molecule.

$$\frac{R}{N} = k$$

where  $k$  is the gas constant per molecule and also universal constant called Boltzmann's constant.

Substituting  $k$  in eqn. (4) gives us

$$\frac{1}{2} M \bar{c}^2 = \frac{3}{2} k T \dots\dots\dots(5)$$

Boltzmann's constant relates energy to temperature and is usually given in joules per degree.

$$k = \frac{\frac{1}{2} m \bar{c}^2}{\frac{3}{2} T}$$

The value of  $k$  is  $1.38 \times 10^{-23} \text{ J K}^{-1}$ .

### 3.10 Deductions from the Ideal Gas Equation.



**i Determination of the value of root mean square (rms) speed ( $\sqrt{C^2}$ )**

You will recall from equation (1) that

$$P = \frac{1}{3} n m \overline{c^2} \dots\dots\dots(i)$$

where  $n$  = number of molecules per unit volume ( $N/V$ ).

$Nm$  = total mass  $M$  of the gas per volume

Substituting the above in equ (i)

$$\text{gives us } p = \frac{1}{3} \frac{m}{V} \overline{c^2} \dots\dots\dots(6)$$

You know that density  $\rho$  is mass per unit volume. Therefore, when we substitute  $\rho$  for  $m/v$  in eqn (6) we have

$$P = \frac{1}{3} \rho \overline{C^2} \dots\dots\dots(7)$$

$$\text{Or } \frac{p}{\rho} = \frac{1}{3} \overline{C^2}$$

which gives us

$$\overline{C^2} = \frac{3p}{\rho} \dots\dots\dots(8)$$

The square root of  $\overline{C^2}$  is called the root – mean – square speed.

That is

$$\sqrt{\overline{C^2}} = \sqrt{\frac{3p}{\rho}}$$

$$\text{But } p = \frac{RT}{V}$$

Hence root-mean –square (rms) speed

$$\sqrt{\overline{C^2}} = \sqrt{\frac{3RT}{M}} \dots\dots\dots(9)$$

Since  $\rho = m/v$  and  $v = 1$  unit volume

To find the value of the root-mean-square.  $\left(\sqrt{\overline{C^2}},\right)$  we substitute



known values of  $\rho$  and  $p$  for hydrogen at STP.

At STP  $\rho$  for hydrogen =  $0.09 \text{ kg m}^{-3}$

$$P = \rho g H$$

Where  $g$  = acceleration due to gravity  $9.81 \text{ ms}^{-2}$

$\rho$  = density of mercury =  $13600 \text{ kg m}^{-3}$

$H$  = barometer height =  $760 \text{ mm} = 0.76 \text{ m}$

Substituting the above in eqn (8) gives us

$$\begin{aligned} \bar{c}^2 &= \frac{3p}{\rho} = \frac{3 \times \rho g H}{\rho \text{ of hydrogen}} \\ &= \frac{3 \times 9.81 \times 13600 \times 0.76}{0.09} \\ &= 304188.48 \\ &= 3.37 \times 10^6 \text{ m}^2 \text{ s}^{-2} \end{aligned}$$

Therefore, the root mean square,  $\bar{c}$  is

$$\begin{aligned} \bar{c} &= \sqrt{3.37 \times 10^6} \\ &= 1838.44 \\ &= 1.84 \times 10^3 \text{ ms}^{-1} \end{aligned}$$

Table 7.2 values of  $\sqrt{\bar{c}^2}$  at  $0^\circ\text{C}$  for the common gases

Gas	$\sqrt{\bar{c}^2}$
Hydrogen	$1.839 \times 10^3 \text{ ms}^{-1}$
Helium	$1.310 \times 10^3 \text{ ms}^{-1}$
Nitrogen	$0.493 \times 10^3 \text{ ms}^{-1}$
Oxygen	$0.461 \times 10^3 \text{ ms}^{-1}$
Carbon dioxide	$0.392 \times 10^3 \text{ ms}^{-1}$

## ii Graham's Law of Diffusion

Graham, an English Chemist, in 1833 discovered that a less dense gas diffuses through a porous material much faster than a denser gas. He carried out an experiment to investigate this relationship and the result of his investigation is commonly referred to as Graham's law of diffusion.

**Graham's law of diffusion states that at constant temperature and pressure the rate of diffusion of a gas through porous material is inversely proportional to the square root of its density.**



In symbols,

$$\text{Rate of diffusion} \propto \sqrt{\frac{1}{\rho}}$$

### iii Avogadro's law

Avogadro (1776 – 1856), an Italian professor put forward a hypothesis that illuminated Chemistry as Newton illuminated mechanics. This hypothesis he puts forward was accepted in the middle of 19<sup>th</sup> century and is now known as the Avogadro's law.

*Avogadro's law states that equal volumes of all gases at the same temperature and pressure contained equal number of molecules.*

We recognize 1cm<sup>3</sup> of gas at STP as containing  $2.6 \times 10^{19}$  molecules.

Avogadro's law enables comparison of gas masses by chemical methods.

### iv Daltons' Law of Partial Pressure

According to Dalton in 1801, the total pressure of a mixture of gases is the sum of the pressures which each individual gas would exert if it were confined alone in the volume occupied by the mixture. This statement first credited to Dalton is generally referred to as Dalton's law of Partial Pressure.

*The law states that the total pressure of a mixture of gases is equal to the sum of the pressures of the individual gases assuming each gas occupies the volume of the mixture at the same temperature.*

We express the law mathematically as

$$P_{\text{total}} = P_A + P_B + P_C \dots\dots\dots P_Z$$

where

total = total pressure of the mixture and  $P_A, P_B, P_C \dots\dots\dots p_Z$  are the partial pressures exerted separately by the individual gases A, B, C, .....Z that make up the mixture.

### v Maxwell's Distribution Law



You are aware that the speed of individual molecules vary over a wide range of magnitude. Therefore, there is a characteristic distribution of molecular speeds for a given gas which depends on the temperature. Even gases which have the same initial molecular speed  $v$  later change their speeds because of collisions. Many of these molecules do not have speeds near zero. (ie  $\ll V_{rms}$ ) or much more than  $V_{rms}$  ( $\gg v_{rms}$ ). Because these extreme speeds require an unlikely sequence of preferential collisions. C Maxwell solved this problem of distribution of speeds in a large number of molecules of a gas in his law – Maxwell's Distribution law

Maxwell's molecular speed distribution law states that for a sample of gas containing  $N$  molecules, that

$$N_v = 4\pi N \left( \frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/kT}$$

Where

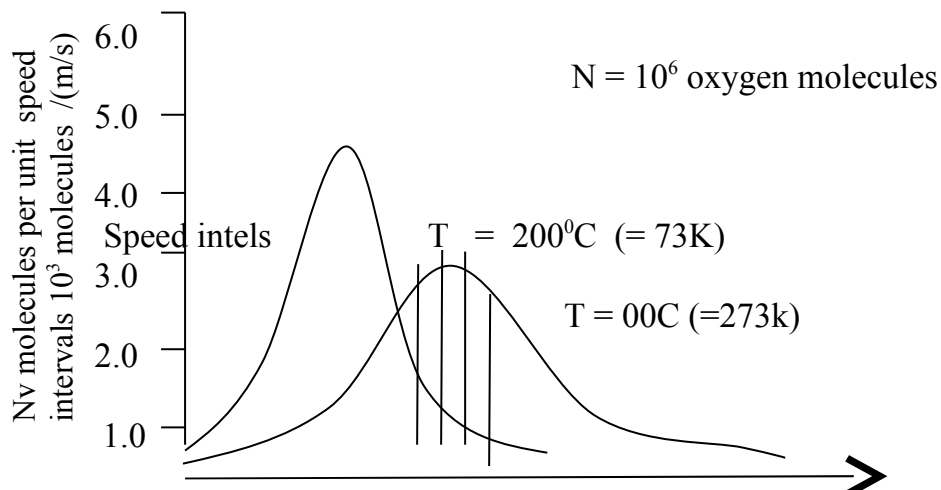
$N(v)dv$  is the number of molecules in the gas sample having speeds between  $v$  and  $v + dv$ . That is

$$N = \int_0^\infty N(v) dv$$

Unit molecules / ( cm/s)

$T$  is the absolute temperature,  $k$  is boltmann's constant and  $m$  is the mass of a molecule.

For a given gas, the speed distribution depends only on temperature.





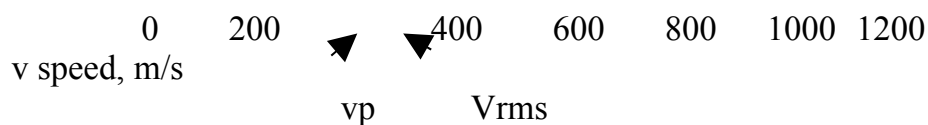


Fig. 7.10: The Maxwellian distribution of speed for oxygen molecules  
Figure 7.10 above shows Maxwell distribution of speeds for molecules of oxygen at two different temperatures. The area under the curve represents the number of molecules at speed between  $V_1$  and  $V_2$ .

## SELF ASSESSMENT EXERCISE 2

1. State the equation of state of an ideal gas.
2. Derive an expression for the root-mean-square (rms) speed  $\sqrt{\overline{C^2}}$ . State your derivation from the equation expressing pressure of a gas as

$$P = \frac{1}{3} n m \overline{C^2}$$

All symbols retain their usual meaning.

3. State
  - (a) Graham's law of diffusion
  - (b) Avogadro's law
  - (c) Dalton's law of partial pressure
  - (d) Maxwell's distribution law

## 4.0 CONCLUSION

In this unit you learned about expansion of gases. You also learned about the kinetic molecular theory of gas and its assumptions. Furthermore, you learned about Boyle's law, Charles' law, Pressure law, their verifications and explanation using kinetic molecular theory. In addition you learned the general gas law, the ideal gas equation, intermolecular energy and forces, pressure calculations and introduction of temperature. Finally, you learned deductions

from the ideal gas equation – the value of root-mean-square  $\sqrt{\overline{C^2}}$  speed, Graham's law of diffusion, Avogadro's law, Dalton's law of partial pressure and Maxwell's distribution law.

## 5.0 SUMMARY

- Boyle's law states that the pressure of a fixed mass of gas is inversely proportional to its volume, provided that the temperature is kept constant.



- $P = \frac{1}{V}$  or  $PV = \text{constant}$
- Where  $P = \text{pressure}$
- $V = \text{volume}$
- of a fixed mass of gas.
- Charles' law states that the volume of a fixed mass of gas increases by  $1/273$  of its volume at  $0^\circ\text{C}$  per degree Celsius rise in temperature, provided its pressure remains constant.
- OR. The volume of a fixed mass of gas is directly proportional to its absolute temperature if its pressure remains constant.
- $\frac{V}{T} = k$
- Where  $V = \text{volume}$ ,  $T = \text{kelvin temperature}$
- Pressure or Gay – Lussac's law states that the pressure of a fixed mass of gas at constant volume increases by  $1/273$  of its pressure at  $0^\circ\text{C}$  for every degree Celsius (or kelvin) rise in temperature.
- $P_t = P_o (1 + t/273)$ .
- Or The pressure of a fixed mass of gas at constant volume is proportional to the absolute temperature of the gas.
- $\frac{P}{T} = \text{constant}$
- The General Gas Equation / law
- $\frac{PV}{T} = \text{constant}$
- The ideal Gas Equation
- $PV = nRT$
- Root - mean - square (rms) speed
- $\sqrt{c^2} = \sqrt{3RT/M}$
- Or  $\sqrt{c^2} = \sqrt{3P/\rho}$



- Graham's law of diffusion states that at constant temperature and pressure the rate of diffusion of a gas through porous material is proportional to the square root of its density
- Rate of diffusion  $\propto \sqrt{1/\rho}$
- Avogadro's law states that equal volume of all gases at the same temperature and pressure contained equal number of molecules.
- Dalton's law of partial pressure states that the total pressure of a mixture of gases is equal to the sum of the pressures of the individual gases assuming each gas occupies the volume of the mixture at the same temperature.
- $P_{\text{total}} = P_A + P_B + P_C + \dots + P_z$
- Maxwell's molecular speed distribution law states that for a sample of gas containing N molecules, that
- $N_v = 4\pi N \left(\frac{m}{2\pi Kt}\right)^{3/2} V^2 e^{-mv^2/KT}$

## 6.0 TUTOR -MARKED ASSIGNMENT

1. State Boyle's law and express the law mathematically. What are the precautions to be taken when verifying this law in the laboratory?
2. (a) Mention five assumptions of the kinetic molecular theory of gases  
(b) Use kinetic molecular theory to explain Charles' law.
3. (a) Write down the ideal gas equation  
(b) A fixed mass of gas occupies 10-x m<sup>3</sup> at S.t.P. What volume does it occupy at 270C if its pressure is 62 cm of mercury?
4. Given the equation

$$p = \frac{1}{3} nmC^2$$

where

n = number of molecules per unit  
 nm = total mass M of the gas  
 per unit volume M of the gas  
 per unit volume

Show that the root mean square speed is given by the expression

$$\sqrt{\quad} \quad \sqrt{\quad}$$



$$C^2 = \frac{3RT}{Mo}$$

## 7.0 REFERENCES/FURTHER READINGS

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## MODULE 4

Unit 1	Reflection of Light
Unit 2	Reflection at Curved (Spherical) Mirrors
Unit 3	Refraction of Light
Unit 4	Refraction at Curved Surfaces (Lenses)
Unit 5	Applications of Light Waves
Unit 6	Dispersion of Light and Colours

## UNIT 1 REFLECTION OF LIGHT

### CONTENTS

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- 3.0 Main Content
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## **1.0 INTRODUCTION**

In this Module, starting from this unit, you will be introduced to light waves. Specifically, you will learn about the concept, nature, sources and properties of light. Also, you will learn about transmission and reflection of light at plane mirror surfaces, laws of reflection and formation of images by plane mirrors. Furthermore, you will study the deviation of light at plane mirror surfaces and of reflected ray by rotated mirror. Finally, you will learn about the applications of reflected light at



plane mirror surfaces especially in simple periscope, kaleidoscope and mirror galvanometer.

## 2.0 OBJECTIVES

At the end of this unit you will be able to:

- list some sources of light you come across in your local environment
- determine the angle of reflection for a given angle of incidence
- draw ray diagrams to show the formation of images by plane mirrors
- explain some practical applications of plane mirrors
- solve simple quantitative problem involving plane mirrors.

## 3.0 MAIN CONTENT

### 3.1 Concept of Light

Light (optics) is a form of energy which causes the sensation of vision. This energy association with light is known as luminous energy and it is the energy which causes the sensation of vision when it falls on our eyes.

#### 3.1.1 Nature of light

As you know, there are two views about the nature of light namely;

- (i) Light as particle (Corpuscular theory);
- (ii) Light as matters (Wave theory);

According to the corpuscular theory light consists of a stream of minute weightless particles given off by the source. On the basis of this theory, interference and diffraction of light cannot be explained while the photoelectric effect could be explained satisfactorily by assuming that light exists in discrete particles known as photons.

On the other hand, the wave theory considered light as a form of disturbance spreading out from its source. On the basis of this theory, the phenomena of interference and diffraction of light could be satisfactorily explained while that of the photoelectric effect cannot be explained by it.

On the basis of these experimental observations, we accept that light sometimes behaves like waves and at others times likes particles. Thus the wave properties of light are used to explain phenomenon such as interference and diffraction while the particle nature of light is used to explain the photoelectric effect. Hence, we speak of the wave – particle



duality of light. These two aspects of light (wave and particle) cannot be observed in a single experiment.

In recent times, however, the wave nature of light has been accepted as a better option. In this regard, light is perceived to consist of waves – like packets of energy called photons with a traveling speed of  $3 \times 10^8 \text{ ms}^{-1}$

### 3.1.2 Sources of Light

Visible light is a form of energy which our eyes can detect. Visible light is divided into two sources namely the luminous sources and the non-luminous sources.

- (i) Luminous sources: These are light sources that generate and emit light by themselves (give out light of their own). Examples are the sun, stars, electric lamp, burning flames, TV screen, fire – flies, some deep – sea fishes and other artificial light sources. The light they produce enter our eyes.
- (ii) Non - luminous sources: These are light sources that do not give out light on their own. They depend on the natural or artificial light sources to illuminate them. They reflect light from luminous objects for them to be seen. Examples include moon, wood, pages of book, road sign. Luminous sources illuminate them and make them appear luminous.

### 3.1.3 Properties of Light

When light strikes an object, it may be, depending on the nature of the material:

- (i) Reflected, absorbed or transmitted;
- (ii) Partly absorbed, partly reflected or partly transmitted.

### 3.1.4 Transmission of Light

Opaque materials – They partly absorb and partly reflect light energy but do not transmit light at the surface. Examples are wood, brick wall or a sheet of cardboard.

Transparent Objects – They transmits most of the light energy falling on it such that the object can be seen. It reflects and absorbs very little. Examples are glass and clear water.



**Translucent Object** – They allow light to energy to pass through them in such a way that the object cannot be seen. Examples include glass sheet used for certain windows and waxed sheet.

### 3.1.5 Rays and Beams of Light

The direction or path along which light energy travels is called a light ray. They are indicated in diagrams by thin lines with arrow heads which indicate the direction of travel of the light. A collection of rays is called a beam. There are three types of beams namely:

- (i) **Parallel beams** – A parallel beam is one in which the rays of light are parallel to one another. Examples of parallel rays are rays from search light.
- (ii) **Convergent beam** – A convergent beam is one in which all the rays of light converge or meet at a point. Convergent beam can be produced using hand lens.
- (iii) **Divergent Beam** – A divergent beam is one in which all the rays of light come from a point and spread out or diverge from that source. Lights from lamps are examples of divergent beam.

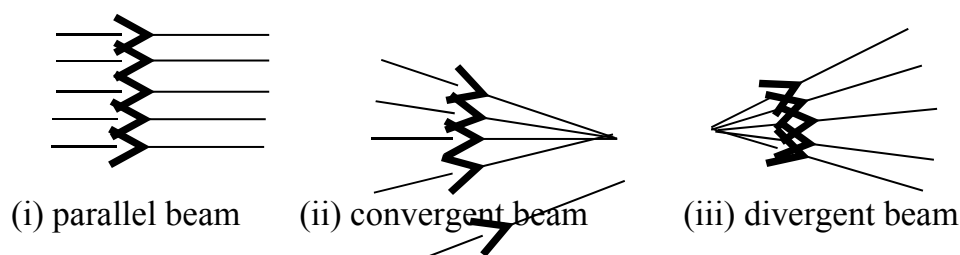
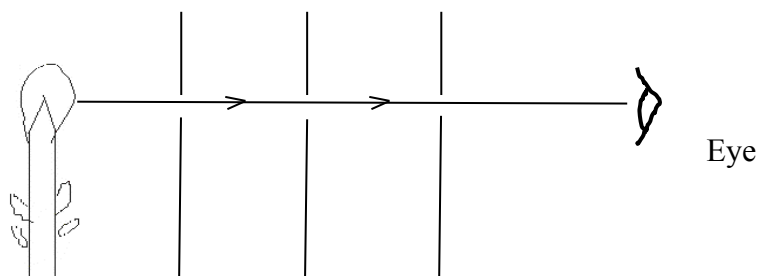


Fig. 1.1: Rectilinear propagation of light

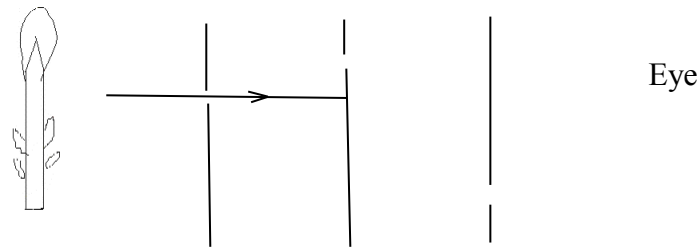
### 3.2 Rectilinear Propagation of Light

You know that rectilinear propagation of light simply means that light travels mainly in a straight line. This is illustrated in Fig. 1.1.



- (a) Candle light can be seen by the eye





(b) Candle light cannot be seen by the eye

Fig. 1.2: Rectilinear propagation of light

Two natural effects result from the rectilinear propagation of light. These are the formulation of shadows and eclipses. Also, the principle of operation of the pin – hole camera depends on the fact that light travels in straight lines.

### 3.2.1 Shadows

A shadow is an area in which light rays from a source cannot reach. It occurs because light travels in a straight line. Shadows are produced by the obstruction of light by an opaque object. For instance, when light rays arrive at an opaque obstacle (one that absorbs Light), the rays just graze the edges of the obstacle and produce the outline of a shadow. The kind of shadow obtained depends on the size of the luminous object (source of light) sending out the rays.



Fig. 1.3 Shows how shadows are formed

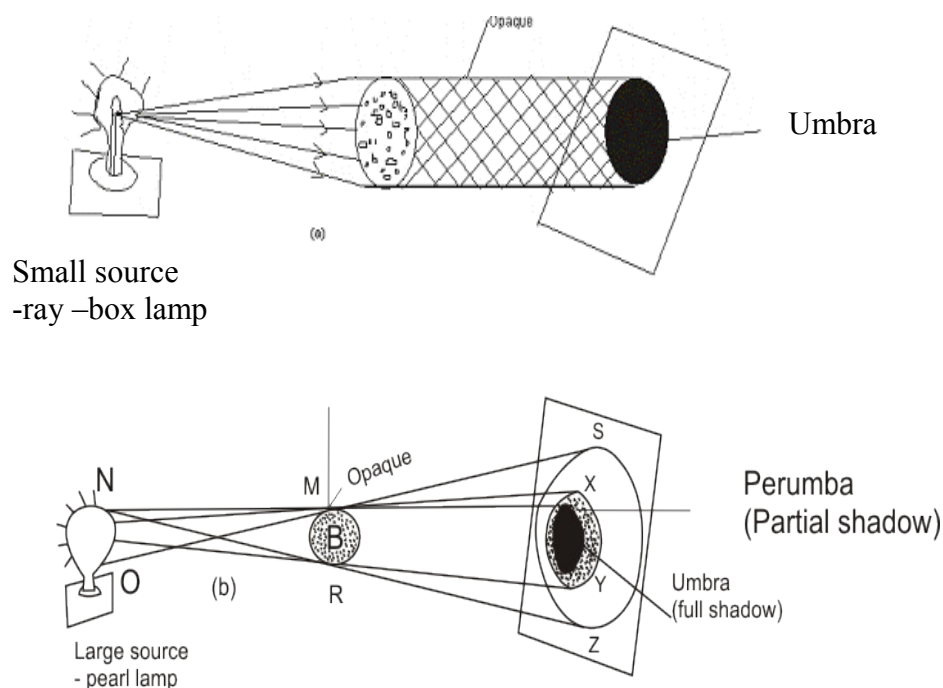


Fig. 1.3: Shadows (a) small source (b) Large source

In Fig. 1.3 a, we used a small light source (point source) of light to produce a sharp shadow of an opaque solid ball B on a screen placed behind B. The shadow is uniformly dark. When the lamp is replaced by a large source such as a pearl lamp P the shadow produced is no longer uniformly dark. This is illustrated in Fig. 1.3b. Light does not reach the region of full darkness (umbra), the inner most shadow region. The edge of the umbra corresponds to the rays NM and OP which grazed B at the edge. As we move away from the edge the darkness diminishes in intensity. Here also, there is an outer light or grey area known as the penumbra or partial shadow. In this region, the light is partially blocked by the opaque object. At point in the screen, the ray PQX indicates that X receives light from small part NP of the pearl lamp while point Y receives light from another small part OQ. This is indicated by the ray QRY. Thus the regions X and Z are in partial shadow (penumbra) as shown by the grazing rays OMS and NRZ. Other points above S and R on the screen are illuminated by the whole of the lamp and as such no shadow is obtained there.



### 3.2.2 Eclipses

Do you know that an eclipse is formed as a result of the shadow cast by one heavenly body on another? That is, the eclipse of the sun and of the moon are direct result of the formation of shadows caused by the revolution of the Earth round the Sun. As we have earlier discussed, the sun is a luminous object while the Moon and the Earth are both non-luminous objects. At certain times during these movements, the three bodies will lie in a straight line. This results in the formation of shadows.

#### (i) Eclipse of the Sun

An eclipse of the sun occurs when the Moon, an opaque body, comes between the Sun and the Earth. This is illustrated in Fig. 1.4.

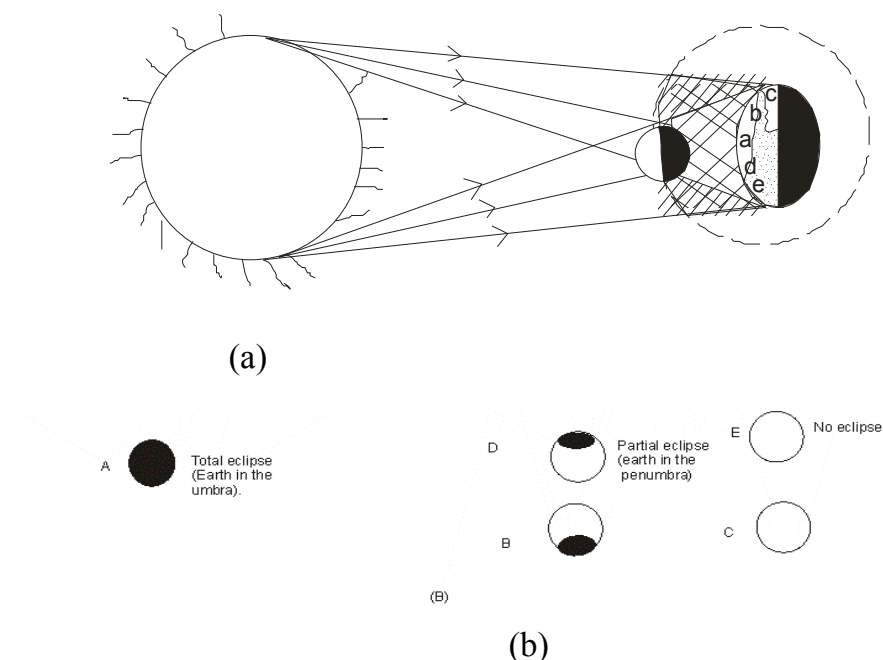


Fig. 1.4: Eclipse of the sun

At this instance, the shadows of the Moon will be cast on the Earth's surface. People living in the part of the Earth directly in the Moon's shadows are cut off from the Sun's view. They are in the Umbra region of the Moon's shadow. This experience, for those at the Umbra (Total darkness) region of the Moon's shadow is called an Eclipse of the Sun or Solar Eclipse and it lasts for just a few minutes.

Those living at the penumbra (partial darkness) regions of the Moon's shadow experience partial Eclipse because at these regions, the sun is only partially covered. The appearance of the sun from various parts of the Earth is illustrated in Fig. 1.4b.



A person at a, as shown in Fig. 1.4a, sees a total Eclipse, another at b and d sees a partial Eclipse while another at c or e sees no eclipse.

### Annular Eclipse

On some occasions, the Earth and the Moon may be in positions such that the rays at the moon's edge intersect before reaching the Earth (Fig. 1.5), thus forming an annular eclipse.

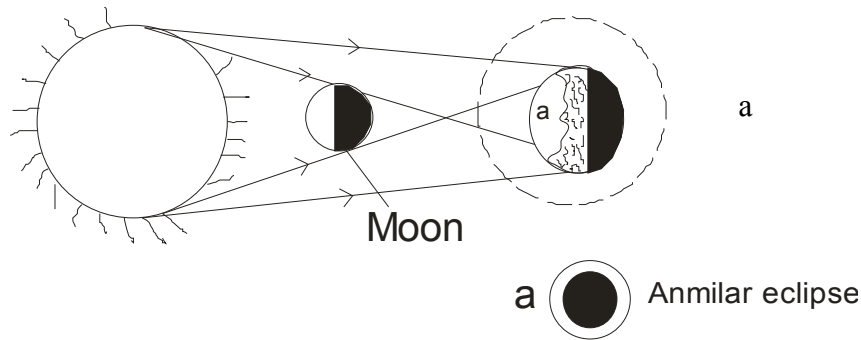


Fig. 1.5.: Annular eclipse

That is, a ring of light is formed around the shadow of the moon. An observer in a position a of the Earth would experience the annular eclipse as a ring of light round the moon.

### (ii) Eclipse of the Moon

An Eclipse of the moon occurs when the Earth comes directly between the Sun and the moon. This is illustrated in Fig. 1.6.

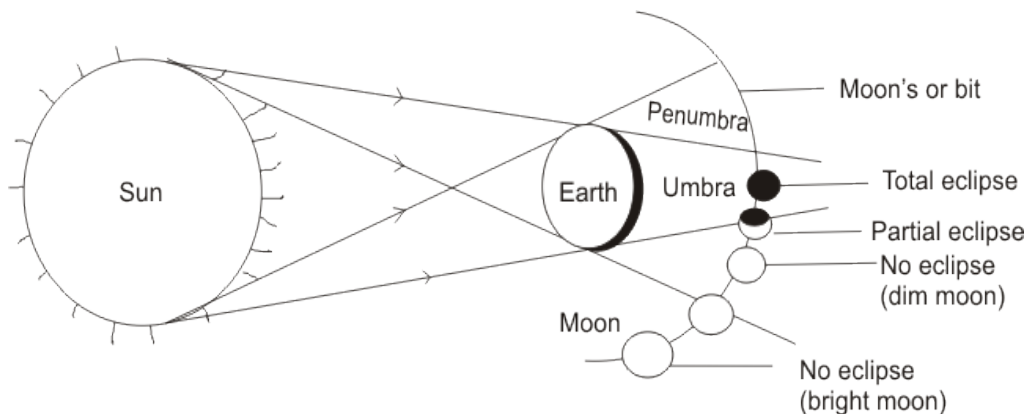


Fig. 1.6: Eclipse of the moon

The Earth, being an opaque body, casts a shadow on the moon. That is, the Moon is in the Earth's shadow. This leads to the formation of the Eclipse of the Moon or Lunar Eclipse just like the Solar Eclipse, Lunar eclipse can be total or partial. Also, since the Moon's orbit round the Earth does not lie in the same plane as that of the Earth's orbit round the



Sun, partial Eclipses of the Moon do not occur every time the Moon and the Sun are on the opposite sides of the Earth.

### 3.2.3 The Pinhole Camera

Are you aware that the pinhole camera is the earliest camera? It makes use of the fact that light travels in straight lines. This camera consists of a light proof box, one end of which has a small hole made with a pin or needle point.

The opposite end has a screen made of tracing paper or ground glass. Light from an object in front of the pinhole passes through it and forms an image on the screen. If the screen is replaced with a photographic paper or film, a picture of an object can be taken.

You can make a simple pinhole camera by removing the cover from a closed box such as the cardboard boxes used for packing sugar cubes or bournvita tin. Remove one face of it and replace it with either semi-transparent or translucent paper to serve as screen thereby sealing it again from outside light. Make a pinhole at the centre of the face of the box (tin) opposite the screen (semi-transparent or translucent paper). Paint the inside of the box (tin) black so as to absorb stray light due to reflection in order not to fall on the screen. Fig. 1.7 represents a simple pinhole camera and how it forms images.

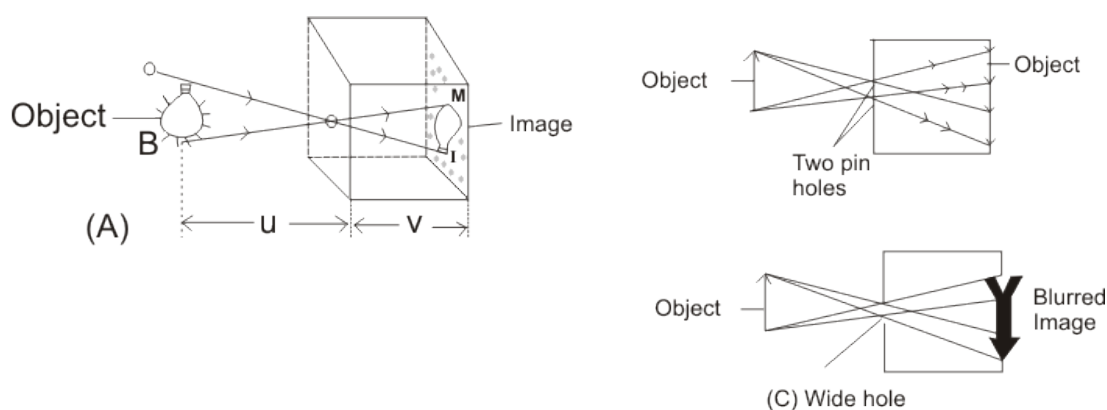


Fig. 1.7: Pinhole camera and images formed

In Fig. 1.7, OB represents the object in front of the camera. Light enters through the pinhole P and an inverted image IM is formed on the screen.

If the hole is very small, the image will be very sharp, clear, but dim due to limited amount of light passing through it. On the other hand, if the hole is large, the image will be bright but blurred.



If the screen is replaced by a photographic film or plate, the pinhole camera can then be used to take photograph of still objects but it requires a long time of exposure to enable the image to be developed on the film or plate. In recent times, however, converging lens of appropriate focal length is used to bring all rays from many pinholes or wide hole on an object to a single image by bringing all rays from a point on an object to a single point image on the screen even when the rays are incident on different parts of the lens. This gives rise to a sharp image of the object on the screen.

### Linear magnification produced by the pinhole camera

#### Definition

Magnification is the ratio of the size (or height) of the image to the size (or height) of the object.

You can compare the size of an object and its image by using the ratio:

$$\text{Or } \frac{\text{Size of image}}{\text{Size of object}} = \text{Linear magnification}$$

$$\text{Or } \frac{\text{Height of the image}}{\text{Height of the object}} = \text{Linear magnification}$$

$$\text{Or } \frac{\text{Distance of the image from pinhole}}{\text{Distance of the object from pinhole}} = \text{Linear magnification}$$

$$\text{Or } \frac{\text{Length of camera}}{\text{Distance of object from pinhole}} = \text{Linear magnification}$$

Symbolically,

$$M = \frac{h_i}{h_o} = \frac{v}{u}$$

Where

M	=	linear magnification
$h_i$	=	height of the image
$h_o$	=	height of the object
v	=	perpendicular distance of the image to the pinhole
u	=	perpendicular distance of the object to the pinhole



### 3.3 Numerical Examples

- (i) The height of an object placed at a distance of 30 cm from the hole of a pinhole camera is 20 cm. If the length of the camera is 15 cm, calculate the magnification produced by the camera and the height of the image.

#### Solution

$$\begin{aligned}\text{Magnification} &= \frac{\text{length of camera}}{\text{distance of object from pinhole}} \\ &= \frac{\text{height of image (} h_i \text{)}}{\text{height of object (} h_o \text{)}}\end{aligned}$$

$$\begin{aligned}\text{Length of camera} &= 15 \text{ cm} \\ \text{Distance of object from pinhole} &= 30 \text{ cm} \\ \text{Height of image} &= h_i \\ \text{Height of object} &= h_o = 20 \text{ cm}\end{aligned}$$

$$\Rightarrow \text{Magnification} = \frac{15}{30} = \frac{1}{2} = 0.5$$

Also

$$M = \frac{h_i}{h_o} = \frac{h_i}{20 \text{ cm}}$$

i.e

$$\begin{aligned}\frac{1}{2} &= \frac{h_i}{20} \\ \Rightarrow 2h_i &= 20 \\ h_i &= \frac{20}{2} = 10 \text{ cm}\end{aligned}$$

- (ii) The distance between the pinhole and the screen is 8 cm, calculate the minimum distance from the pinhole of a body 2.0 m tall. If a full length picture of him is required on a photographic plate of length 25.0 cm.

#### Solution

$$\begin{aligned}\text{Magnification (} m \text{)} &= \frac{\text{height of the image (} h_i \text{)}}{\text{Height of the object (} h_o \text{)}} \\ &= \frac{\text{distance of the Image from pinhole (} v \text{)}}{\text{distance of the object from pinhole (} u \text{)}}\end{aligned}$$



$$\begin{array}{ll}
 \text{Image distance} & v = 8\text{cm} \\
 \text{Object distance} & u = ? \\
 \text{Image height} & h_i = 25\text{ cm} \\
 \text{Object height} & h_o = 2.0\text{ m} = 200\text{ cm}
 \end{array}$$

$$\text{Magnification} = \frac{h_i}{h_o} = \frac{v}{u}$$

$$\Rightarrow \frac{h_i}{h_o} = \frac{v}{u}$$

$$\frac{25\text{ cm}}{200\text{ cm}} = \frac{8\text{ cm}}{u}$$

$$\begin{aligned}
 25u &= 200 \times 8 \\
 u &= \frac{200 \times 8}{25} = 64\text{ cm}
 \end{aligned}$$

### SELF ASSESSMENT EXERCISE 1

- (1) List at least five light sources you come across in your immediate environment.
- (2) With the aid of diagrams explain how total and partial eclipse of:
  - (i) the Sun
  - (ii) the Moon
 are formed
- (3) Construct a simple pinhole camera using improvised box or closed can. Use the camera to focus on a named object and answer the following questions.
  - (i) Is the image of the object erect?
  - (ii) How sharp is the image?
  - (iii) It is observed that the pinhole camera is not generally used in taking photographs of objects. Why?
- (4) An object stands 15 m away from a pinhole camera which is 10 cm long if the height of the object is 25 m, what is the height of its image?

### 3.4 Reflection of Light at Plane Surfaces

Earlier in this unit, you have learned that we see non-luminous objects because of the light they reflect. As you know, the light originates from some luminous object such as the Sun, an electric lamp or other



luminous sources. When light rays fall on the surface of an object, it is absorbed, transmitted or reflected. Sometimes, however, a combination of the above processes may occur. When light strikes on a surface, the way it is reflected depends on the nature of the surface. There are two types of reflections namely Regular and Diffused reflection resulting from two types of surfaces namely smooth or polished surface and rough or irregular surfaces. The two types of reflection are illustrated in Fig. 1.8 below.

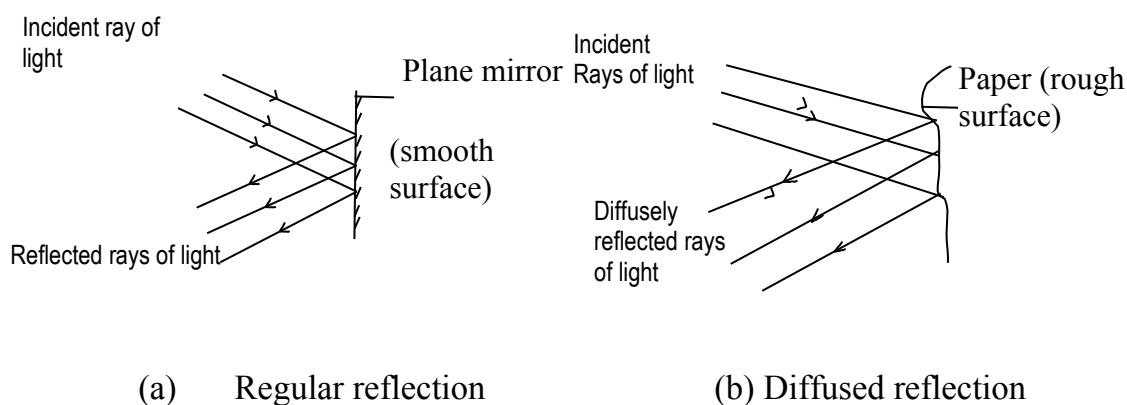


Fig. 1.8: Regular and diffused reflection

### Regular Reflection

When parallel beam of light strikes smooth or polished surfaces such as plane mirror, the rays are reflected parallel to one another in one definite direction from the point where they strike the surface as a result of the nature of the surface. This is termed Regular Reflection (Fig.1.8a).

### Diffused Reflection

When parallel beam of light strikes rough surfaces such as paper, person's face or clothing materials, the rays are scattered because of the lack of smoothness or grain of the surface. Parallel rays of light incident on such Surfaces are reflected in different directions. This reflection is termed scattered, irregular or diffused reflection (Fig. 1.8b).

## 3.5 Laws of reflection

There are two laws of reflection at plane surfaces. These are:

- (i) The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.
- (ii) The angle of incidence is equal to the angle of reflection.



The laws are illustrated in Fig. 1.9

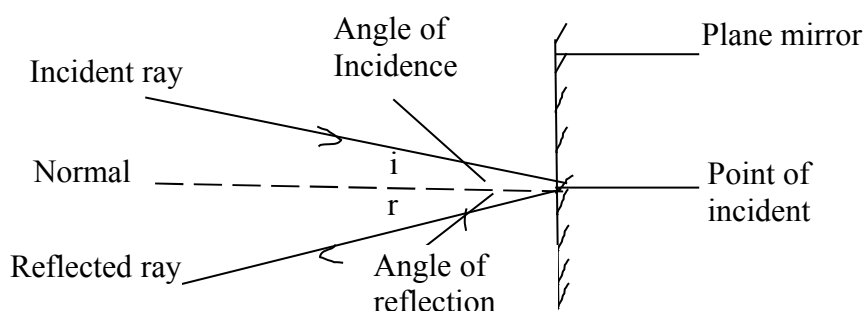


Fig.1.9: Laws of reflection

Plane mirror surfaces consist of plane glasses silvered at the back-Light rays are reflected at the silvery at the back by preventing light rays that strikes the glass from passing through the glass.

Plane mirrors are used in investigating the laws of reflection in school laboratories.

**(xiii) Verification of the first law of reflection**

**The apparatus is set up as shown in Fig. 1.10.**

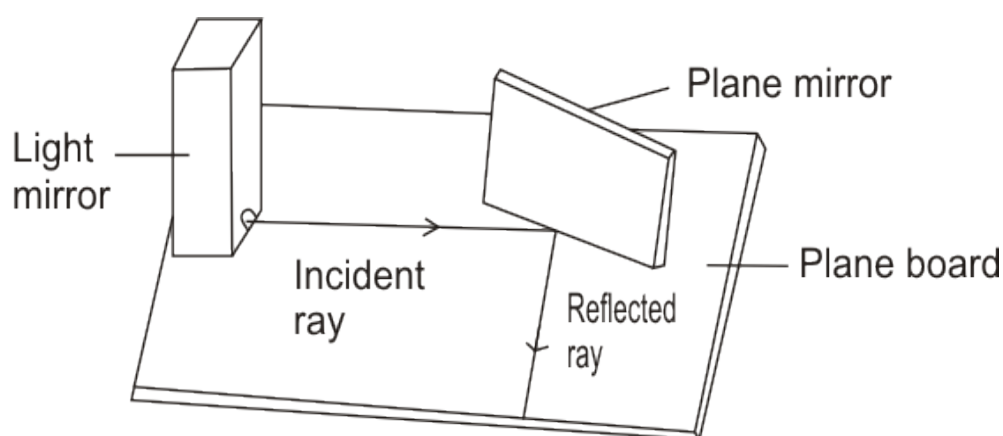


Fig. 1.10: Verification of the first law of reflection

Direct the incident ray from a light source (ray box or candle light) to traverse along the surface or the plane of the board. You will observe that it is only when the incident ray is visible on the board that the reflected ray will also be distinctly visible on the board. This shows that the incident and reflected rays are in the same plane. Since the mirror is placed perpendicular to the board, the normal to the mirror also lies in the same plane as the incident and reflected rays.

**(xiv) Verification of the Second Law of Reflection**



### Using Optical pins

Fix a sheet of paper firmly on a drawing board. Mount a plane mirror vertically along a line AB on the paper and support it with plasticine or wooden block (Fig. 1.11a).

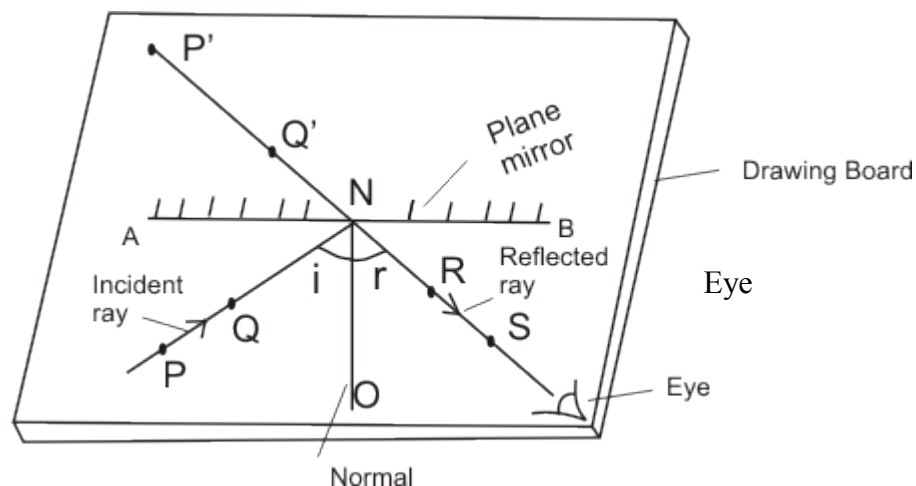


Fig. 1.11a: Verification of the second law of reflection using optical pins

Draw a normal ON. That is a line ON perpendicular to the mirror at N to bisect AB at right angles. Draw lines (one at a time) on the paper making angles such as  $20^\circ$ ,  $30^\circ$ ,  $40^\circ$ ,  $60^\circ$  and  $70^\circ$  to ON.

Fix two pins P and Q in front of the mirror along the first line making angle  $20^\circ$  to ON (see Fig. 1.11a). Use two other pins RS to fix at points in front of the mirror where the eye sees them in line with the images  $P'$  and  $Q'$  of P and Q in the mirror. The line PQ represents the reflected ray.

Remove the pins and produce lines PQ and RS to meet at a point N on AB measure the angle of incidence  $i$  (the angle between the incident ray and the normal) and the angle of reflection  $r$  (the angle between the reflected ray and the normal).

Repeat the experiment for angles  $30^\circ$ ,  $40^\circ$ ,  $60^\circ$ , and  $70^\circ$ , measure and record the angles of incidence  $i$  and the corresponding angles of reflection  $r$  in each case. Tabulate your result.

You will observe that each set of angles, angle of incidence  $i$  and the corresponding angle of reflection  $r$ , are always equal.

- Using a Ray Box  
set up the apparatus as before (Fig. 1.11a)



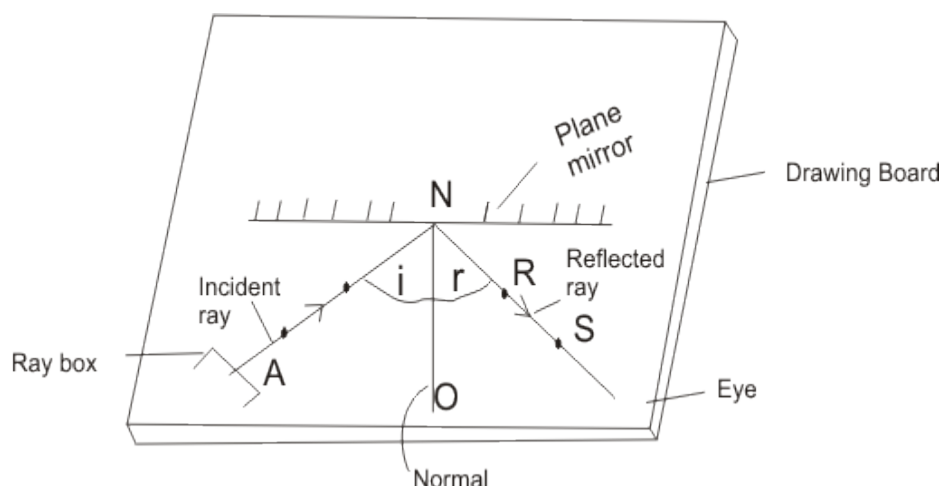


Fig. 1.11b: Verification of the second law of reflection using a ray box

Place a ray box in such a way that a single ray follows the drawn line for angle  $20^\circ$ , line AN. Mark the reflected ray BN with two dots. Use the edge of the incident ray and the corresponding edge of the reflected ray if the ray is thick to obtain accurate result. Draw other lines for angles  $30^\circ$ ,  $40^\circ$ ,  $60^\circ$  and  $70^\circ$  in turn, using the ray box, direct the ray (incident ray) along the drawn lines and mark each reflected ray direction as you did for angle  $20^\circ$ .

Measure and record the angles of incidence  $i$  and the corresponding angles of reflection  $r$ . tabulate your readings.

You will also observe here that in all cases, the angle of incidence is equal to the angle of reflection  $r$ .

Note that the angles of incidence and reflection are always measured from the normal ON (perpendicular) to the mirror.

### 3.5.1 The Principles of Reversibility of Light

You must have observed in the experimental verification that a light ray directed to wards a mirror say AN will be reflected along BN in accordance with the laws of reflection of light. This observation is illustrated in Fig. 1.12. If light ray is incident along BN, it will emerge along AN after reflection.



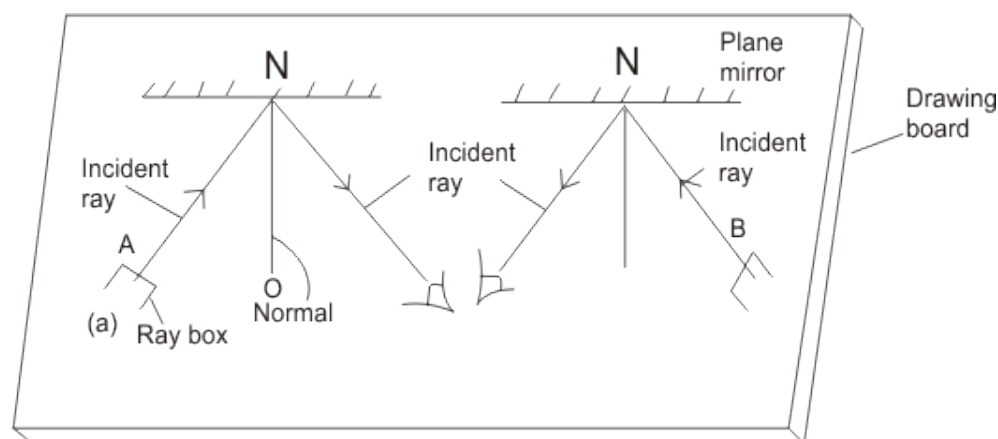


Fig. 1.12: Principle of reversibility of light

Thus, the path of the light ray is seen to be reversible. This observation is a general principle in optics and is known as the principle of reversibility of light.

### Definition

The principle states that the path of light ray is reversible. It therefore follows that if a person standing at point A can see the image of a person standing at point B, the person at point B, can likewise see the image of the person at A.

## 3.6 Formation of Images at Plane Mirror Surfaces

If you look at the front of a plane mirror, you will see the image of yourself. Your image appears to be behind the mirror but it is not actually there, it cannot be caught in a screen.

Now consider a situation where a candle light is placed in front of a plane mirror to serve as an object (Fig. 1.13).

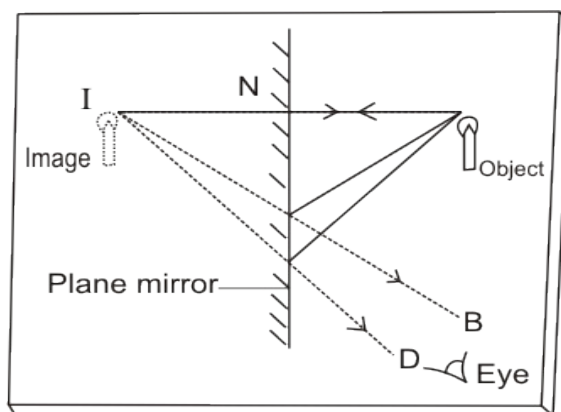


Fig. 1.13: Image of an object O observed in a plane mirror



The upright image of the candle light appears to an observer as if it is located at I behind the mirror from which the light rays enter the eye. This observation can be explained thus: Two incident rays OA and OC from the tip O, of the candle light strike the plane mirror at A and C and are reflected as AB and CD respectively in accordance to the laws of reflection. After reflection, the rays appear to the observer's eye as if the rays are coming from the tip of the image I of the candle light in the mirror. You can see that I is the point of intersection of the two reflected rays produced backwards. Also a ray ON normal to the mirror is reflected back along NO. This ray meets the other two rays at I when produced backward and the distance ON is equal to NI.

Actually, no rays come from I because AB and CD are diverging from each other according to the laws of reflection. The image of the candle light in the mirror is said to be a virtual image and cannot be received on a screen.

- An image is a picture of an object formed by a reflecting surface.
- A real image can be received on a screen.
- It is produced by actual intersection of rays of light.
- A virtual image cannot be received on a screen it is produced by apparent intersection of rays of light.

### **3.7 Locating Images in Plane Mirrors**

#### **3.7.1 Method I**

The image of an object O such as a pin in a plane mirror M can be located as described below:

- (i) Mount a plane mirror on a flat board and erect an optical pin O in front of the mirror. View the image of the object pin O in the mirror (Fig. 1.14)

Erect two other pins A and B in line with the image I of the pin observed in the mirror. Repeat with two more pins in a different position such as C and D. After taking the pins away, produce the lines AB and CD until they intersect. This point of intersection behind the mirror is the position of the image I. Measure the distances ON and IN. You would see that ON is equal to IN



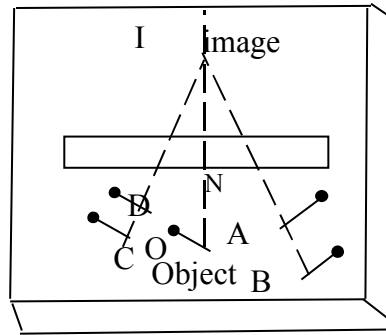


Fig. 1.14: Locating images in plane mirrors

### 3.7.2 Method II

It is also possible to locate the image position by the no – parallax 1 method. You can do this by using two optical pins  $P_1$ , and  $P_2$  mounted on a cork as shown in Fig. 1.15.

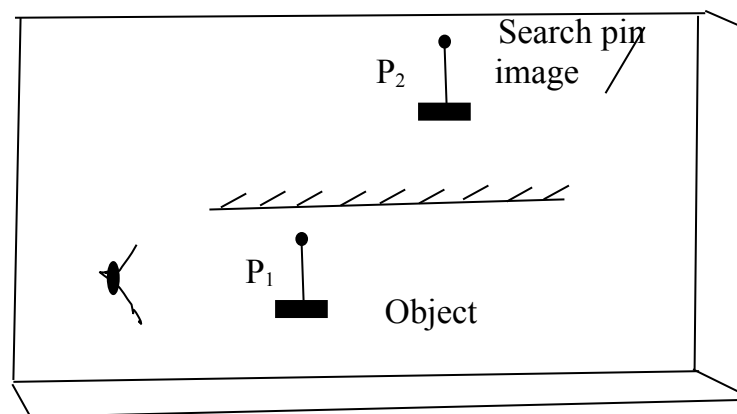


Fig. 1.15: No parallax method

### 3.7.3 Image of Large Object –Lateral Inversion

The image of large object is drawn by taking one point after another on the object and locating its corresponding image using either method I or Method II above.

#### Lateral Inversion

This is the effect of plane mirror on large objects placed in front of it. The image appears like a reversal of the object (Fig. 1.16).



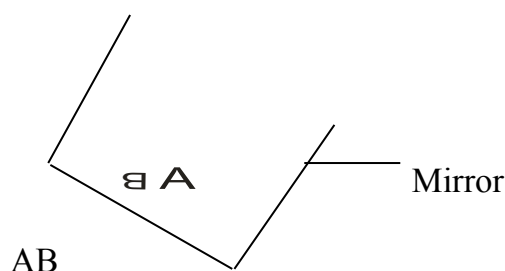


Fig. 1.16: Image of large object showing lateral inversion

Lateral inversion is due to the fact that the object is perpendicularly opposite its image behind the mirror.

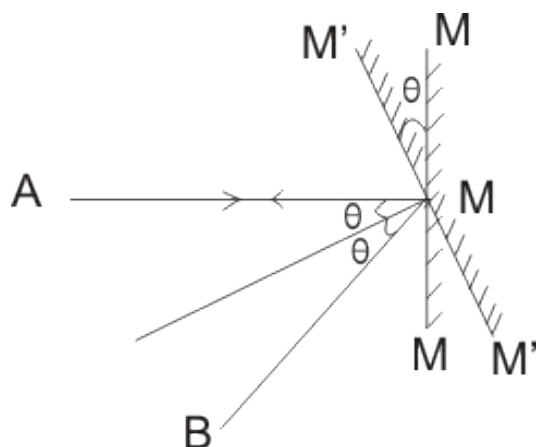
### 3.7.4 Characteristics of Images Formed by Plane Mirror

From the forgoing you must have noted the following characteristics of images formed by plane mirror.

- (i) It is the same size as the object;
- (ii) Each part is the same distance behind the mirror as the object is in front of it;
- (iii) It is laterally inverted (ie turned around);
- (iv) It is erect (ie upright)
- (v) It is virtual

### 3.8 Deviation of Reflected Ray by Rotated Mirror

If a mirror is turned through an angle, the reflected ray of any light incident on it is turned through twice that angle. That is, if the direction of an incident ray on a mirror is kept constant and the mirror is rotated through an angle, the reflected ray will be rotated through an angle, the reflected ray will be rotated through twice that angle. This is illustrated in Fig. 1.17. The mirror initially in the position  $MM$  is rotated through an angle  $\theta$  to the position  $M'M'$





The incident ray AP is normal to MM and is reflected back along its own path. When the mirror is rotated to the position M<sup>1</sup>M<sup>1</sup>, the normal to the mirror becomes PN at that point of incidence P. This makes an angle of  $\theta$  with PA. Thus the angle of incidence on M<sup>1</sup>M<sup>1</sup> is  $\theta$  as well as the angle of reflection. Hence, the direction PA to the direction PB through an angle of rotation equal to  $2\theta$ .

### 3.9 Images Formed by Inclined Mirrors

The number of images formed by inclined mirrors depends on the angle of inclination of the mirrors. For instance, when two mirrors are inclined at an angle  $\theta$ , the number of images,  $n$ , formed by the two mirrors is given by the formula.

$$n = \frac{360^\circ}{\theta} - 1$$

Where  $n$  = number of images formed;  
 $\theta$  = angle between the two mirrors;

and  $\frac{360^\circ}{n}$  is always approximated or rounded up to the nearest higher whole number when a fraction is obtained.

For example, if two mirrors are inclined at an angle of  $90^\circ$ , then, the number of images formed by the mirrors is

$$n = \frac{360^\circ}{90^\circ} - 1 = 4 - 1$$

i.e (4-1) images = 3 images

This is illustrated in Fig. 1.18.

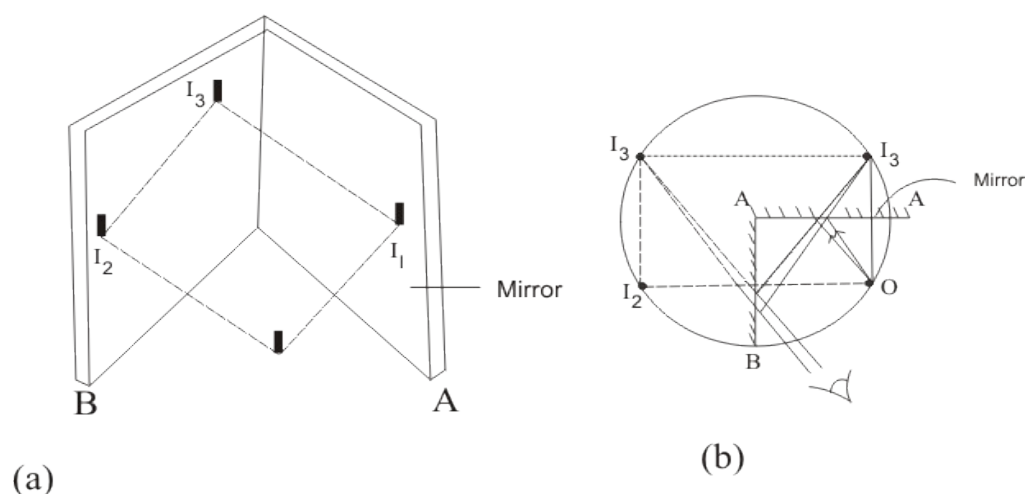


Fig. 1.18: Images formed in two mirrors inclined at right angles



As illustrated in Fig. 1.18 above, two mirrors are inclined at right angles to each other and a candle stick is placed between them. As you can see, the number of images formed is three and these are:

- (a) The image ( $I_1$ ) of the candle formed by mirror AC;
- (b) The image ( $I_2$ ) of the candle formed by mirror BC;
- (c) The coincident Image ( $I_3$ ), which is a combination of the images of  $I_1$  and  $I_2$  formed by both mirrors.

### 3.10 Images Formed by Parallel Mirrors

Have you ever noticed that when two mirrors A and B are placed parallel to each other and an object O is placed between them then that multiple images are formed? This is illustrated in Fig. 1.19.

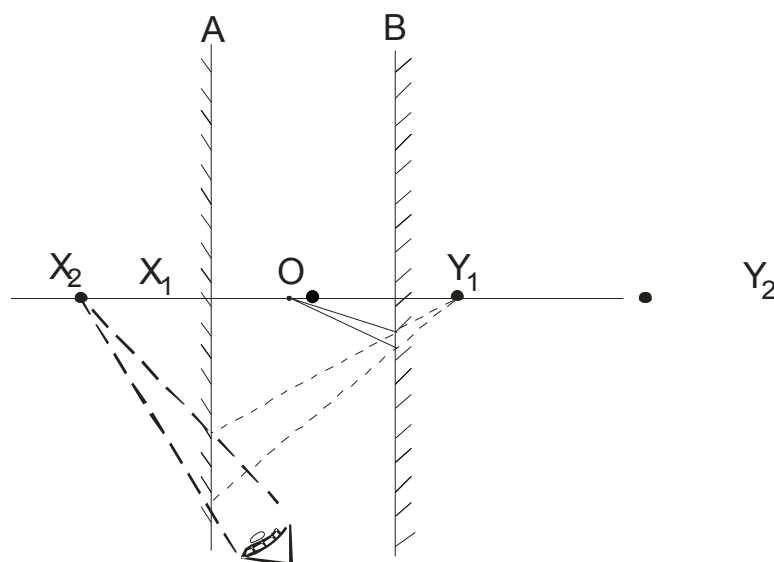


Fig. 1.19: Multiple images by two parallel mirrors

As illustrated, when an object O is placed between two parallel mirrors A and B, multiple images of the object will be formed. In the Fig. 1.19,  $X_1$  is the image of the object O formed by A and  $Y_1$  is the image of the object O formed by B.  $X_2$  is the image of image  $Y_1$  formed by mirror A and similarly,  $Y_2$  is the image of image  $X_1$  formed by mirror B. That is, as the images act as objects for further formation of images in the mirrors, an infinite number of images are formed by the parallel mirrors. However, as the number of images increases, the images become fainter because a little light is diffused and absorbed at each reflection.



### 3.11 Applications of Reflection of Light at Plane Mirror Surfaces

Some of the applications of the reflection of light at plane mirror surfaces are:

- (i) Driving mirrors – Plane mirrors are used as driving mirrors in vehicles
- (ii) Looking glass - The common looking glass is a plane mirror.
- (iii) The Sextant – This is an instrument used by navigators for finding the angle of elevation of the Sun and other celestial bodies it depends on the constant deviation of rays striking two mirrors inclined at a fixed angle.
- (iv) Kaleidoscope – This is a toy which makes use of two plane mirrors inclined at an angle of  $60^\circ$  to each other. It produces different image patterns each time it is shaken. Designers use it in looking for new color pattern.

#### Periscope

The instrument consists of two mirrors fixed facing each other at an angle of  $45^\circ$  to the line joining them. A ray from an object enters the eye of an observer by successive reflections from the mirrors A and B. This enables the observers to see an image of the object.

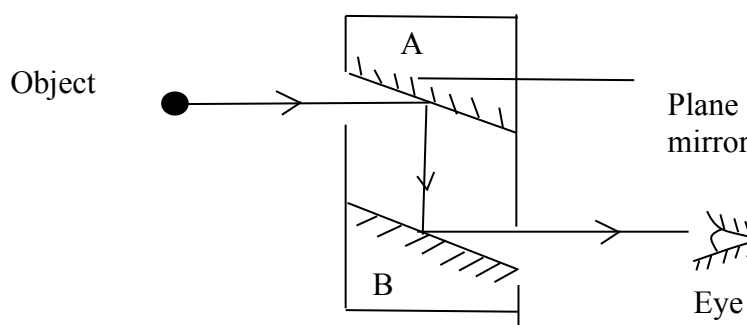


Fig. 1.20: A simple periscope

Simple periscope is used in submarine to enable men below the water surface see what is happening above the water surface. It is also used to see over peoples head or spy over a tall wall or around corners. That is, it enables the use to see over obstacles.

#### (vi) Uses of Plane Mirrors in Meters

A plane mirror placed below the pointer in the same plane as the scale of commercial meters helps to overcome errors in reading due to parallax between the point and the scale



A plane mirror is also used to measure the small angle of rotation of the coil in a mirror galvanometers used for the measurement of very small electric current.

## SELF ASSESSMENT EXERCISE 2

1. With the aid of the ray diagrams, distinguish between regular and diffuse reflections.
2. State the laws of reflection. How would you verify the second law in the laboratory?
3. With the aid of ray diagram, illustrate the formation of image by a plane mirror. What are the characteristics of images formed by such mirrors?
4. The angle between a plane mirror and an incident ray is  $40^\circ$ . What is the;
  - (i) angle of incidence
  - (ii) angle of reflection
  - (ii) total angle through which the ray is deviated?
5. Explain why
  - (i) You cannot see the image of yourself in this papers page.
  - (ii) Inside a barber's shop, Ada, observed the image of her new hairstyle formed by a plane mirror kept 2 m away from her eyes.

## 4.0 CONCLUSION

In this unit you studied reflection of light at plane surfaces. You learned about sources of light, shadows, eclipses and the pinhole camera. You also learned how to draw ray diagrams to show the formation of images by plane mirrors as well as the laws of reflection. In addition, you learned about the deviation of light at plane mirror surfaces and of reflected ray by rotated mirror and how to solve quantitative problems involving plane mirrors. Finally, you learned some practical applications of plane mirrors.



## 5.0 SUMMARY

- Light is perceived to consist of wave like packets of energy called photons with a traveling speed of  $3 \times 10^8 \text{ ms}^{-1}$
- Visible light is a form of energy which our eyes can detect. It is divided into two sources namely the luminous source and the non-luminous sources.
- Luminous sources generate and emit light by themselves. Non luminous sources do not give out light on their own but depend on natural or artificial light sources to illuminate them.
- When light strikes an object, it may be, depending on the nature of the material:
  - (i) reflected, absorbed or transmitted;
  - (ii) partly absorbed, partly reflected or partly transmitted.
- A ray is the path taken by light energy. A beam is a collection of rays.
- Shadows and eclipse occur because light travels mainly in a straight line.
- The laws of reflection are:
  - (i) The incident ray, the reflected and the normal at the point of incidence all lie in the same plane.
  - (ii) The angle of incidence is equal to the angle of reflection.
- An image is a picture of an object formed by a reflecting surface
- A real image can be received on a screen and is produced by actual intersection of rays of light.
- A virtual image cannot be received on a screen and is produced by apparent intersection of rays of light.
- The image formed in a plane mirror is virtual, laterally inverted, the same size as the object and as far behind the mirror as the object is in front.
- The number of images formed by inclined mirrors depends on the angle of inclination of the mirrors
- When two mirrors are inclined to each other at an angle  $\theta$ , the number of images in formed by the two mirrors is given by the formula.

$$n = \frac{360^\circ}{\theta} - 1$$



## 6.0 TUTOR-MARKED ASSIGNMENT

1. The heights of an object placed at a distance of 30 cm from the hole of a pinhole camera is 20 cm. If the length of the Camera is 15 cm, calculate the magnification produced by the Camera and the height of image
2. State the lens of reflection of light and illustrate the laws with the aid of a diagram.
3. An object is placed 5 cm in front of a plane mirror. With the aid of ray diagram, locate and state the characteristics of the image formed.

## 7.0 REFERENCES/ FURTHER READINGS

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## **UNIT 2 REFLECTION AT CURVED (SPHERICAL) MIRRORS**

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### **1.0 INTRODUCTION**

In this unit you will focus attention on curved mirrors and learn about formation of images by curved mirrors. Also, you will, learn how to draw ray diagrams to show how images are formed by concave mirrors, use ray diagrams to locate image position at different object positions. Furthermore, you will learn practical applications of curved mirrors especially in car headlamps and driving mirrors. Finally you will learn about the sign convention, the mirror formula and its application.



## 2.0 OBJECTIVES

By the time you finish this unit, you will be able to:

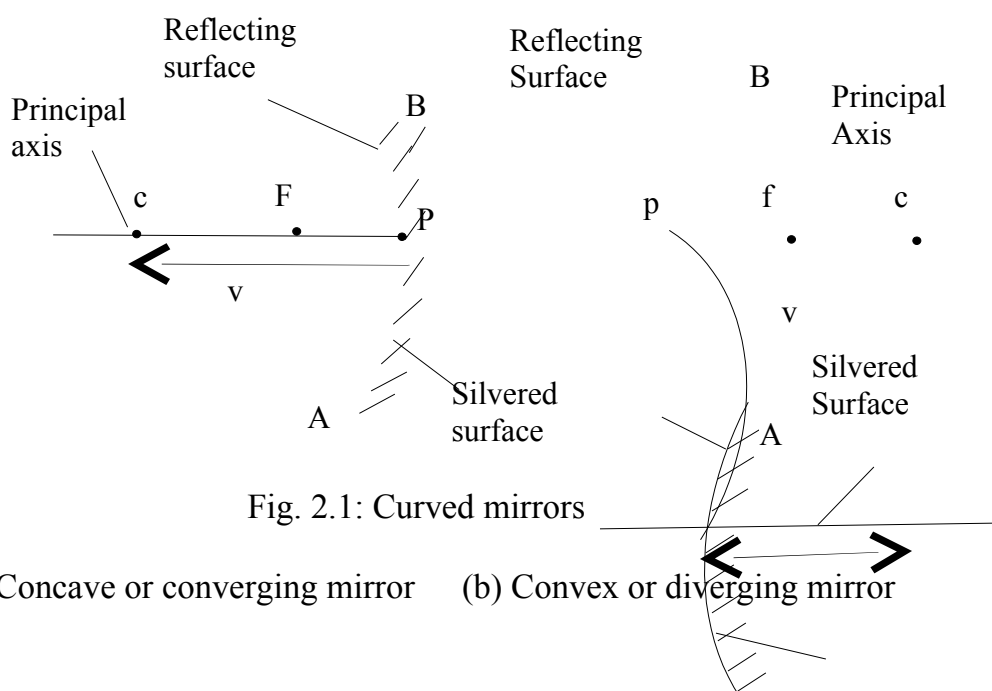
- Draw ray diagrams to show the formation of images by curved mirrors;
- Explain some practical applications of curved mirrors;
- Locate image positions by using curved mirror;
- Investigate the relationship between object distance ( $u$ ), image distance ( $v$ ) and focal length ( $f$ ) of the curved mirror.

## 3.0 MAIN CONTENT

### 3.1 Types of Curved Mirror

We have two types of curved mirrors namely the concave and the convex mirrors. They are made silvering a glass surface which is part of a sphere.

If the outside surface of the spherical part is silvered and the inside surface of the reflecting part, the resulting mirror is called a concave mirror. On the other hand, if the inside surface is silvered and the outside surface is the reflecting part, the resulting mirror is called convex mirror. This is illustrated in Fig. 2.1

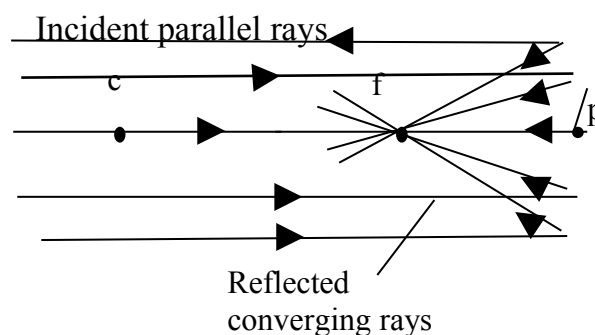




### 3.1.1 Terms Used in Curved Mirrors

We use the following terms illustrated in Fig. 2.1 in describing the essential parts of a spherical mirror.

- The aperture: the width (AB) of the mirror
- The pole (p): the centre of the reflection surface of the curved mirror.
- The Centre of Curvature (c) = the centre of the sphere of which the mirror forms a part
- The Radius of curvature (r) = the distance CP. It is the radius of the sphere of which the mirror forms a part.
- The Principle Axis: the line PC from the pole to the centre of curvature.
- The Principal Focus (f): The principal focus (f) of a curved mirror is that point on the principal axis to which incident rays parallel and close to the principal axis converge or from which they appear to diverge after reflection.
- The Principal Focus of a concave mirror is said to be a real focus because the reflected rays pass through it while that of a convex mirror is a virtual focus because the reflected rays do not pass through it. This is illustrated in Fig. 2.2.
- The Focal Length (f): the distance (f-p) from the principal focus to the pole. It is half the radius of curvature. That is  $f = r/2$ .





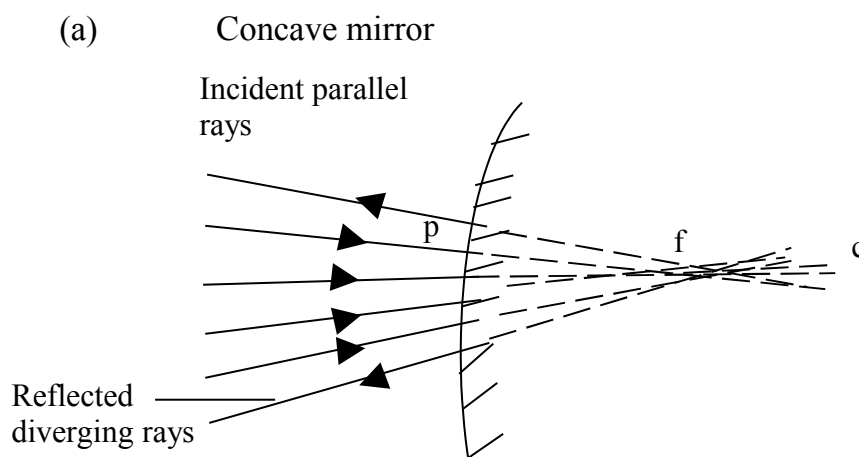


Fig. 2.2: Principal focus of concave and convex mirrors

### Principal rays for locating images in a curved mirror

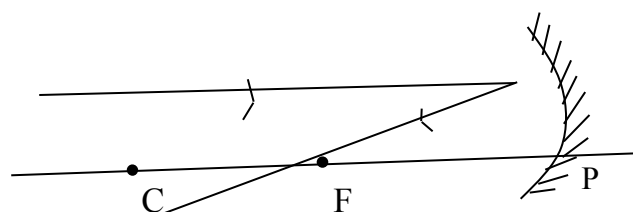
When ever you want to construct ray diagrams, you will use the following principal rays to locate the image position of your object.

#### (i) For a concave mirror

##### Rule1

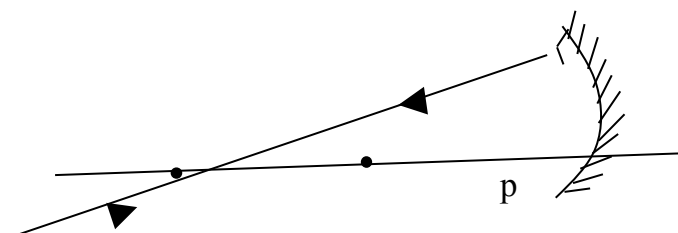
- Incident rays parallel to the principal axis are reflected through the principal focus.

##### Rule1



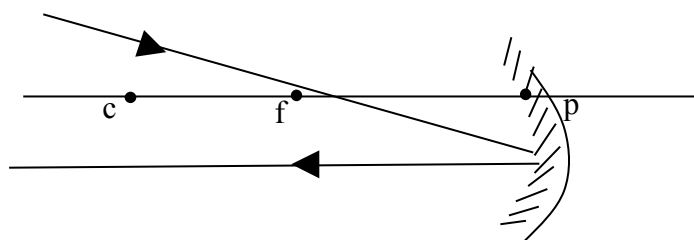
**Rule2<sub>1</sub>** Incident rays passing through centre of curvature meet the mirror at right angle and so are reflected back along their own paths.

##### Rule2





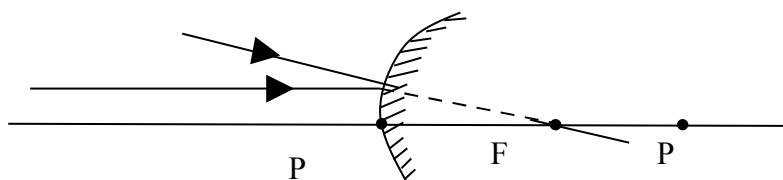
**Rule3:** Incident rays passing through the principal focus are reflected parallel to the principal axis.



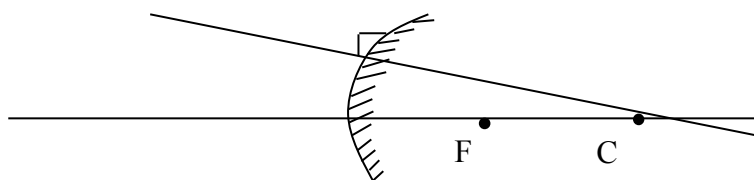
Any two of the rays are sufficient for you to locate an angle.

(ii) For a convex mirror

**Rule1:** Incident rays parallel to the principal axis are reflected so that they appear to come from the principal focus.



**Rule2:** Incident rays traveling towards the centre of curvature meet the mirror at right angles and so are reflected back along their own paths.



### 3.2 Image Formed by Curved Mirrors

The size, nature and position of an image formed by a curved mirror depend on the position of the object from the pole of the mirror. You can study there by the use of an illuminated object, a concave mirror and a screen.

To do this, you will place the concave mirror on a stand and allow light from the illuminated object to fall centrally on it by the adjustment of the height of the mirror. The mirror is slightly turned to one side so as to form an image where possible on the screen. You can do this for various object positions.



If you use a convex mirror, you will obtain an erect and virtual image smaller than the object position.

Also, you can always obtain the images formed by curved mirrors by scaled ray diagrams. To do this, you will make use of the ‘principal rays’ discussed in 3.1.3. The point I to which these reflected rays intersect represents the required image of O, the top of the object lies on the axis, since the ray XCP strikes the mirror normally at p and is reflected straight back. This is illustrated in Fig. 2.3 for concave mirror and 3.4 for convex mirror.

### 3.2.1 Images Formed by Concave Mirror

(a) Object beyond c

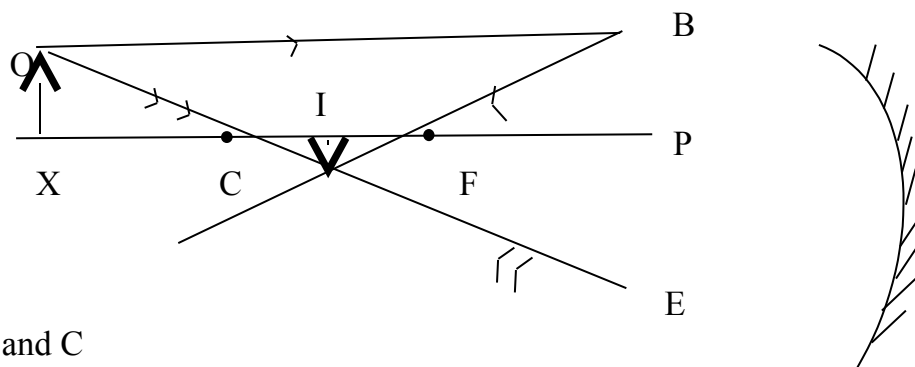


Image is: between F and C

- Real
- Inverted
- Diminished

(b) Object at C

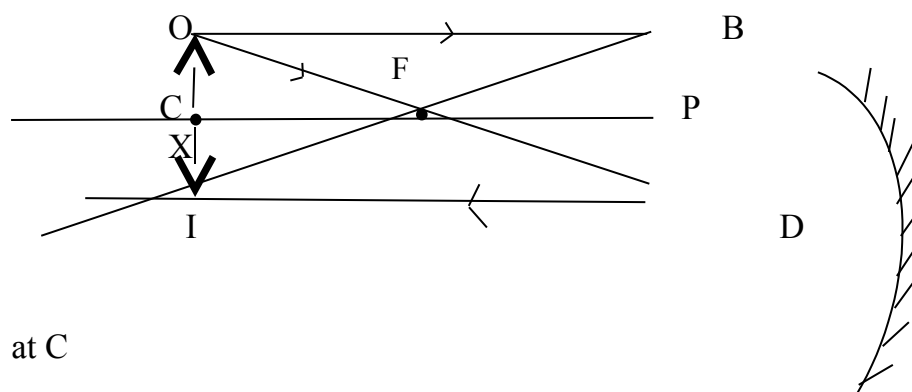


Image is: at C

- Real
- Inverted
- Same size as object



(c) Object between C and F

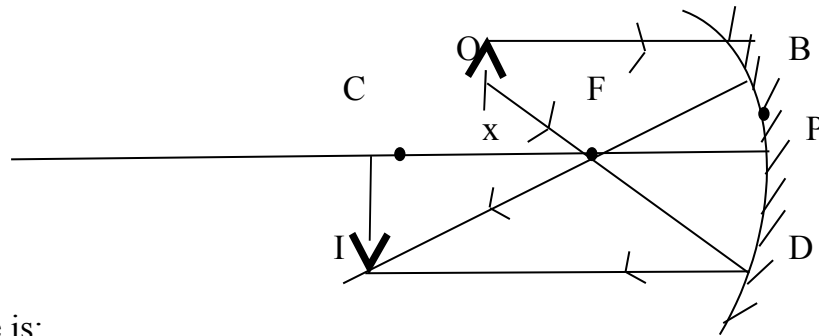


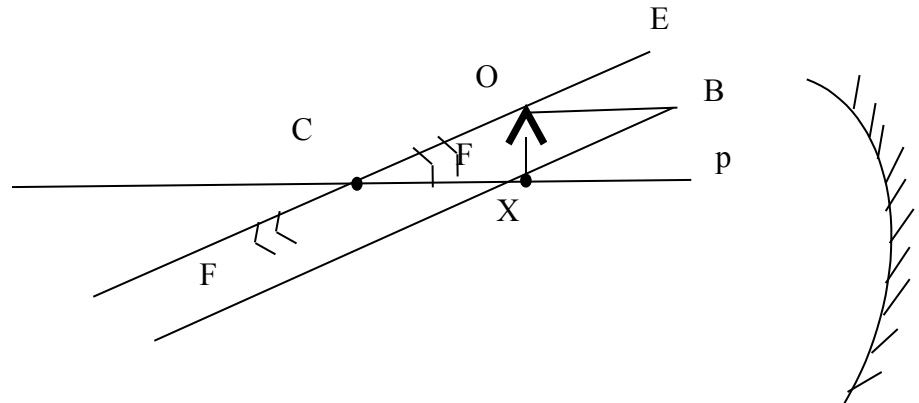
Image is:

- Beyond c
- Real
- Inverted
- Magnified

(d) Object at F

Image is:

- at infinity
- Real
- Inverted
- Magnified



(e) Object between f and p

Image is:

- behind the mirror
- virtual
- erect
- magnified

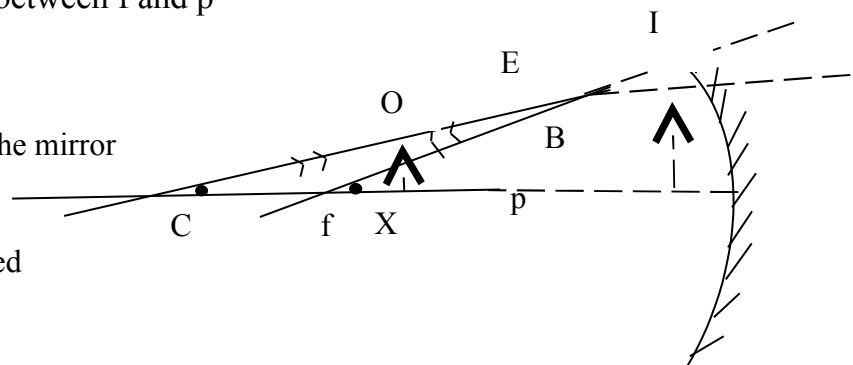


Fig. 2.3: Images formed by concave mirror

You will observe that in the ray diagrams, OB represents rays parallel and close to the principal axis (rule 1) which after reflection passes through f.

Ray OC passes through the centre of curvature C (rule 2) and is reflected back on its own path.



### 3.2.2 Images Formed by Convex Mirror

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Fig. 2.4: Image formed by concave mirror

From Fig. 2.4 you see that convex mirror produced only virtual erect and diminished images. The image is formed at the intersection of the reflection rays produced backwards (in broken lines) and is formed behind the mirror (virtual). This is the case for all positions of the object in front of the mirror.

### 3.2.3 Distinction between Real and Virtual Images

You will now note the following distinction between real and virtual images.

Real Image	Virtual Image
(a) can be received on a Screen.	Cannot be received on a Screen
(b) Produced by actual inter-section of rays of light	Produced by apparent intersection of rays of light.

### 3.3 Uses (Applications ) of Curved Mirrors

Here, we are going to discuss some practical applications of images formed by curved mirrors at various objects by curved mirrors at various object positions.

- Object between F and P of a concave mirror  
The concave mirror, at this object position is used as dentist mirror, make – up mirror and showing mirror because the image formed in this position of object is enlarged, erect and virtual.
- Object at f of a concave mirror  
At the object position, the concave mirror is used for torch. Also, parabolic (not spherical) concave mirrors are used in car headlamps and searchlight. They produce a wide parallel beam of light of constant intensity when a small light source is placed at its focus. Spherical concave mirrors are not used because they do not provide parallel beam of constant intensity (Fig. 2.5).



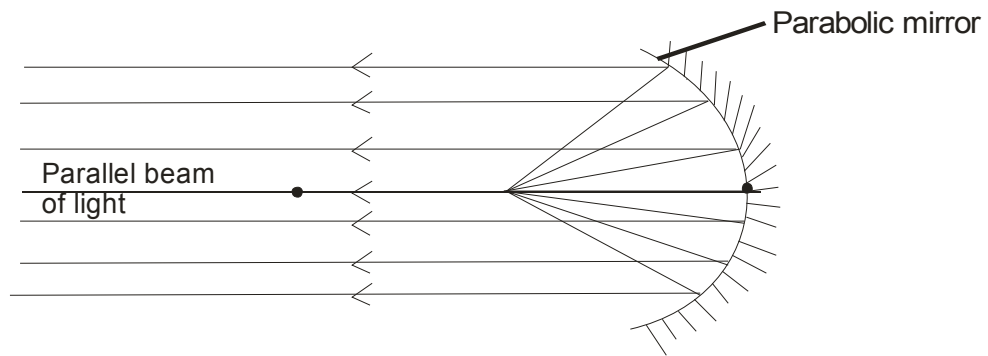


Fig. 2.5: Parabolic mirror

- Object between  $f$  and  $c$   
At the object position, concave mirrors are used in some kind of flood light
- Object at  $c$   
Concave mirror is used as projector lamp in this object position.
- Concave mirrors are also used as reflectors in reflecting telescope and microscopes.
- Convex mirrors are used as driving mirrors because they always give an erect image of an object behind the driver. In addition, they provide a wider field of view than a plane mirror of the same aperture. This makes it possible for the driver to see objects with in a large angle.

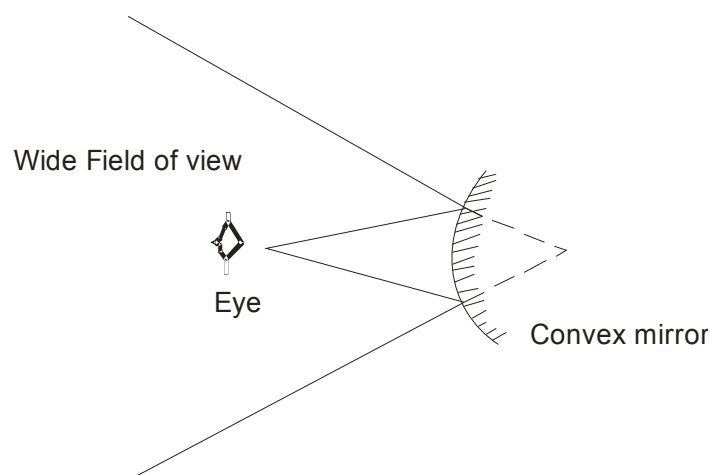


Fig. 2.6: Field of view with a convex driving mirror



## SELF ASSESSMENT EXERCISE 1

- (1) With the aid of ray diagrams, illustrate the characteristics of image formed by a concave mirror when an object is placed
  - (i) Between F and P
  - (ii) at F,
  - (i) between F and C
  - (ii) at C;
  - (iii) beyond C, and
  - (iv) at infinity
- (2) In what situations would the image formed by a convex mirror be useful?
- (3) Distinguish between real and virtual images.
- (4) What is meant by the radius of curvature and the principal focus of a concave mirror?

### 3.4 Measurement of Focal Length of a Concave Mirror

#### 3.4.1 Quick but Approximate Method

A simple and quick method of finding an approximate value of the focal length of a concave mirror is to focus the image of a distant object on a screen. To do this, you will hold a concave mirror with its reflecting surface facing a distant object such as a window pane at an angle (Fig. 2.7). You will also hold a screen, may be a piece of cardboard sheet. Move the screen in front of the mirror until the sharpest image of the mirror window forms on it. Measure the mirror distance between the screen and the mirror. This value is the approximate value of the focal length of the mirror.

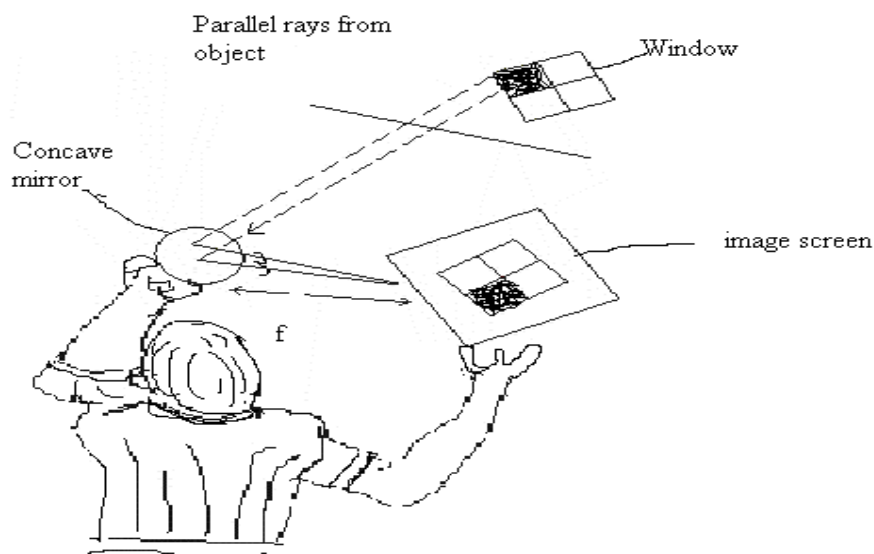


Fig. 2.7: Focal length by quick and approximate method



Since the mirror is at a considerable distance away from the windows, the rays from it may be considered as parallel rays when they strike the mirror. They will therefore come to a focus after reflection at the principal focus.

### 3.4.2 The Mirror Equation Method

In this method you will make use of the relationship between the object distance  $u$ , the image distance  $v$  and the focal length of a spherical mirror the relationship is expressed in the equation.

$$\frac{1}{U} + \frac{1}{V} = \frac{1}{F} \quad \text{-----} \quad (1)$$

In using this equation, you will first obtain the approximate focal length of the concave mirror using the method in paragraph 3.4.1 above. You will need an illuminated object (the cross wire of a ray box or a candle stick), a screen, and a concave mirror. Set up the apparatus as shown in Fig. 2.8

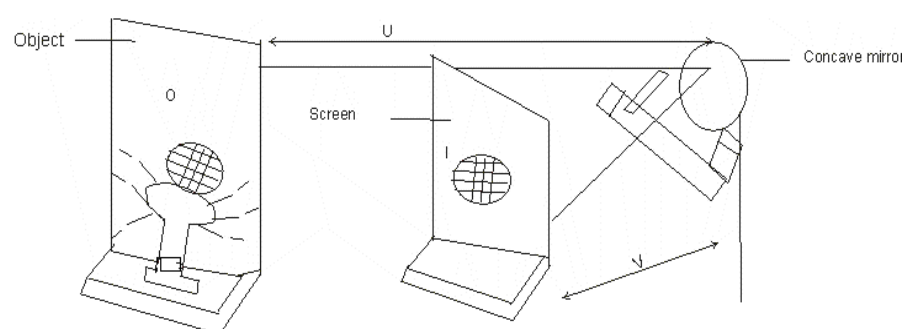


Fig. 2.8: Focal length by the mirror equation method

Place the illuminated object at distance of about  $\frac{3}{2} f$  in front of the mirror and adjust the screen position until a sharp image is obtained. Measure and record the object distance (distance from the object to the concave mirror),  $u$  and the image distance (distance from the screen to the concave mirror),  $v$ . Repeat the procedure for at least six object distance,  $u$ , and obtain the corresponding values of image distance,  $v$ . Tabulate your readings as in the Table 2.1 below.

Table 2.1: Values of  $u$  and the corresponding values of  $v$



	u	v	uv	U+V	$\frac{uv}{u+v}$
1					
2					
3					
4					
5					
6					

Table B.

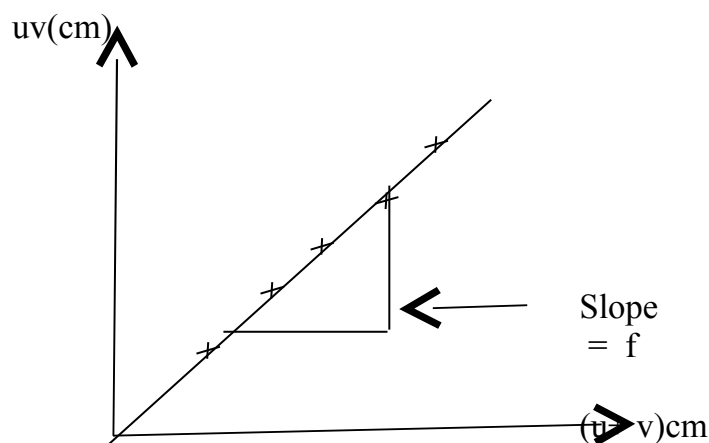
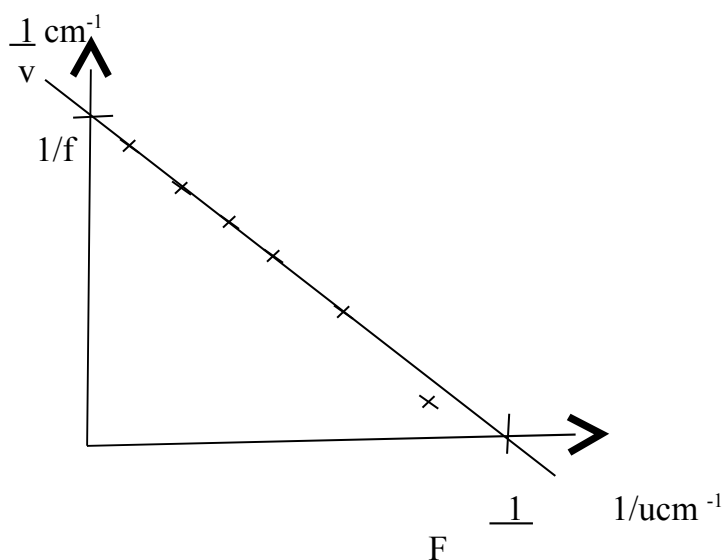
	U	V	1/u	1/v	$\frac{1}{u} + \frac{1}{v}$
1					
2					
3					
4					
5					
6					

Table A.

You can now obtain the value of  $F$  from your readings in Table 2.1 in the following ways.

- Add the values of  $uv / u+v$  in Table A and find the average. The result is the experimental value of  $f$ .
- Add the values of  $\frac{1}{u} + \frac{1}{v}$  in Table B and find the average. The result is  $1/f$  from which you will determine the value of  $F$ .  $F$  is the reciprocal of the value obtained.
- Plot a graph of  $uv$  against  $u + v$ . You will obtain a straight line graph. The slope of this graph is equivalent to  $f$ , see Fig. 2.9 (a)
- Plot a graph of  $1/v$  against  $1/u$ . You will obtain a graph with intercepts on both axes. Each of the intercepts gives you a value of  $1/f$ . From this value, find the reciprocal to obtain the value of  $F$  (Fig. 2.9b).



Fig. 2.9(a): Graph of  $uv$  against  $(u + v)$ Fig. 2.9(b): Graph of  $1/v$  against  $1/u$ .

### 3.5 The Mirror Formula and Sign Conventions

You can solve any problem on spherical mirror of small aperture by drawing to scale ray diagrams as in paragraph 3.2 above.

You can also use an equally valid method, the algebraic formulae which connect the object distance  $u$ , image distance  $v$ , focal length  $f$ , and magnification  $m$ .

For a concave mirror,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{----- (1)}$$

from experimental determination of focal length it was found that



$$1/v + 1/u = 1/f$$

Also the magnification

$$m = v/u \text{ ----- (2)}$$

For us to extend the usefulness of the above equations to virtual images as well as to convex mirrors, it is necessary to adopt what is called a sign convention. The sign convention tells us when to add a position (+) or a negative (-) sign to each of the distances. Two commonly used type sign conventions are presented below for you to study and use.

### 3.5.1 The Sign Convention

Real is positive	New Cartesian
<ul style="list-style-type: none"> <li>• Real objects and real images are considered to be a positive distance from the mirror.</li> <li>• Virtual images are at a negative distance from the mirror.</li> <li>• Focal length of a concave mirror is positive</li> <li>• Focal length of a convex mirror is negative</li> </ul>	<ul style="list-style-type: none"> <li>• Distances measured to the right of the mirror are positive.</li> <li>• Distances measured to the left of the mirror are negative.</li> <li>• Focal length of convex mirror is negative</li> <li>• Focal length of a convex mirror is positive.</li> <li>• Distances are measures from the Pole of the mirror. The mirror is positioned with its face to the left.</li> </ul>

### 3.5.2 Mirror Formulae

The mirror equations are:

(i)  $1/u + 1/v = 1/f$  (Real is positive sign convention)  
 or  
 $1/v - 1/u = 1/f$  (New Cartesian sign convention)

Where

$u$  = object distance from the mirror  
 $v$  = object distance from the mirror  
 $f$  = focal length of the mirror.

(ii)  $m = \frac{\text{height of image}}{\text{height of object}} = v/u$



height of object  
where

$M$  = Linear magnification

$$(iii) \quad f = r/2$$

Where

$r$  = Radius of curvature of mirror

$f$  = Focal length of mirror.

### Application of the Mirror Formulae in Solving Numerical Problems

#### Work Examples

- (1) An object 3 cm tall is placed 12 cm from a concave mirror of focal length 8 cm. find the image position and its height.

#### Solution

$$u = 12, f = 8.$$

Using the mirror formula

$$1/u + 1/v = 1/f \text{ (Real is positive convention).}$$

Substituting in the above equation gives

$$1/12 + 1/v = 1/8$$

$$1/v = 1/8 - 1/12$$

$$1/v = 1/24$$

$$v = 24$$

The image position is 24 cm.

Magnification

$$M = v/u = 24/12 = 2$$

$$\text{Height of the image} = (3 \times 2) \text{ cm} = 6 \text{ cm.}$$

- (2) An Object is magnified five times by a concave mirror of radius of curvature 30 cm. What is the object position if the image is (a) real and (b) virtual?

#### Solution

- (a) When the image is real

$$M = v/u = 5/1$$

$$\text{If } u = x$$

$$v = 5x \text{ (image is real).}$$



Using the mirror formula:

$$\begin{aligned}\frac{1}{u} + \frac{1}{v} &= \frac{1}{f} \\ r &= 30\text{cm} \\ \Rightarrow f &= r/2 = 30/2 = 15\text{cm}\end{aligned}$$

Substituting in the formula gives

$$\begin{aligned}\frac{1}{x} + \frac{1}{5x} &= \frac{1}{15} \\ \frac{5+1}{5x} &= \frac{1}{15}\end{aligned}$$

$$\begin{aligned}6/5x &= 1/15 \\ 5x &= 6 \times 15 = 90 \\ x &= 90/5 = 18\text{ cm}.\end{aligned}$$

The object position when the image is real is 18 cm.

(b) When the image is virtual

$$\begin{aligned}m &= v/u = 5/1 \\ f &= r/2 = 30/2 = 15\text{cm}\end{aligned}$$

$$\begin{aligned}\text{If } u &= x \\ V &= -5x \text{ (image is virtual)}.\end{aligned}$$

Substituting the values in the mirror formula we have

$$\begin{aligned}\frac{1}{x} - \frac{1}{5x} &= \frac{1}{15} \\ \frac{5-1}{5x} &= \frac{1}{15} \\ 4/5x &= 1/15 \\ 5x &= 4 \times 15 = 60 \\ x &= 60/5 = 12\text{cm}\end{aligned}$$

The object position when the image is virtual is 12 cm.

### 3.6 Defects of Spherical Mirrors

You will observe spherical aberration when the aperture of a spherical mirror is large. You will notice that a beam of light parallel to the axis does not converge to the principal focus. Such a mirror brings rays close to the principal axis to a focus further away from the pole of the mirror than the focus. In addition, it is found that the reflected rays are tangents to a curve known as a caustic curve. The inability of such spherical



mirror of wide aperture to bring all parallel rays to the same focus is known as spherical aberration. In order to avoid this phenomenon, spherical mirrors of small apertures are usually used. Also, parabolic mirrors, where all parallel rays are brought to a single focus are used. To avoid blurred images due to spherical aberration, parabolic mirrors are used to produce parallel beams in search lights, torch lights and car headlamps.

## SELF ASSESSMENT EXERCISE 2

1. Describe an experiment to measure the focal length of a concave mirror.
2. Explain the phenomenon spherical aberration.  
How is it corrected in practice?
3. A small object is placed 24 cm from a concave mirror of focal length 16 cm. What is the position of the image?
4. An object is placed 50 cm in front of a concave mirror and a real image is produced 20 cm from the mirror.
  - (a) What is the magnification produced?
  - (b) What magnification will be produced if the object is placed 30 cm from the mirror?

## 4.0 CONCLUSION

In this unit you studied reflection of light at curved surfaces. You learned about types of curved mirrors, terms used in curved mirrors, and principal rays for locating images in a curved mirror. Also, you learned how to locate images formed by curved mirrors by scale drawings and calculations. In addition you learned how to find the focal length of concave mirrors experimentally. Finally, you learned the applications of curved mirrors and the defects of spherical mirrors.

## 5.0 SUMMARY

- A concave mirror converges incident parallel rays while a convex mirror diverges incident parallel rays.
- A concave mirror has a real focus while a convex mirror has a virtual focus.
- Concave mirrors produce different images when the object is placed in different positions.
- Convex mirrors always produce virtual diminished and erect images.



- The mirror equations are:
  - (i)  $1/u + 1/v = 1/f$
  - (ii)  $m = v/u$
  - (iii)  $f = r/2$
- Some practical applications of;
  - (a) Concave mirrors are:
    - (i) As shaving mirror, dentist mirror, make up mirror, projector lamp and some kind of flood lights.
    - (ii) Parabolic mirrors are used in searchlights, torches and car headlamps
  - (b) Convex mirrors are used as driving mirrors.

## 6.0 TUTOR-MARKED ASSIGNMENT

1. With the aid of ray diagram illustrate the characteristics of the image formed by a concave mirror when an object is placed between c and f.
2. What is the defect of a spherical mirror? How is it corrected?
3. Which of the types of curved mirrors is used as a driving mirror and why?
4. An object is placed 20 cm in front of a convex mirror and an image is produced 4 cm behind the mirror. Calculate the focal length of the mirror

## 7.0 REFERENCES/FURTHER READINGS

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## UNIT 3      REFRACTION OF LIGHT

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    - 3.3.2      Relation of Refractive Index to Real and Apparent Depths
  - 3.4      Measurement of Refractive Index
    - 3.4.1      Measurement of Refractive Index of Solid (glass)
    - 3.4.2      Measurement of Refractive Index of Liquid (water)
    - 3.4.3      Determination of the Refractive Index Glass by the Method of Minimum Deviation of Light through a Glass Prism
  - 3.5      Critical Angle and Total Internal Reflection
    - 3.5.1      Critical Angle and Total Internal Refraction
    - 3.5.2      Relationship between Critical Angle  $c$  and Total Internal Reflection
    - 3.5.3      Applications of Total Internal Reflection
  - 3.6      Numerical Examples
- 4.0      Conclusion
- 5.0      Summary
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### 1.0      INTRODUCTION

In this unit, you will learn the refraction of light through glass blocks and prisms. You will also learn the laws of refraction – the statements of the law and their verifications. Furthermore you will learn about real and apparent depths in solids and liquids and the relationship between them. In addition, you will learn the measurement of refractive index of glass and liquid using Snell's law and 'real and apparent depths'. Finally, you will learn about critical angle and total internal reflection as the applications of total internal reflection.



## 2.0 OBJECTIVES

By the time you finish this unit, you will be able to:

- explain how the direction of light changes as it travels from one medium into the other
- measure angles of incidence and refraction and hence deduce a value for the refractive index of a given material
- explain the meaning of critical angle and total internal reflection stating the conditions under which they occur
- establish the relationship between critical angle and refractive index and apply it to the solution of simple problems
- trace light rays through a triangular prism and obtain graphically the value of the angle of minimum deviation.

## 3.0 MAIN CONTENT

### 3.1 Refraction of Light

#### 3.1.1 Concept of Refraction

If you consider the path taken by a ray of light as it travels from one medium to another say from air to water, you will observe that the direction of the ray changes instantly at the boundary separating the two media (air and water). This instant change in the direction of light ray at the boundary of two transparent media is referred to as the refraction of light. This phenomenon called refraction is evident in the following every day observations:

- i. The bottom of clear pond appearing to be nearer the surface than it really is;
- ii. A rod or spoon appears bent or broken when it is partially immersed in water;
- iii. When a thick glass block is placed on a printed page, the letters under the block appear raised relative to those not under the block.

A rod appears bent in water



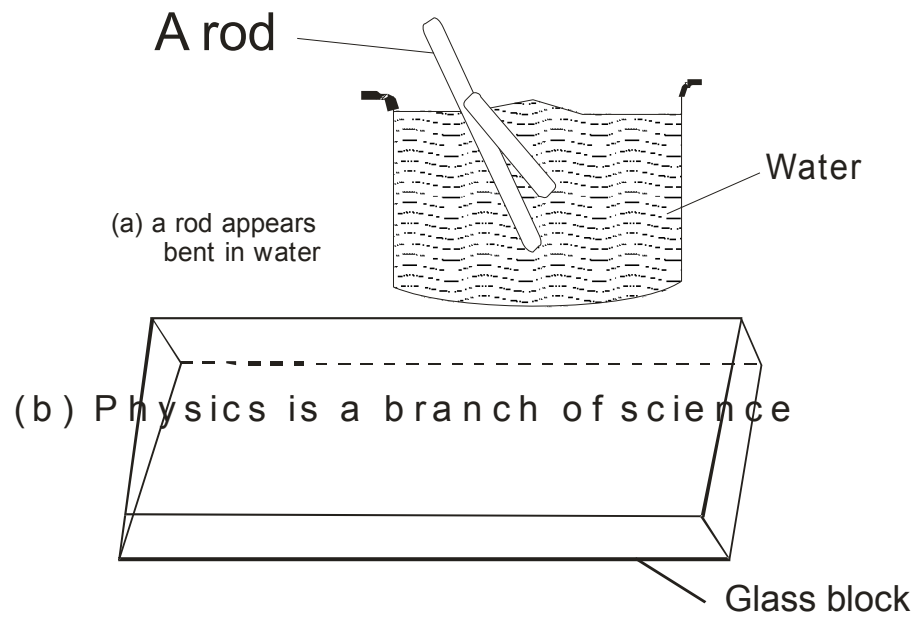


Fig. 3.1: Demonstration of refraction effects

### Definition

Refraction is the bending of a light ray as it crosses the boundary between two media of different densities thereby causing a change in its direction.

Refraction is due to the difference in the speed of light in different media.

If we consider a ray of light traveling from a medium A to another medium B as shown in Fig. 3.2

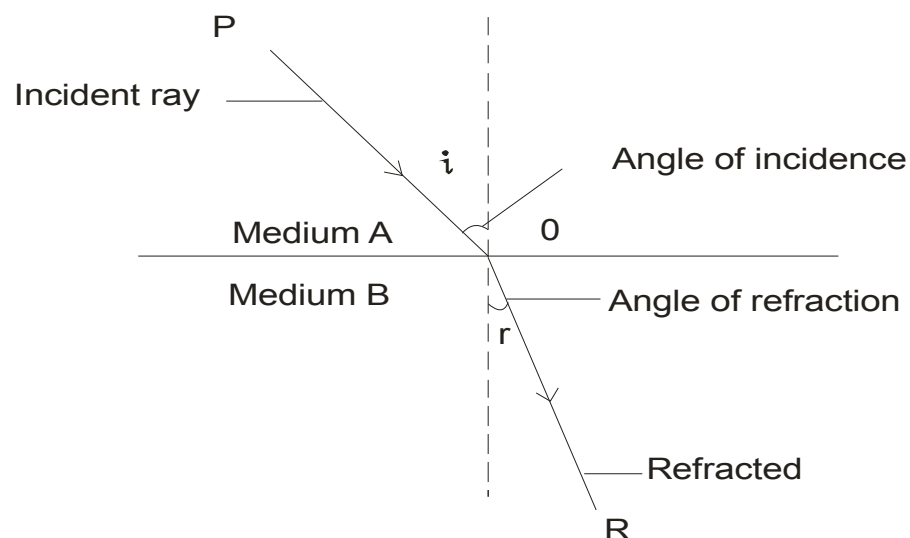


Fig. 3.2: Ray of light traveling from one medium to another



The incident ray PO is the path along which the light travels in the first medium.

The angle of incidence ( $i$ ) is the angle which the incident ray makes with the normal to the surface at O. The incident ray is refracted as it enters the second medium and it travels along the path OR. This ray OR is called the refracted ray. The angle between the refracted ray and the normal to the surface at O is called the angle of refraction ( $r$ ).

Speed of light is more in medium of less optical density. Rays of light traveling from low optical dense medium to light optical dense medium bend towards the normal. On the other hand the rays of light traveling from high optical dense medium bend away from the medium.

### 3.1.2 Refraction of Light through Rectangular Glass Block

Let us study the refraction of light at air glass boundary using a rectangular glass block. To do this, you will place a thick rectangular glass block (or slab) horizontally on a cardboard sheet and draw the outline of the block with a pencil. Direct a light ray from a ray box into the glass block surface AB along a line PO inclined at an acute angle to the normal ON. The light ray is refracted in the glass CD. Mark two points along the path of this ray. Join the point by a line SR and produce it to meet the glass – air boundary at the point R. Remove the glass block. Using a pencil, draw a line to join the points R and O. This line RO represents the path of the light ray in the glass block.

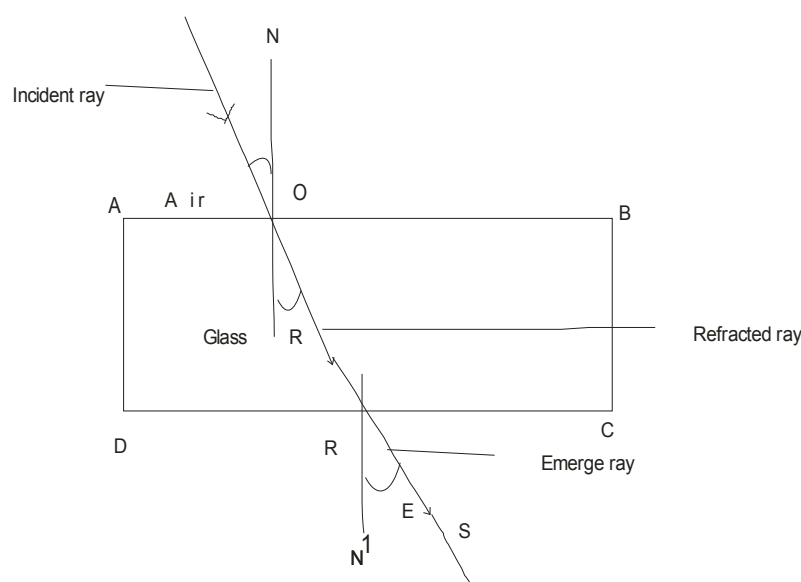


Fig.3.3: Refraction through a rectangular glass block

You will notice that the line RS is parallel to the line PO (Fig. 3.3). The line PO is known as the incident ray, OR is the refracted ray while RS is



the emergent ray. The angle ( $i$ ) between the incident ray and the normal at O is the angle of incidence. The angle ( $r$ ) between the refracted ray OR and the normal at O is the angle of refraction. The angle ( $e$ ) between the emergent ray RS and the normal at R is the angle of emergence.

You may have also noticed that when the light travels from air to glass, it is refracted towards the normal; when it travels from glass to air, it is refracted away from the normal. This is a general phenomenon earlier mentioned and is due to the different in densities of the two media in which the incident and refracted rays travel. That is:

- When a ray of light travels from a less dense medium to a denser medium, the ray will be refracted towards the normal.
- A light ray passing from a denser medium to a less dense medium will be refracted away from the normal.

Note that if the incident ray is along the normal NO- at an angle of  $90^\circ$  to the side AB, ( $i = 0$ ). It will pass straight through the glass and emerges on the surface CD without a change of direction.

Note also that there is no deviation or change in the direction of the emergent ray when compared with the incident ray.

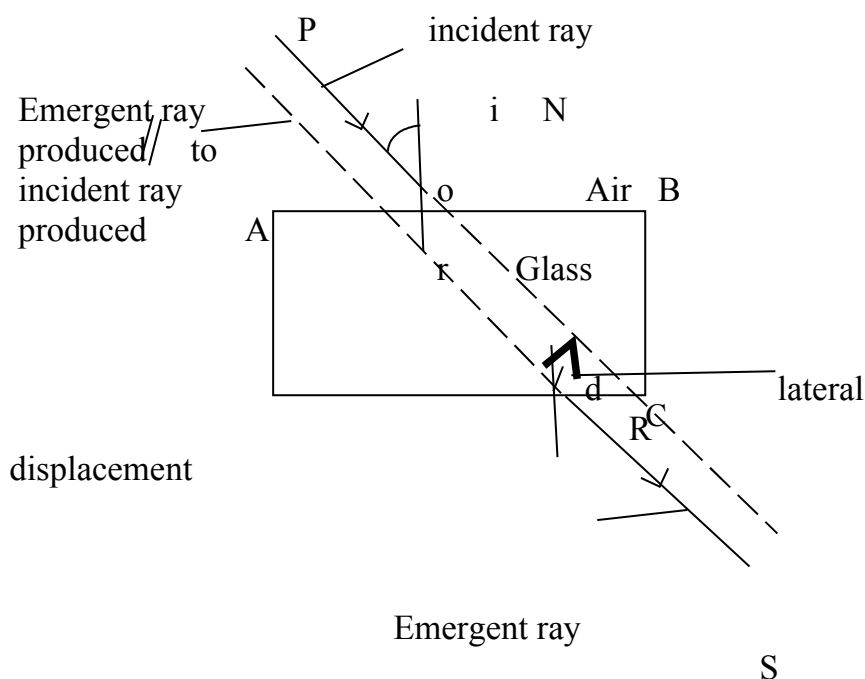


Fig. 3.4: Lateral displacement

The above figure shows that the incident ray produced is parallel to the emergent ray produced; thus resulting in the lateral displacement  $d$ .



### 3.1.3 Refraction of Light Through Triangular Prism

Using a pencil, draw the outline ABC of a  $60^\circ$  prism placed on a white sheet of cardboard paper. Remove the prism and use a protractor to draw a normal ON to AB at a point O between A and B. Now, consider the passage of light rays through this triangular glass prism (Fig. 3.5).

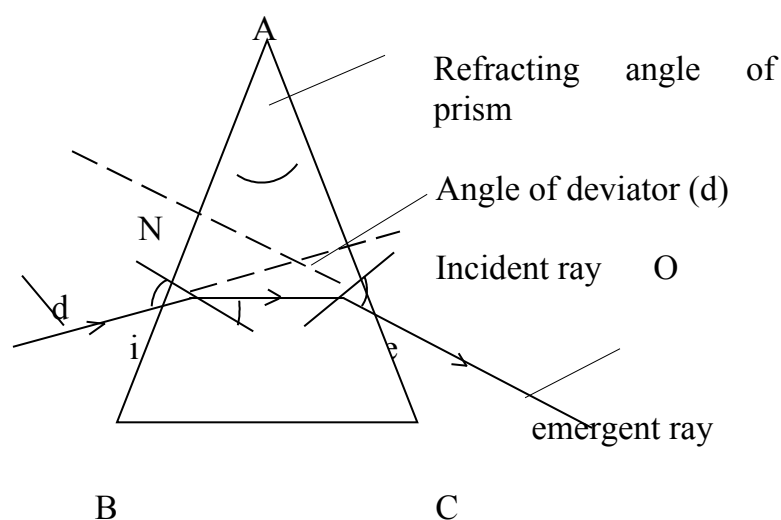


Fig. 3.5: Refraction through triangular prism

Consider one such ray PQ incident on the prism side AB. The incident, the refracted and the emergent rays are as shown in Fig 3.5. Here, the emergent ray is not parallel to the incident ray but it is at an angle to it. The prism deviates the incident ray through an angle  $d$  and is known as the angle of deviation. It is the angle between the incident ray and the emergent ray of light passing through the prism. This angle of deviation depends on:

- (i.) The angle of incidence
- (ii) the refracting angle (A) of the prism and
- (iii) the refractive index of the material of the prism.

It has been proved from experimental results that the angle of deviation ( $d$ ) varies with the angle of incidence ( $i$ ) as shown in Fig. 3.6



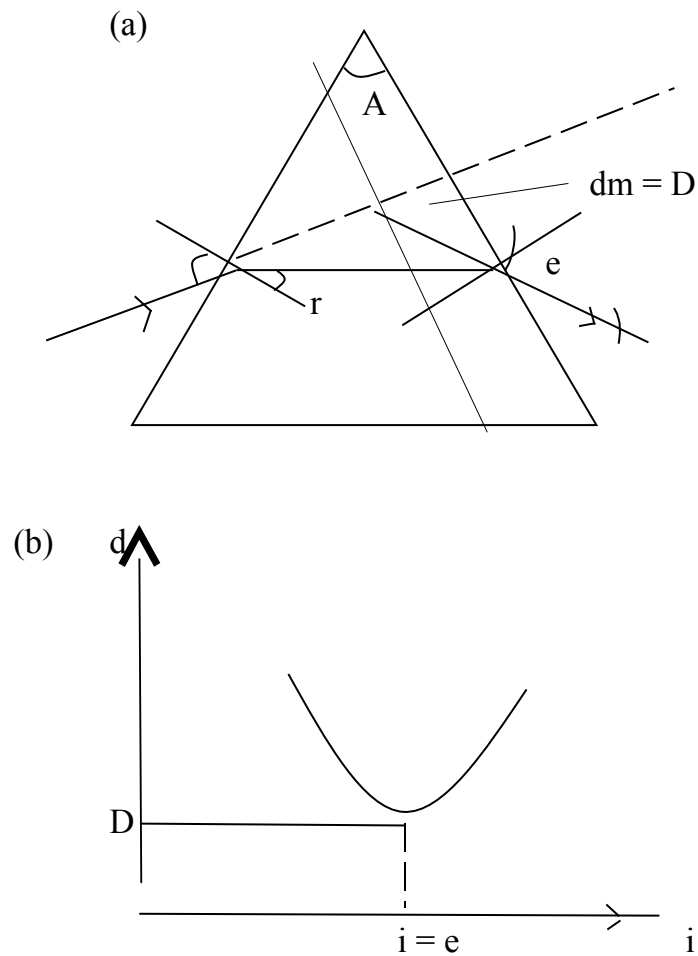


Fig. 3.6: At minimum deviation  $i = e$

Figure 3.6 shows that there is a minimum angle of deviation ( $D$ ) which occurs when the incident and emergent rays are equally inclined to their respective surfaces. Also, the ray passes symmetrically through the prism and the angle of incidence equal angle of emergence ( $i = e$ ).

At minimum deviation, the refractive index ( $n$ ) of the glass, the refracting angle ( $A$ ) of the prism and the angle of minimum deviation ( $D$ ) are related by the equation.

$$n = \frac{\sin \frac{1}{2} (A + D)}{\sin \frac{1}{2} A}$$

### 3.2 Laws of Refraction

The phenomenon of the refraction of light was first observed in 300 BC by Euclid, a Greek mathematician. An Egyptian Ptolomy was the first to



record the values of the angles of incidence and refraction but he could not find the relationship between these angles. However in 1621 a mathematician, willebrod Snell formulated the exact relationship between the angles of incidence and refraction. Hence, the law is known as Snell's law of refraction.

### 3.2.1 Statements of Laws of Refraction

There are two laws governing the refraction of light. These are:

- i The incident ray, the refracted ray and the normal at the point of incidence all lie on the same plane.
- ii The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media.

The second law is the Snell's law and it may be expressed as:

$$\frac{\sin i}{\sin r} = \text{constant}$$

For a given pair of media

where  $i$  = angle of incidence  
 $r$  = angle of refraction  
 $n$  = the refractive index of the second medium with respect to the first medium.

The refractive index  $n$  is a number which gives a measure of the refraction or bending of light as it travels from one medium to another.

When light travels from air to glass, the refractive index of glass is given by

$${}_a\eta_g = \frac{\text{Sine of angle of incidence in air}}{\text{Sine of angle of refraction in glass}}$$

When light travels from glass to air then the refractive index  ${}_g\eta_a$  is given by

$${}_g\eta_a = \frac{\text{Sine of angle of incidence in glass}}{\text{Sine of angle of refraction in air}}$$

From the principle of the reversibility of light we have that

$${}_a\eta_g = 1/{}_g\eta_a$$

Also, because refraction is due to the change in the speed of light as it travels from one medium to another, the refractive index is given by



$${}_a n_g = \frac{\text{speed of light in air (vacuum)}}{\text{speed of light in glass.}}$$

### 3.2.2 Verification of Snell's Law

Snell's law can be verified by performing an experiment using a rectangular glass block. To do this, place the glass block on a sheet of paper and trace the outline with a pencil. Draw a normal ON and the draw lines making angles of  $30^\circ$ ,  $40^\circ$ ,  $50^\circ$ ,  $60^\circ$  and  $70^\circ$  with then normal ON. Direct a single ray of light from a ray box along one of the lines making angles of say  $30^\circ$  with the normal.

This is now the incident ray PO. Mark two point TS where the light emerges from the glass block with a pencil. This ray TS is the emergent ray – join the two points and produced the line to cut the glass block at R (fFig. 3.7). Remove the glass block and join the points.

OR. Draw a normal YRZ at point R where the emergent ray line or represents the refracted ray in the glass.

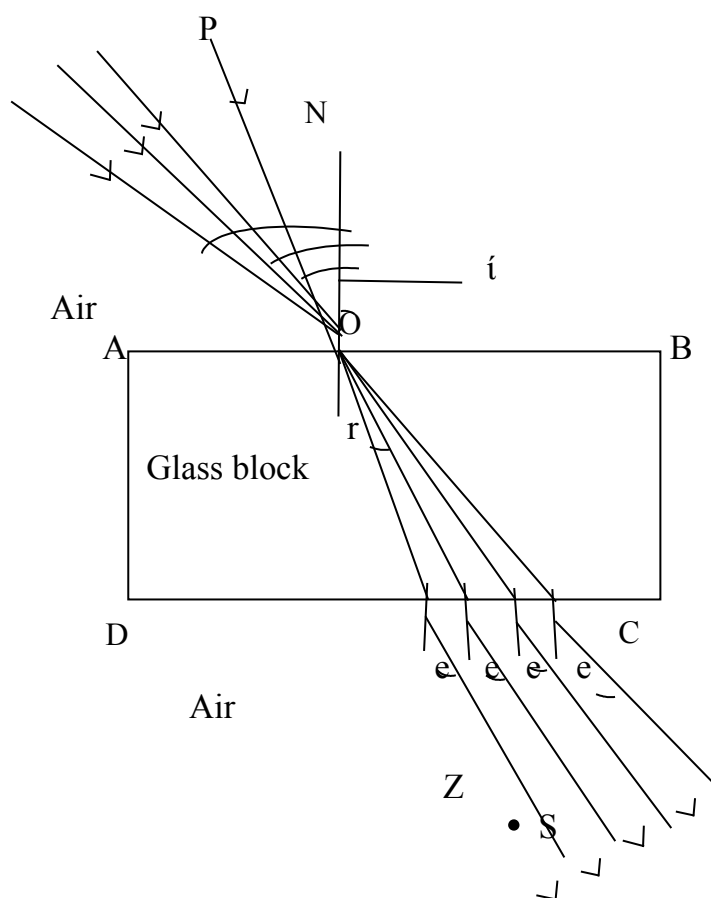


Fig.3.7: Verification of laws of refraction



Measure the angle of incidence  $i$  and the angle of refraction  $r$ . Repeat the experiment for other angles of incidence and obtain the corresponding angles of refraction. Measure and tabulate the angle of incidence ( $i$ ) and their corresponding angles of refraction ( $r$ ) as shown in Table 3.1 below.

Table 3.1: Experimental values of the incident angle ( $i$ ) and their corresponding angles of refraction ( $r$ ) for a glass block

Angle of incidence ( $i$ )	Angle of refraction ( $r$ )	$\sin i$	$\sin r$
$30^\circ$			
$40^\circ$			
$50^\circ$			
$60^\circ$			
$70^\circ$			

Evaluate and record  $\sin i$  and  $\sin r$ . Use the values to plot  $\sin i$  on the vertical axis and  $\sin r$  on the horizontal axis. Obtain the slope of the graph as shown in Fig. 3.8 below.

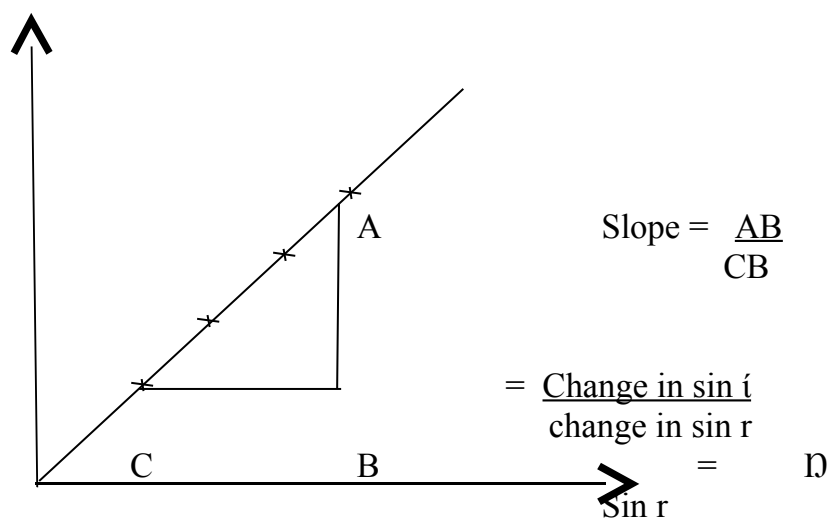


Fig. 3.8.: Graph of  $\sin i$  against  $\sin r$

When  $\sin i$  is plotted against  $\sin r$ , a straight line graph passing through the origin is obtained as shown in Fig. 3.8 which proves that the ratio  $\frac{\sin i}{\sin r}$

is a constant. That is  $\sin i / \sin r = \text{constant}$ . Hence,

Snell's law is verified. The slope of the graph gives the refractive index for the glass block used.

- (ii.) We can also verify Snell's law with optical pins in the experiment instead of a ray box. Here optical pins are used to represent the



incident ray while sighting pins are used to locate the emergent ray. Two pins MN are fixed on the line normal MO making angle  $30^\circ$  with the normal NO to represent the incident ray. A sighting pin X is moved along CD until it appears to be in same straight line as MN when viewed through the glass block. A second pin Y is fixed so that X and Y appear coincident with the image of M and N in the glass block.

This is illustrated in Fig. 3.9

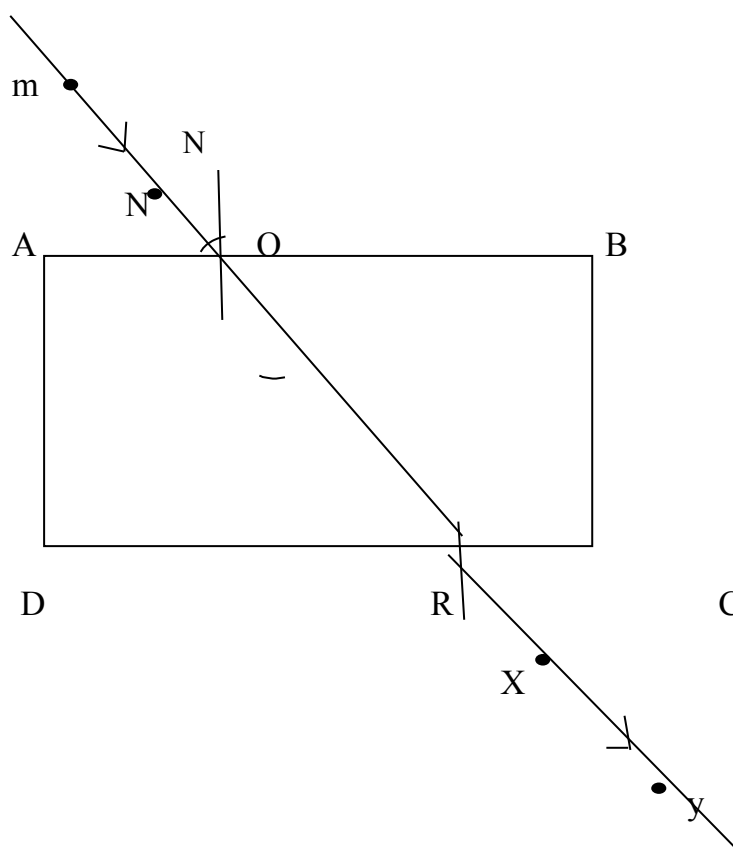


Fig. 3.9: Verification of Snell's law

The glass block is then removed and the incident ray is drawn through M and N while the emergent ray is drawn by joining the point O at which the line MN meets the glass surface to the point R where the emergent ray leaves the glass surface. The angles of incidence and refraction are measured and the produce and repeated for various angles of incidence.

This is followed by plotting the graph of  $\sin i$  against  $\sin r$ . Again, the graph is a straight line passing through the origin indicating  $\sin i / \sin r$  is a constant and the slope gives the value of the refractive index. Thus, Snell's law is also verified and found to be true.



### 3.3.1 Real and Apparent Depths

There appears to be an upward displacement of the bottom of a vessel containing water when viewed upward. An object inside water appears to be closer to the surface than it actually is. That is, the depth appears to be shallower than the actual depth. This apparent depth is caused by refraction (Fig. 3.10)

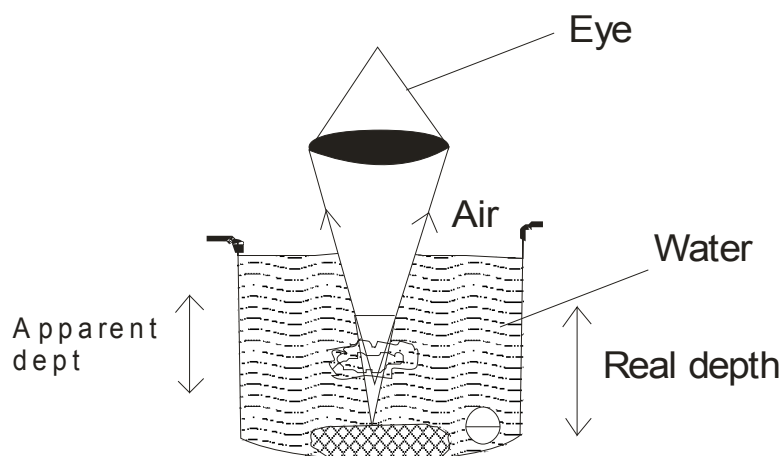


Fig. 3.10: Real and apparent depths

The rays from an object O under water do not travel straight to the eye but are refracted away from the normal as they leave the water surface. When these rays are produced backwards, they meet at I where the rays appear to be coming from –the virtual image of O. Thus, the observer sees the object as though it was at I.

The apparent depth is the distance of I from the surface of the water while the distance of the object O from the surface is called the real depth.

In the same way, if a thick glass block is placed on a printed page, the letters under the block will appear raised relative to those not under the block. This is already illustrated in Fig. 3.1b.

### 3.3.2 Relation of Refractive Index to Real and Apparent Depths

We can show how the real and apparent depths are related to the refractive index of the medium. You know that the refractive index for rays of light traveling from air to water is given by the relation

$$n = \frac{\sin i \text{ (in air)}}{\sin r \text{ (in water)}}$$



$\sin r$  (in water)

Suppose an incident ray from an object O in water bends away from the normal as it travels from water to air in accordance to refraction effects. This is illustrated in Fig. 3.11.

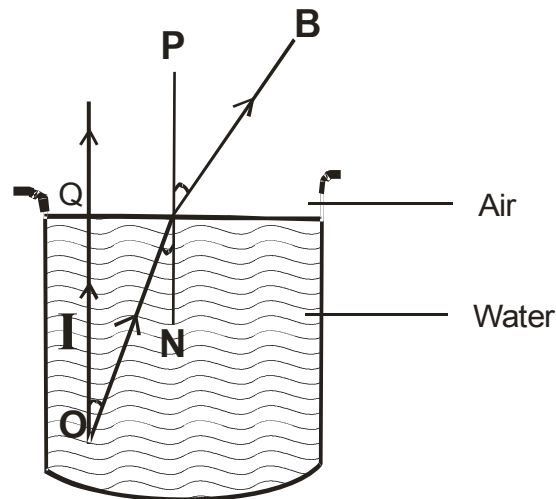


Fig. 3.11: Relation between refractive index and real and apparent depths (Points Q and A are close to each other)

From the principle of reversibility of light, light traveling in the opposite direction from air to water would follow the same path.

Recall that:

$$\text{Refractive index, } \eta = \frac{\sin i}{\sin r}$$

This implies that

$$\eta = \frac{\sin \angle \text{PAB}}{\sin \angle \text{OAN}}$$

But

$$\angle \text{PAB} = \angle \text{QIA} \text{ (corresponding angles)}$$

and

$$\angle \text{OAN} = \angle \text{QOA} \text{ (alternate angles)}$$

Therefore

$$\begin{aligned} \eta &= \frac{\sin \angle \text{QIA}}{\sin \angle \text{QOA}} \\ &= \frac{\text{QA} / \text{AI}}{\text{QA} / \text{AO}} = \frac{\text{QA} \times \text{AO}}{\text{AI} \times \text{QA}} \end{aligned}$$



$$= \frac{AO}{AI}$$

Since the eye sees both rays OQ and OA, they must be close to each other.

Hence

$$AL = QI \text{ and } AO = QO$$

$$\Rightarrow \eta = \frac{QO}{QI}$$

That is,

$$\eta = \frac{\text{Real depth}}{\text{Apparent depth.}}$$

The above is a general formula and can be used to calculate the apparent depth if the refractive index and the real depth of a medium are known.

### 3.4 Measurement of Refractive Index

#### 3.4.1 Measurement of Refractive Index of Solid (glass)

Using

- (i) Snell's law
- (ii) Real and apparent depths
- (i) By Snell's law

This follows same procedure as the verification of Snell's law already discussed and illustrated in section 3.2.2.

- (ii) By real and apparent depths  
The refractive index of glass can be determined by the real and apparent depth method. This is illustrated in Fig. 3.12.

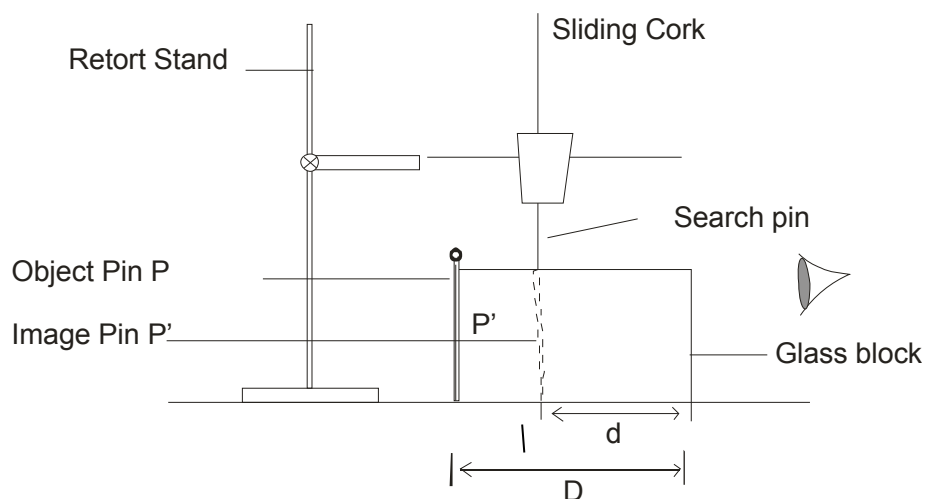




Fig. 3.12: Finding the refraction index of glass block by real and apparent depth methods

This is done by fixing an object pin P with plasticine vertically touching one side of a rectangular glass block mounted on a cardboard paper placed on a laboratory table. A search pin on a sliding cork is then mounted above the rectangular glass block using retort stand and clamp as shown in Fig. 3.12. The object pin P is then viewed through the block from the opposite side of the pin as the search pin is moved back and forth until there is no parallax between the object pin P and its image P<sup>1</sup> as seen through the glass block. The real thickness (depth) D and the apparent thickness (depth) d of the glass block are measured. The refractive index n of the glass is then calculated using the formula:

$$\text{Refraction index, } n = \frac{\text{real thickness(depth)}}{\text{apparent thickness (depth)}} = \frac{D}{d}$$

### 3.4.2 Measurement of Refractive Index of Liquid (water)

#### Using

- (i) Snell's law
- (ii) Real and apparent depths

- (i) By Snell's Law

A straight – sided transparent Perspex box of water is placed on a sheet of cardboard paper and its outline ABCD is drawn using a sharp pencil. A ray box is then adjusted to give a narrow beam of light. The narrow beam of light is directed to strike the box of water surface AB at point O at an acute angle such that the incident ray PO makes an angle of incidence i say 60° to the normal ON.

The light beam comes out into the air at R in the same direction as the incident beam at O but is displaced sideways. The point Q and R are marked on the emergent ray. The water box is then removed and the points PORQ are joined as shown in Fig. 3.13.



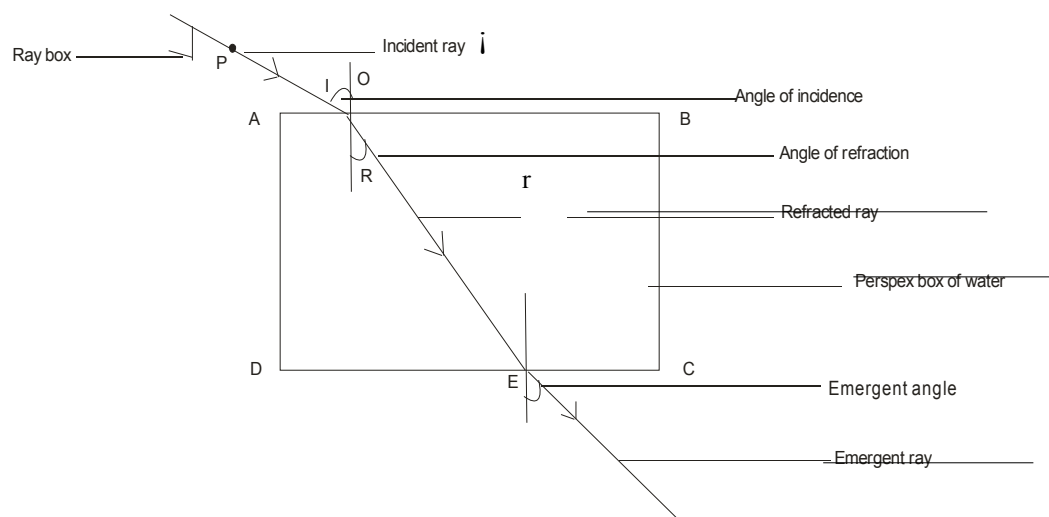


Fig. 3.13: Finding the refractive index of water using Snell's law

The angle of incidence  $i$  and the corresponding angle of refraction  $r$  are measured and recorded. Following  $r$  are same procedure, four different values of the angles of incidence say  $50^\circ$ ,  $40^\circ$ ,  $30^\circ$  and  $20^\circ$  and their corresponding angles of refraction are obtained. The result is then tabulated in a composite table such as Table 3.2 below.

Table 3.2: Experimental values of the incident angle ( $i$ ) and their corresponding angles of refraction ( $r$ ) for a box of water

Incident Angle $i^\circ$	Refracted Angle $r^\circ$	Sin $i$	Sin $r$	$n = \frac{\sin i}{\sin r}$
$60^\circ$				
$50^\circ$				
$40^\circ$				
$30^\circ$				
$20^\circ$				
		Average $\eta$		

The refraction index is obtained by either plotting a graph of  $\sin i$  against  $\sin r$  as in section 3.2.2 or finding the average of  $\eta$ .

(ii) By real and apparent depths.

We can also find the refractive index of a liquid (water) by the real and apparent depth methods illustrated in Fig. 3.13



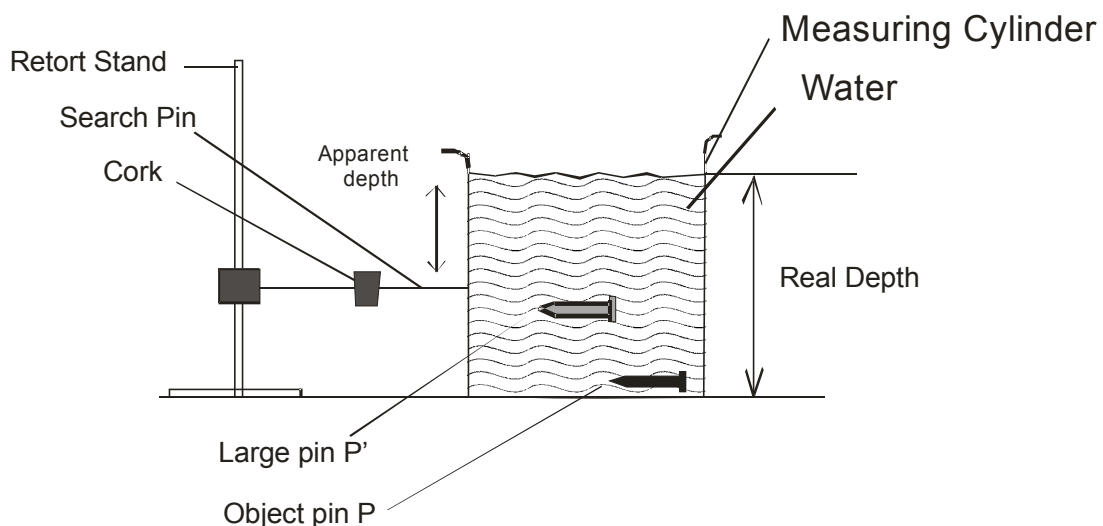


Fig. 3.14: Finding the refractive index of water by real and apparent depth methods

In this method an optical pin is placed at the bottom of a beaker to serve as the object P. Water is then poured into the beaker until it is about three – quarters full. A search pin is attached horizontally to a sliding cork and it is moved up and down until it coincides with the apparent position of the pin as seen from above the liquid. At this point there is no parallax between the object pin p and the image pin P<sup>1</sup>. The real and apparent depths of the pin from the surface of the water are then measured and the refractive index of the liquid (water) is calculated using the formula:

$$\text{Refractive index, } \eta = \frac{\text{real depth}}{\text{apparent depth}}$$

### 3.4.3 Determination of the Refractive Index of Glass by the Method of Minimum Deviation of Light through a Glass Prism

In this experiment, the path of a single ray of light through a triangular prism is traced using either the optical pins no – parallax method or a ray box. The angle of incidence  $i$  of say  $30^\circ$  of the ray of light at the face AB and the corresponding angle of emergence  $e$  at the face AC are obtained, measured and recorded by drawing normal at G and L respectively. This is done by joining the emergent ray HK and producing it to meet the incident ray FG produced at a point M. the angle of deviation  $d$  is also measured and recorded. This illustrated in Fig. 3.14.



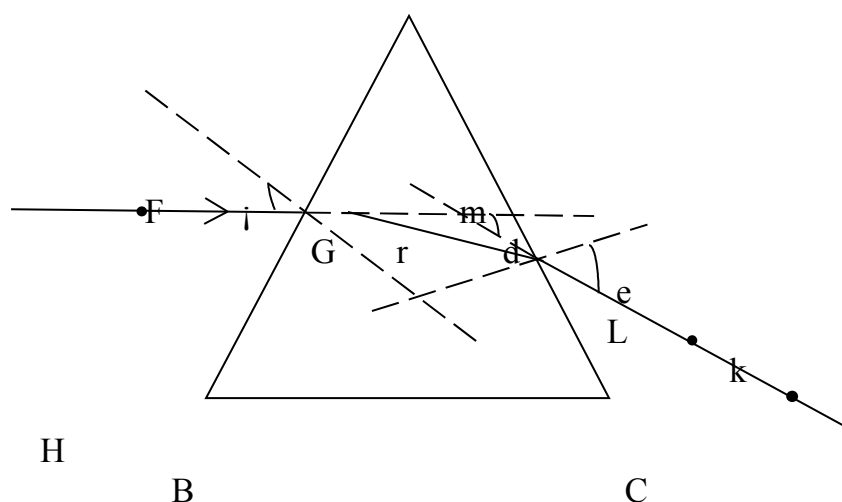


Fig. 3.15: Determination of refractive index of glass using triangular prism

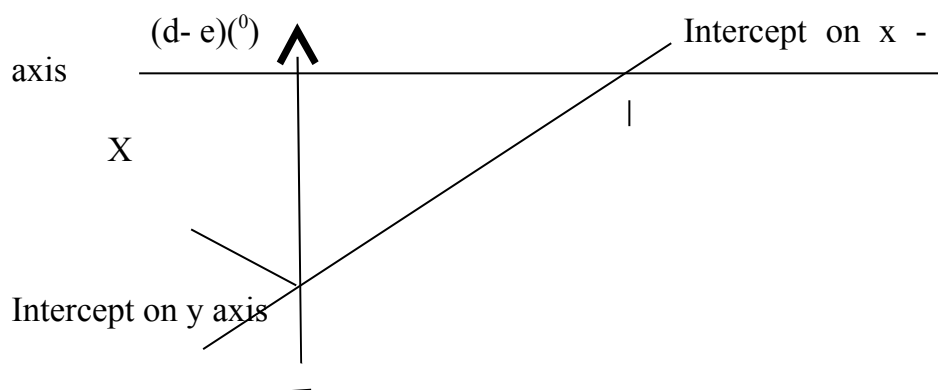
The above procedure is repeated for several other values of angles of incidence  $i$  say  $40^\circ$ ,  $45^\circ$ ,  $50^\circ$ ,  $55^\circ$  and  $60^\circ$ , and the corresponding values of emergent angles  $e$  and angles of deviation  $d$  are measured and recorded in each case. The result is tabulated in the headings shown in Table 3.3 below:

Table 3.3: Experimental values for determining refractive index using triangular prism

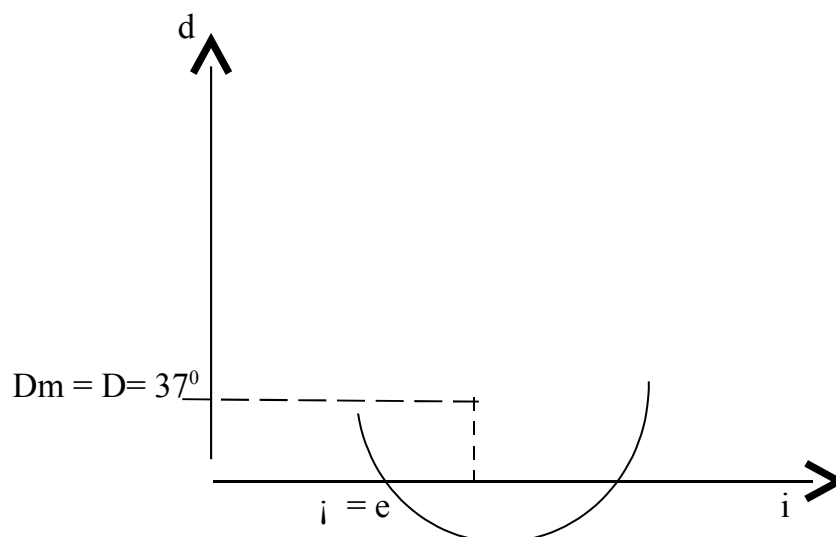
Angle of incidence $i$ ( $^\circ$ )	Angle of emergent $e$ ( $^\circ$ )	Angle of deviation $d$ ( $^\circ$ )	$(d - e)$ ( $^\circ$ )
30			
40			
45			
50			
55			
60			

A graph of  $(d - e)$  on the vertical axis against  $i$  on the horizontal axis is then plotted from which the intercepts on both axes are determined. The mean of these intercepts is determined, using the magnitude of the intercepts (ie disregarding the negative sign). This gives the value of  $A$ , the refracting angle of the triangular prism. That is, the mean of the values of the magnitude of the intercepts on both axes is equal to the refracting angle  $A$  of the triangular prism.



Fig. 3.16: Graph of  $(d-e)$  against  $i$ 

A graph of  $d$  on the vertical axis against  $i$  on the horizontal axis is also plotted. From this graph (Fig.3.17), the minimum value of  $d$  (ie  $D$ ) is determined and the value of  $i$  at this minimum deviation  $D$  is read from the graph.

Fig. 3.17: Graph of  $d$  against  $i$  showing minimum deviation  $D$ 

These values of  $A$  and  $D$  are then substituted in the equation

$$\eta = \frac{\sin \frac{1}{2} (A + D)}{\sin \frac{1}{2} A}$$

From which the value of the refraction index  $n$  of the glass is then calculated.

The result of experiments show that as the angle of incidence increases, the angle of deviation,  $d$ , decreases, until at a certain minimum value,  $37^\circ$ , the angle of deviation begins to increase for any further increase in the angle of incidence. Hence the graph is a curve.



In this experiment the refractive index  $n$  of the prism material can also be worked out from the values of angles of incidence  $I$  and the corresponding angles of refraction  $r$  from the relation.

$$\text{Refractive Index, } n = \frac{\sin i}{\sin r}$$

Either by plotting the graph of  $\sin i$  against  $\sin r$  as in section 3.2.2 or by finding the mean value of the ratio of  $\frac{\sin i}{\sin r}$  for each set of reading as in

section 3.4.2

### 3.5.1 Critical Angle and Total Internal Reflection

When ever light is incident on the interface between two media, there is usually a weak reflected ray in addition to the refracted ray (Fig. 3.18)

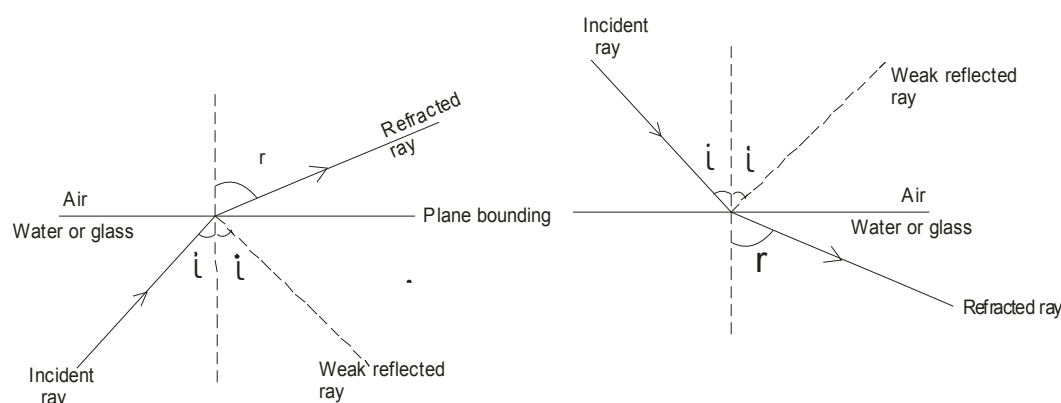


Fig. 3.18: Refraction and partial reflection

A striking change occurs when the situation is reversed optically so that light passes from glass to air.

When light passes through an optical dense medium to meet the surface of less dense one, as the angle of incidence  $I$  is increased, the angle of refraction  $r$  also increases until it is  $90^\circ$ . In that case, the refracted ray is along the interface (surface) of the two media.

- Critical angle  $c$  is the angle of incidence in the denser medium for which the angle of refraction in the less dense medium is  $90^\circ$ . The value of the critical angle  $C$  depends on the nature of the two media or their relative refractive indices. (It is about  $42^\circ$  for crown glass the most common type of optical glass).
- Total internal reflection occurs when the incident angle  $I$  is more than the critical angle  $C$ . At this point all incident light is



reflected back to give strong reflected ray. There is no refraction. Hence total internal reflection (Fig. 3.19).

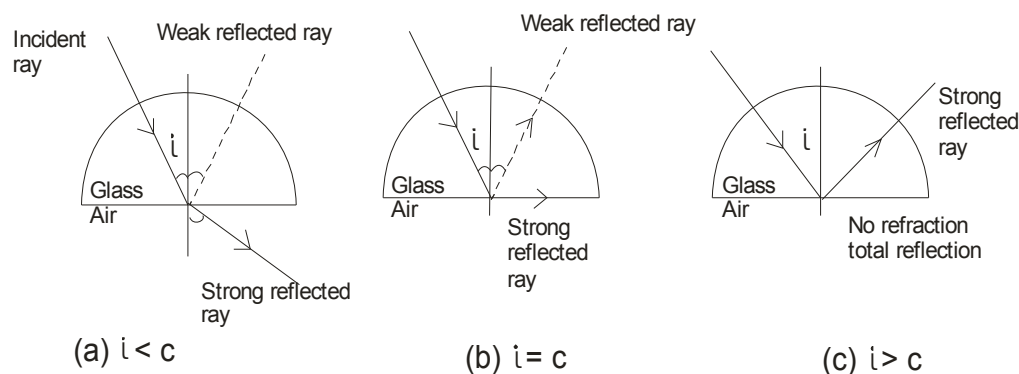


Fig. 3.19: Illustration of total internal reflection

Total internal reflection cannot occur when light is incident in air on a glass block. This is because the angle of refraction in the glass is always less than the angle of incidence. Therefore, even when the angle of incidence in air is nearly  $90^\circ$ , a refracted ray is obtained.

Reflection always occurs at air- glass boundary but this reflection is only partial because most of the light passes into the glass (water). It is therefore important to note that: For total internal reflection to occur

- Light must travel from an optically denser medium to an optically less dense medium;
- The angle of incidence in the denser medium must be greater than the critical angle.

### 3.5.2 Relationship between Critical Angle C and Total Internal Reflection

You will recall that from Snell's law we have for a ray passing from:

- air to glass,  $n_g = \frac{\sin i}{\sin r}$

Therefore, applying the principle of reversibility of light, for a ray passing from:

- Glass (denser medium) to air (less dense medium),  
 $n_a = \frac{\sin r}{\sin i}$



That is, the refractive index  $n$  of the denser medium is given by

$$n = \frac{\sin r}{\sin i}$$

When  $i = C$ , the critical angle  
 $r = 90^\circ$

$$\text{Hence } n = \frac{\sin 90^\circ}{\sin C} = \frac{1}{\sin C}$$

$$\begin{aligned} \text{Since } n &= 1/\sin C \\ \text{ie } n \sin C &= 1 \\ \sin c &= 1/n \end{aligned}$$

In general, refractive index  
 $n = 1/\sin(\text{critical angle})$

$$\Rightarrow \sin C = 1/n$$

Generally, the critical angle in the denser medium is given by  
 $\sin C = n_a/n_b$

Where  $n_b$  is the absolute refractive index for the optical denser medium.

$n_a$  - is the refractive index for the less dense medium.

Since  $C$  cannot be greater than unity  $n_b$  must always be greater than  $n_a$ .

### 3.5.3 Applications of Total Internal Reflection

Total internal reflections are the basis for explaining the working principle of some of the phenomena we experience in nature and some optical instruments. For instance the working of prism periscope, prism binoculars, bicycle reflectors, reflectors on road, light guides optical fiber are explained on the basis of total internal reflection. Also, the phenomena of image, fish and cat's eyes view are some natural observations that are explained on the basis of total internal reflection.

Now let us consider some of them:

#### (i.) Optical fiber

The principle of total internal reflection is used in optical fibers. The optical fiber (Fig.3.20) is made up of a fine strand of glass coated with another type of glass of lower refractive index. It is constructed in such a



way that light rays falling on the interphase between the two types of glass always undergo total internal reflection as shown in Fig. 3.20.

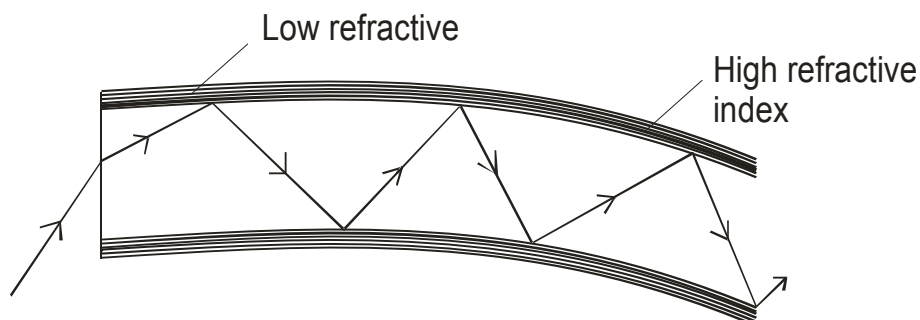


Fig. 3.20: Optical fiber action

When light enters one end of the fibre at an angle less than say  $60^\circ$ , it is refracted towards the normal by the inner glass. The refracted ray impinges on the boundary between the two types of glass and undergoes total internal reflection. The light emerges at the other end after undergoing many repeated total internal reflections at the walls of the optical fibre.

### Uses

- They can be used as light guides to enable the image of an object at one end to be seen at the other.
- They can be fitted with an objective and an eyepiece and inserted into the stomach or throat to study the internal structure.
- They are used for electrical and engineering purposes such as in telecommunications and TV transmission, the electrical impulses are converted into light signals which are then transmitted via these fibres.

### (ii) Mirage

Mirage is one of the natural phenomena that can be explained on the basis of total internal reflection. This occurs at the boundary of a hot and a cold gas. On a hot day for instance the air in contact with the ground gets very much hotter than the layer above the ground. The hotter air is optically less dense than the colder air above the ground. The refractive index of air depends on its intensity. Hence, the lower hotter layers have a lower refractive index than the layers above them. Rays of light passing through these different layers of air are refracted due to the gradual change in density. Therefore a ray from part of the sky passing from colder to warmer air layer (from denser to a less dense medium) bends gradually from the incident direction until it enters a layer of air near the ground at B where total internal reflection occurs. This ray is then reflected upwards into the dense air as illustrated in Fig. 3.2.



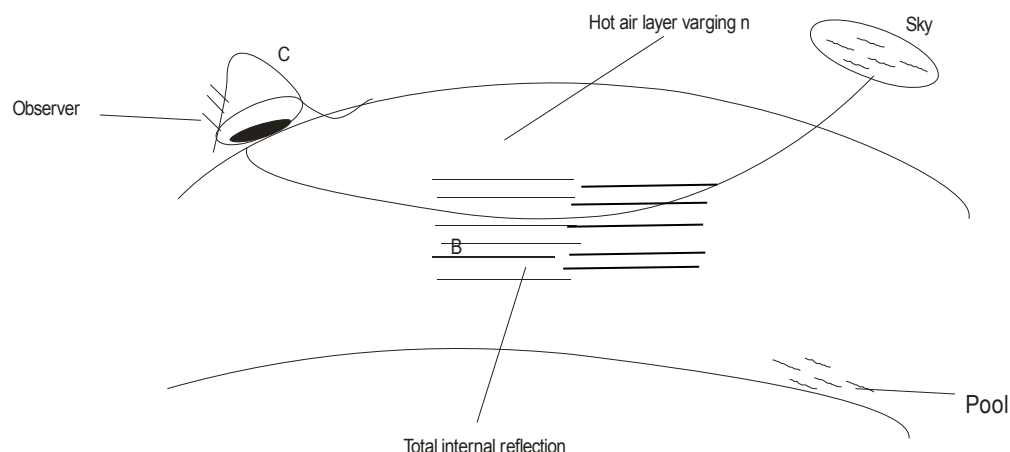


Fig. 3.21: Image

After undergoing refraction in a gradual upward direction, the ray enters the eye of an observer at C as if it comes from A. It gives the illusion of a reflecting pool of water in the road some distance away. I am sure you must have experienced this illusion on a hot day while on a car moving on a tarred road.

People may sometimes see illusions of inverted images of trees in a pool of water below the actual tree for the reason explained above especially around hot deserts because of the intense heat there.

### 3.6 Numerical Examples

- (i.) The apparent depth of a liquid in a big jar is 6 m when its real depth is 8 m. What is the refractive index of the liquid?

#### Solution

$$\text{Refractive index of liquid} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\text{ie } n = \frac{\text{real depth}}{\text{apparent depth}}$$

$$= \frac{8}{6}$$

$$= \frac{4}{3}$$



- ii The angle of incidence of a ray of light striking an equilateral triangular prism is  $45^\circ$ . If the refractive index of the prism is 1.5, and the refracting angle is  $60^\circ$ , calculate
- the angle of refraction at the first face,
  - the angle of emergence

### Solution

(a) let  $r$  be the angle of refraction at the first face, AB

$$\text{using } n = \frac{\sin i}{\sin r}$$

$$1.5 = \frac{\sin 45^\circ}{\sin r}$$

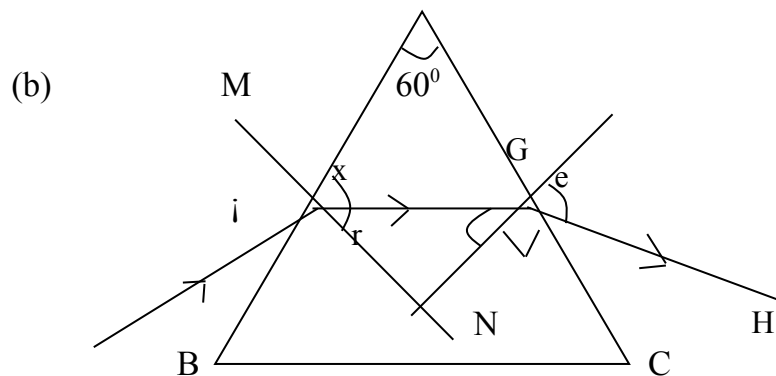
$$\sin r = \frac{\sin 45^\circ}{1.5}$$

$$= \frac{0.7071}{1.5}$$

$$= 0.4714$$

$$\therefore r = \sin^{-1} 0.4714$$

$$\text{Angle of refraction } r = 28.13^\circ$$



Let the emergent angle be  $e$  as shown above.

$$x = 90^\circ - r \text{ (MN normal to AB)}$$

$$= 90^\circ - 28.13^\circ$$

$$= 61.87^\circ$$



$$90^\circ + y = 60^\circ + x \quad (\text{exterior angles of triangle AFG})$$

$$y = 60^\circ + 61.87^\circ - 90^\circ$$

$$= 31.87^\circ$$

Note that the incident ray at the second face AC is FG and the refracted ray is GH. Thus applying the principle of reversibility of light, a light ray incidence on AC along HG will be refracted along GF.

Hence

$$\text{Refractive of index prism} = \frac{\sin e}{\sin y}$$

$$\text{ie} \quad 1.5 = \frac{\sin e}{\sin y}$$

$$\sin e = 1.5 \times \sin 31.87^\circ$$

$$= 1.5 \times 0.5280$$

$$e = \sin^{-1} 0.7920$$

$$\text{Angle of emergence } e = 52.37^\circ$$

### SELF ASSESSMENT EXERCISE 1

1. What do you understand by the term refraction of light?
2. With the aid of a diagram explain the refraction of light through rectangular glass block.
3. State the laws of refraction and explain how you can measure the refractive index of water using Snell's law.
4. What do you understand by the concepts? Critical angles and total internal refraction? What is the relationship between the two concepts?
5. Mention five applications of total internal reflection and explain any two of them.

### 4.0 CONCLUSION

In this unit you studied the refraction of light. You learned the refraction of light at both the rectangular and triangular glasses. Also you learned about the laws of refraction and how to verify them. In addition, you learned about real and apparent depths and how to use them in finding the refractive index of both water and glass block.



Finally, you learned about the critical angle and total internal reflection as well as the applications of total internal reflection.

## 5.0 SUMMARY

- When light passes from one medium to an optically denser medium it is refracted towards the normal. When it passes to an optically less dense medium it is refracted away from the normal.
- Laws of refraction
  - (i) The incident ray, the refracted ray and the normal at the point of incidence all lie in the same plane.
  - (ii) The ratio of the sine of angle of incidence to the sine of the angle of refraction is a constant for a given pair of media. (Snell's Law).
- The ratio  $\frac{\sin i}{\sin r}$  is called the refractive index  $n$ .

$$n = \frac{\sin i}{\sin r}$$

- The ratio real depth / apparent depth is equal to the refractive index  $n$  at near normal incidence.
- Critical angle  $C$  is the angle of incidence in the denser medium for which the angle of refraction in the less dense medium is  $90^\circ$ .
- Total internal reflection occurs when the incident angle  $i$  is more than the critical angle  $C$  and light travels, from denser to less dense medium.

## 6.0 TUTOR -MARKED ASSIGNMENT

1. State the laws of refraction and illustrate the second law with the aid of diagram.
2. With the aid of diagram explain the meaning of real and apparent depth.
3. Define the following terms
  - (i) Critical angle
  - (ii) Total internal reflection.

## 7.0 REFERENCES/ FURTHER READINGS

- Anyakoha, M.W. (2000). New School Physics for Senior Secondary Schools Onitsha: Africana FEP.
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## **UNIT 4    REFRACTION    AT    CURVED    SURFACES (LENSES)**

### **CONTENTS**

- 1.0 Introduction
- 2.0 Objectives
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  - 3.1 Refraction through Lenses
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    - 3.1.2 Terms used in Lenses
    - 3.1.3 Rules for Constructing Images Formed by
  - 3.2 Formation of Images by Lenses
    - 3.2.1 Formation of Images by Convex (converging) Lens
    - 3.2.2 Formation of Images by Concave (diverging) Lens
  - 3.3 Practical Applications (use) of Convex (converging) Lens
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- 6.0 Tutor-Marked Assignment
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### **1.0 INTRODUCTION**

In this unit, you will learn about lenses, how to obtain images through lenses, the relationship between object distance, image distance and the focal length of a lens. Also, you will learn the derivation and use of the lens formula and how to solve problems on lenses both graphically and analytically.



## 2.0 OBJECTIVES

By the time you finish this unit, you will be able to:

- obtain images due to light rays through converging and diverging lenses using:
  - (a) ray tracks; and
  - (b) ray tracing methods.
- derived and use the lens formula to solve numerical problems on lenses
- use the ray – box to show relationship between object distance and the focal length of a lens
- solve problems on lenses both graphically and analytically.

## 3.0 MAIN CONTENT

### 3.1 Refraction through Lenses

Refraction through lenses involves the change in direction of light rays. A lens is any transparent material with two faces of which at least one is curved. It causes a beam of light passing through it to either converge or diverge.

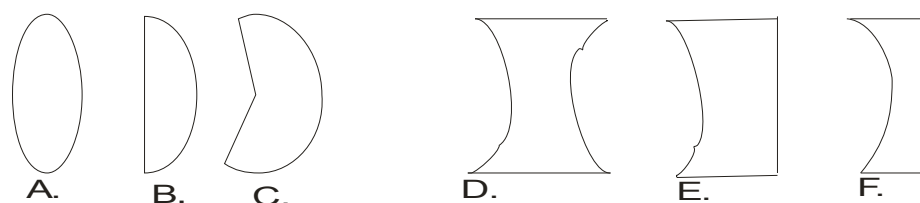
#### 3.1.1 Types of Lenses

We have two types of lenses namely convex lens and concave lens.

- A convex lens is a converging lens. It makes a parallel beam of light falling on it to converge. It is thicker at its centre than at its edges.
- A concave lens is a diverging lens. It makes parallel beams of light falling on it to appear to diverge from a point. It is thinner at its centre than at the edges.

We have various forms of convex and concave lenses but the most commonly used ones in school laboratory experiments are the biconvex and the biconcave lenses. Plano-convex and Plano – concave lenses are used in optical instruments while concavo – convex and convexo – concave are used as contact lenses to correct defective vision. Most lenses are made from glass, a few are made with quartz or plastic. The eye has a crystalline biconvex lens. These are shown in Fig. 4.1.





Convex lenses

Concave Lenses

(Converging Lenses)

(Diverging Lenses)

- |                      |                       |
|----------------------|-----------------------|
| (a) biconvex         | (d) biconcave         |
| (b) Plano – convex   | (e) Plano - concave   |
| (c) Concavo - convex | (f) convexo – concave |

Fig. 4.1: Types of lenses

### 3.1.2 Terms Used in Lenses

We have some important parameters associated with a lens. They are defined as follows:

- The optical centre (c) (Fig. 4.2) of a lens is the point through which rays of light pass without being deviated by the lens.
- The principal axis is the line passing through the optical centre of the lens and joining the centre of curvature of its surfaces.
- The principal focus (f) of a converging or convex lens is the point to which all rays parallel and close to the principal axis converge after refraction through the lens.
- The principal focus of a diverging or concave lens is the point from which all rays parallel and close to the principal axis appear to diverge after refraction through the lens.
- A lens has two principal foci because light may pass through a lens from either direction.
- Focal length (f) is the distance between the optical centre and the principal focus of the lens.
- The power (p) of a lens is equal to the reciprocal of the focal length and is measured in diopeters when f is in meters. The shorter the focal length f the greater will be the power of the lens.
- The principle of reversibility of light states that, the path of a light ray is reversible. Because of the principle of reversibility of light, rays originating from f and passing through the convex lens will emerge parallel to the principal axis.



- The principal focus of a converging lens is real, that of a diverging lens is virtual.
- Lenses whose thicknesses are small in comparison with their focal lengths are referred to as thin lenses. For such lenses, the two foci are symmetrically placed with respect to the optical center of the lens.

The terms associated with lenses are illustrated in Fig. 4.2 below.

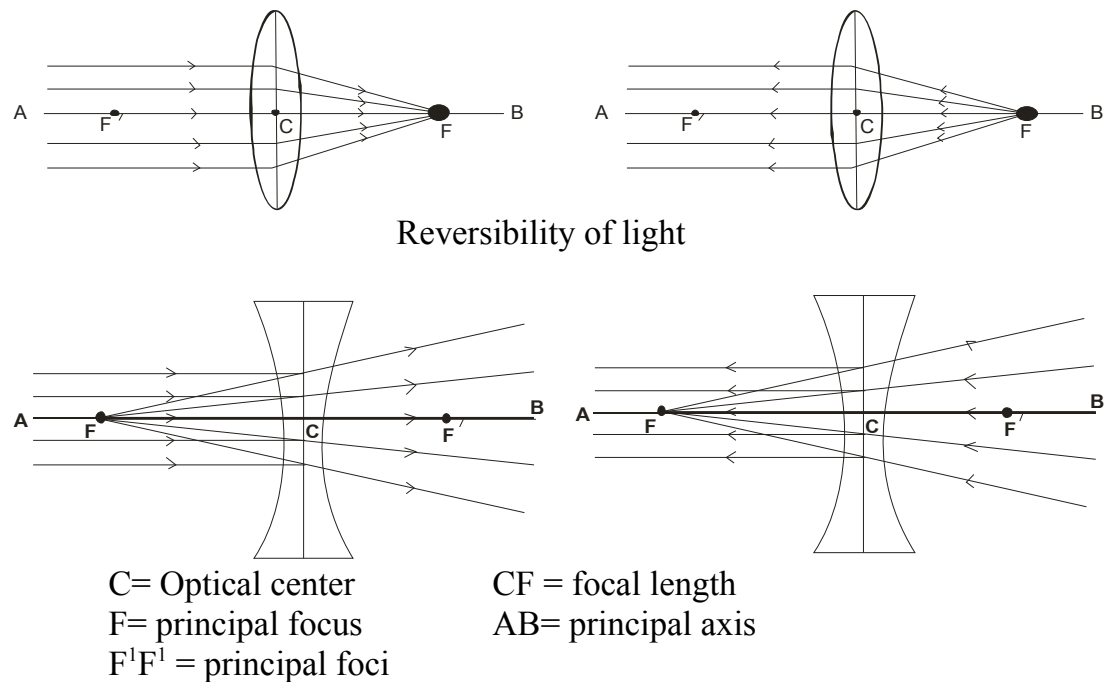


Fig. 4.2: Terms associated with lenses

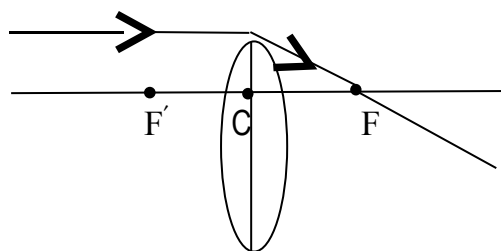
### 3.1.3 Rules for Constructing Images Formed by:

- (i) Convex (Converging) lens
  - (ii) Concave (Diverging) lens
- (i) Rules for convex (converging) lens.

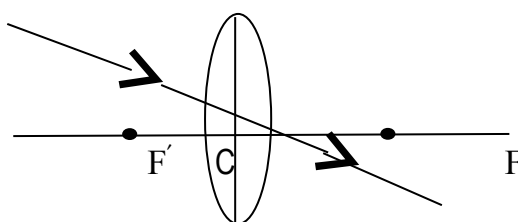
You will obey the following rules when constructing images formed by convex (converging) lens.

Rule 1: Incident rays parallel to the principal axis are refracted through the principal focus.

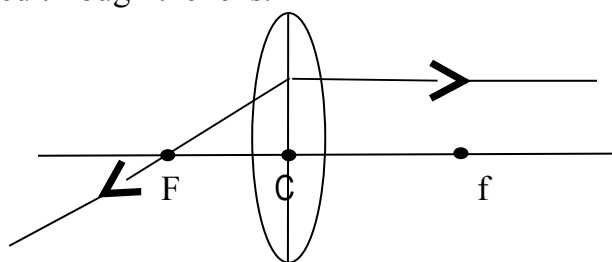




Rule 2: Incident rays passing through the optical center are undeviated



Rule 3: Incident rays passing through the principal focus emerge parallel to the principal axis after being refracted through the lens.

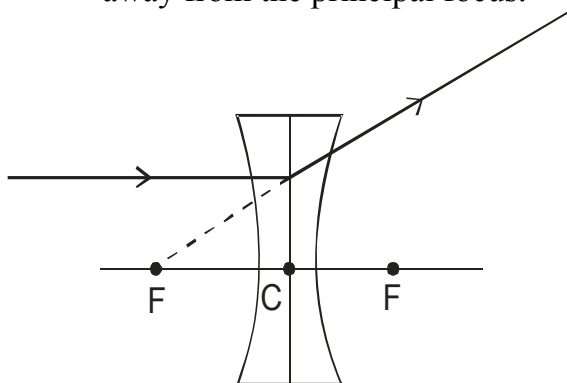


You can use any two of the three rays to locate an image.

(ii) Rule for concave (diverging) lens.

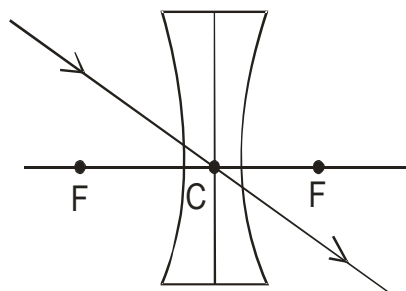
The following rules will guide you when constructing images formed by concave (diverging) lens.

Rule 1: Incident rays parallel to the principal axis are refracted away from the principal focus.





**Rule 2:** Incident rays passing through the optical centre are undeviated.



### 3.2 Formation of Images by Lenses

You should note that light rays incident on a converging lens converge to form a real image on the other side of the lens after passing through the lens. For a diverging lens, the rays from an object diverge to form a virtual image of the object. When you know the distance of an object from the lens and the focal length of the lens, you can obtain the position and nature of the image by the construction of ray's diagrams. To do this, you will employ the rules for constructing images formed by lenses discussed in 3.1.3 above.

#### 3.2.1 Formation of Images by a Convex (Converging) Lens

You will locate the position and determine the nature of the image formed by a convex lens by following these construction steps.

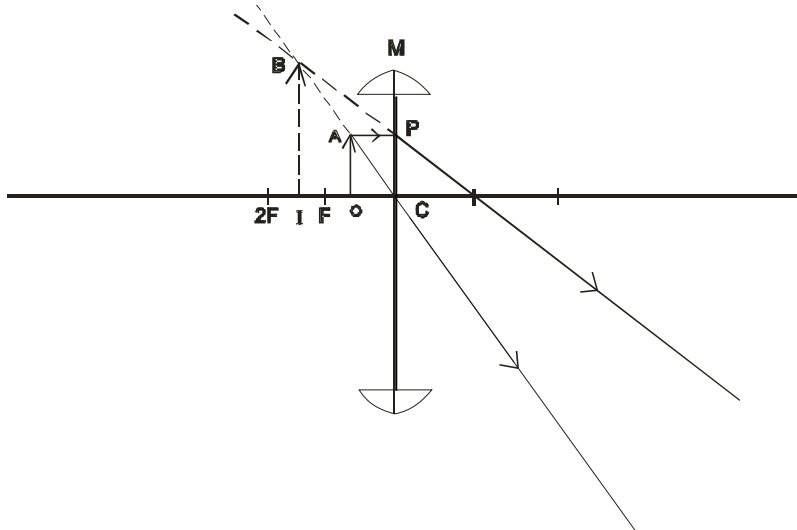
- (i) Choose a suitable scale for the construction
- (ii) Draw a straight vertical line MW (Fig. 4.3.) to represent the position of the lens and mark the position of the focus F according to the scale. Represent the focal length with CF.
- (iii) Represent the object by drawing an arrow line OA perpendicular to OC, OC should correspond to the distance of the object from the lens
- (iv) and OA, the height,
- (v) Draw a ray AP parallel to the principal axis from the top of the OA, According to rule 1, ray AP is refracted through F<sup>1</sup>
- (vi) Draw another ray AC through the centre of the lens C. This ray passes straight through without changing direction (rule 2).
- (vii) The point of intersection B of the two emerging rays PB and CB represents the top of the image.
- (viii) Now, complete the image by drawing B1 normal to the principal axis.
- (ix) Measure the image distance from C and the image height B1, convert to the actual values using your scale.



Ray diagrams for various object positions are presented in Fig. 4.3.

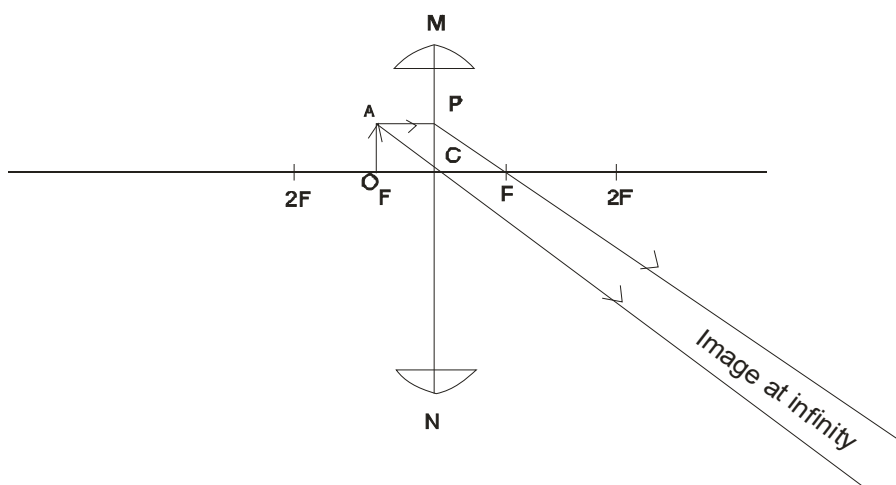
(a) Object between  $F$  and Lens image is:

- Erect
- Magnified
- Virtual
- Beyond the object on same side as object



(b) Object at  $F$ : image is:

- At infinity
- Real
- Inverted
- Magnified

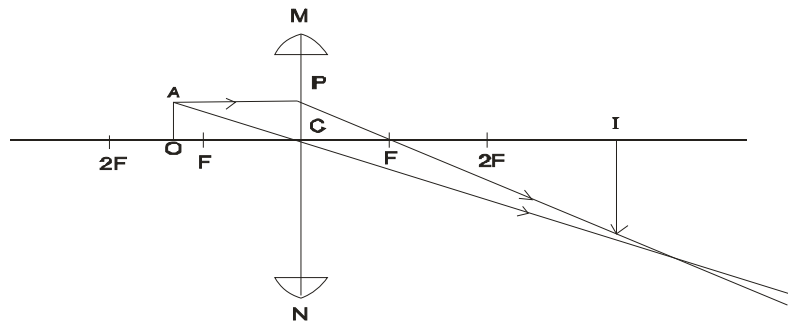


(c) Object between  $F$  and  $2F$  image is:

- Inverted

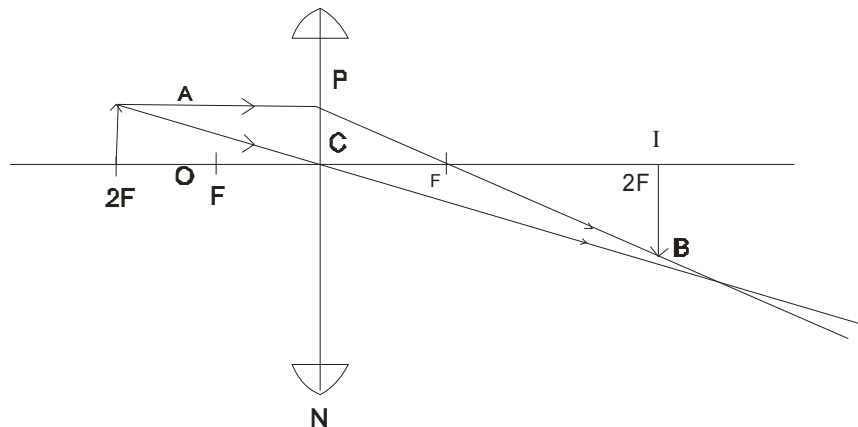


- Magnified
- Real
- Beyond  $2f$



(d) Object at  $2F$  Image is

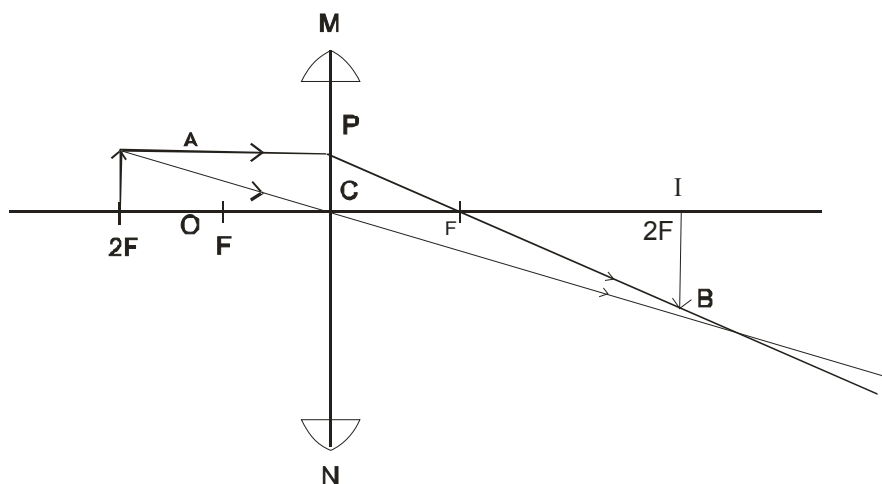
- Inverted
- Real
- Same size as object
- At  $2F$  the other side of the lens.



(e) Object beyond  $2f$  Image is

- Inverted
- Diminished
- Real
- Between  $F$  and  $2F$  on the other side of lens.





(f) Object at infinity image is

- Inverted
- Diminished
- Real
- At F on the other side of lens.

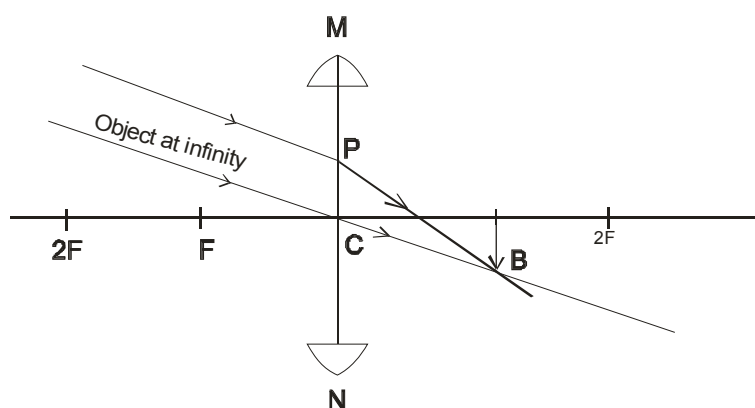


Fig. 4.3: Image formed by convex (converging) lens

### 3.2.2 Formation of Images by a Concave (Diverging) Lens

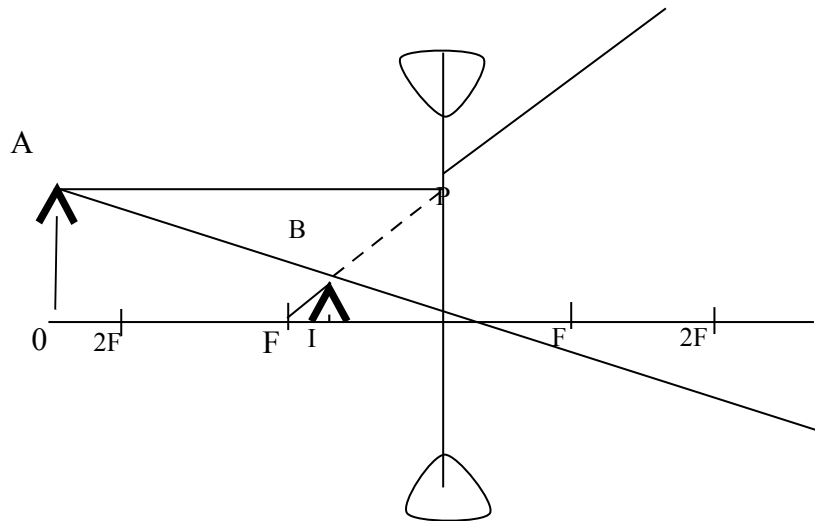
You will observe by drawing ray diagrams for a concave lens that the image is virtual, erect and diminished. It is always on the same side of the lens as the object and between the object and the lens. These image characteristics are for all object positions. This is illustrated in Fig. 4.4. A concave lens is usually used in spectacles for the correction of short sightedness because of its qualities.

(a) Object beyond  $2f$  image is:

- Erect
- Diminished



- Virtual
- Between F and C



(b) Object between F and C image is

- Erect
- Diminished
- Virtual 2F
- Between F and C

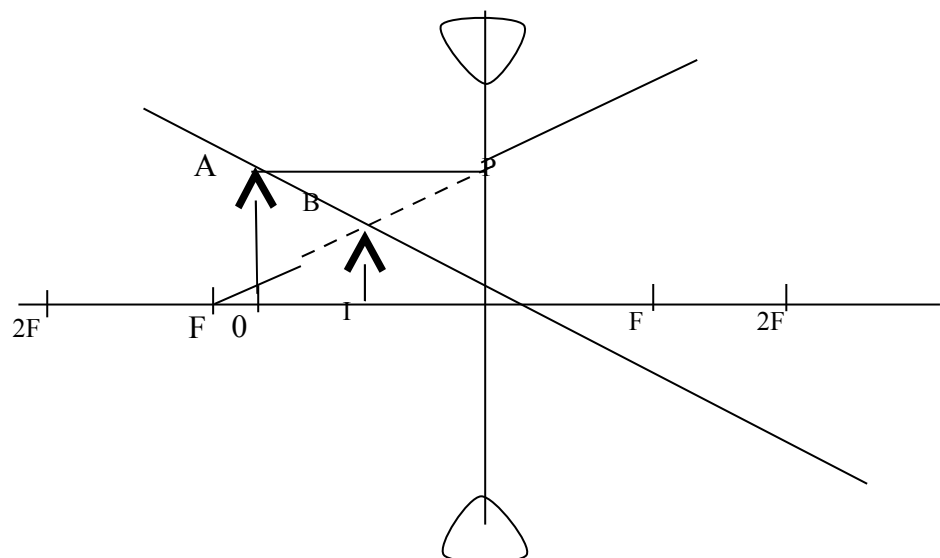


Fig. 4.4: Images formed by concave (diverging) lens

These ray diagrams were constructed using the rules listed in section 3.1.3 (ii).



### 3.3 Practical Applications (uses) of the Convex (Converging) Lens

We use the convex (converging) lens in many optical instruments. Some of them are listed below.

- When the object is between  $F$  and lens, the lens is used as eye lens of many instruments, magnifying glass, spectacles for long sight.
- When object is at  $F$ , it is used at theatre and stadium spotlights.
- When object is between  $F$  and  $2F$ , it is used as objective lens of microscopes and projectors.
- When object is at  $2F$ , it's used in copying camera.

### 3.4 Practical Application (use) of the Concave (Diverging) Lens

A concave lens is usually used in spectacles for short sight.

#### SELF ASSESSMENT EXERCISE 1

1. Define
  - (i) The optical centre
  - (ii) The principal focus
  - (iii) The focal length of a converging lens.
2. What do you understand by?
  - (i) The principle of reversibility of light
  - (ii) The power of a lens.
3. List the similarities and differences between the action of a concave mirror and a convex lens.
4. What are the applications of convex and concave lenses?
5. With the aid of rays diagrams, illustrate the characteristics of images formed by a convex lens when the object is placed at:
  - (i)  $F$
  - (ii)  $2F$
  - (iii) Between  $F$  and  $2F$
6. With the aid of ray diagrams illustrate the characteristics of images formed by a concave lens when an object is placed at two different positions from the lens.

### 3.5 Lens Formulae and Sign Conventions



### 3.5.1 Sign Conventions

You have already learned the sign conventions in Module 4 Unit 2 for the case of curved mirrors. A similar situation applies for lenses. We have that for;

- New Cartesian convention
  - i The optical centre is taken as the origin for measuring all distances.
  - ii Distances measured against the incident light are negative.
  - iii Distances measured in the same direction as the incident light are positive.

If the object is always placed to the left of the lens, this convention implies that all distances to the left of the lens are negative, while all those to the right are positive.

- Real – is – positive convention
  - (i) The optical centre is taken as the origin for measuring all distances.
  - (ii) Distances of real objects and images are positive.
  - (iii) Distances of virtual objects and images are negative.

When you use the real – is – positive convention, you will take the focal length of converging lens to be positive because the lens has a real principal focus. On the other hand, you will take the focal length of a diverging lens to be negative because the lens has a virtual principal focus.

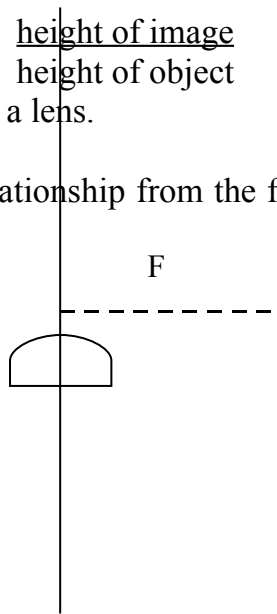
### 3.5.2 Linear Magnification

You will recall that the linear magnification ( $m$ ) produced by mirrors is the ratio.

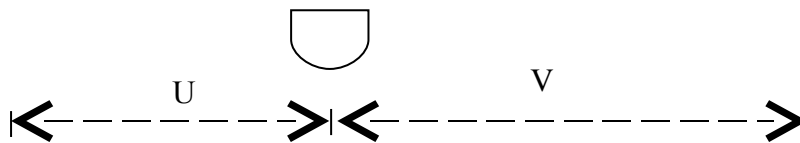
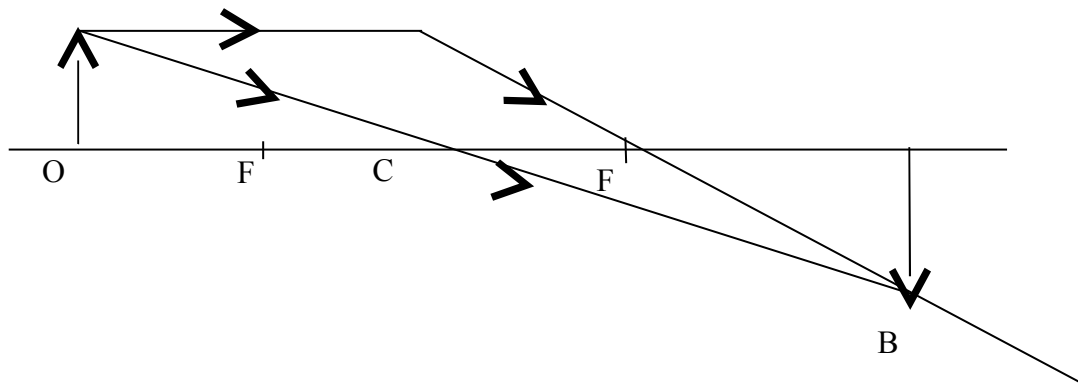
$$M = \frac{\text{height of image}}{\text{height of object}}$$

This is also true for a lens.

You will easily see this relationship from the following ray diagrams in Fig. 4.5.

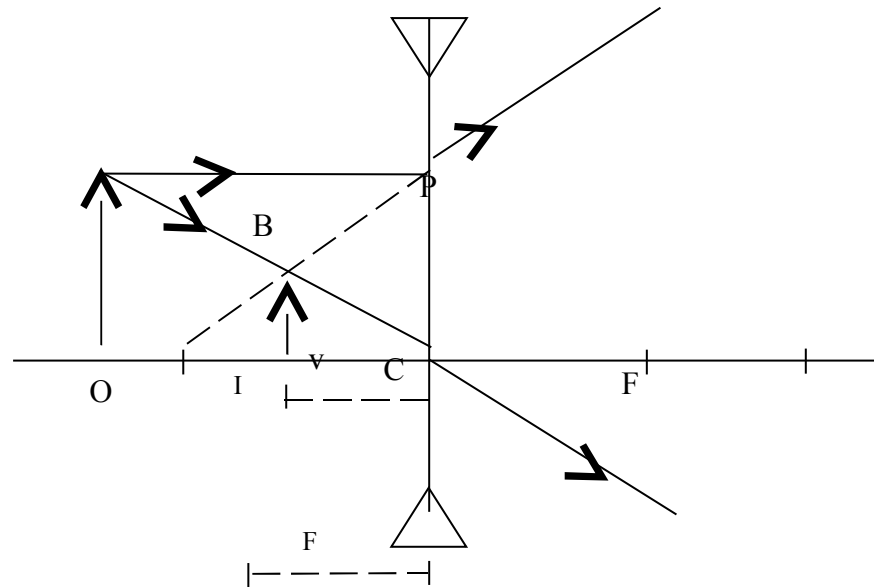








## (a) Converging lens



## (b) Diverging lens

Fig.4.5: Ray diagrams for lens formula

You can see from Fig. 4.5 (a) and (b) that triangles CIB and COA are similar.

It therefore follows that:

$$M = \frac{\text{height of image}}{\text{height of object}} = \frac{IB}{OA} = \frac{IC}{OC}$$

That is,

$$M = \frac{\text{distance of image from lens}}{\text{distance of object from lens}}$$

In symbols,

$$M = \frac{V}{U}$$

Where,

$$\begin{aligned} M &= \text{linear magnification} \\ V &= \text{image distance} \\ U &= \text{object distance} \end{aligned}$$



### 3.5.3 Lens Formulae

We can derive the formulae relating  $u$ ,  $v$ ,  $m$  and  $f$  by either theoretically from the geometry of ray diagrams or practically from the result of experiments.

Here, our derivation is theoretical from the geometry of ray diagrams. Also, the real – is positive sign convention is adopted.

Recall that for thin lenses, distances are measured from the optical centre of a lens. Distances measured to real foci, images and objects are positive, distances measured to virtual foci, images and objects are negative.

From figure 4.5, you can see that:

triangles CIB and COA are similar

also,

triangles FIB and FCP are similar

Therefore,

for the converging lens,

The magnification

$$M = \frac{IB}{OA} = \frac{V}{U}$$

because

$\triangle FIB$  and  $\triangle FCP$  are similar.

Also,

$$M = \frac{IB}{CP} = \frac{FI}{FC}$$

because

$\triangle SIB$  and  $\triangle FCP$  are similar

From figure 4.5 (a)

$$\text{and } \begin{aligned} FI &= v - f \\ FC &= f \end{aligned}$$

This implies that

$$M = \frac{v - f}{f}$$



Therefore,

$$M = \frac{v}{u} = \frac{v-f}{f}$$

From the above, we have that

$$\frac{v}{u} = \frac{v-f}{f}$$

$$vf = uv - uf$$

Dividing through by v gives us

$$\frac{vf}{v} = \frac{uv}{v} - \frac{uf}{v}$$

$$f = u - \frac{uf}{v}$$

Dividing through by u gives us

$$\frac{f}{u} = \frac{u}{u} - \frac{uf}{vu}$$

$$\frac{f}{u} = 1 - \frac{f}{v}$$

Dividing through by f gives us

$$\frac{f}{uf} = \frac{1}{f} - \frac{f}{vf}$$

$$1/u = 1/f - 1/v$$

Rearranging, we have,

$$1/u + 1/v = 1/f$$

The above relation is the lens formula.

For the diverging lens, you can also, by a similar procedure, using the two pairs of similar triangles obtain two expressions for the magnification m.

That is,

$$m = -v/u = \frac{-f+v}{-f}$$

or

$$m = v/u = \frac{-f+v}{f}$$

The above will lead you to

$$1/v + 1/u = 1/f$$

as in that of converging lens.



### 3.6 Determination of the Focal Length of a Converging Lens by the Measurement of Object and Image Distances

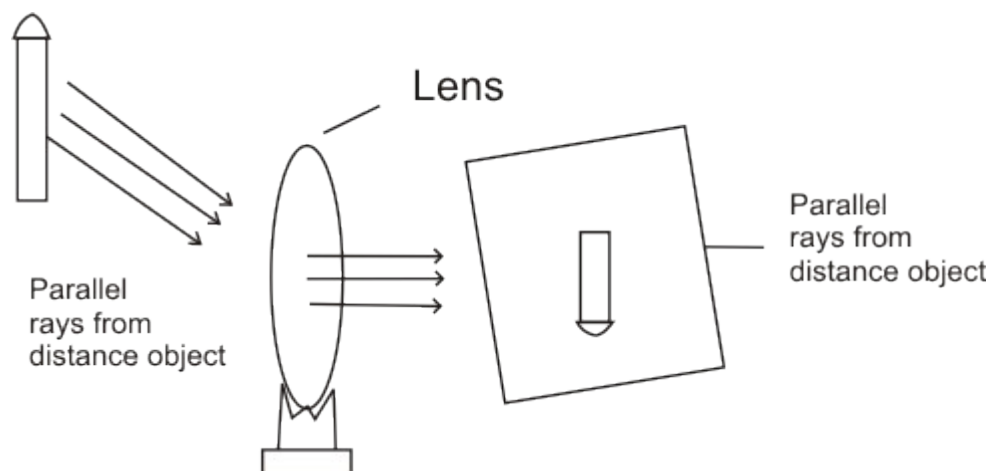


Fig. 4.6 (a): Image of distant object: Quick and approximate methods

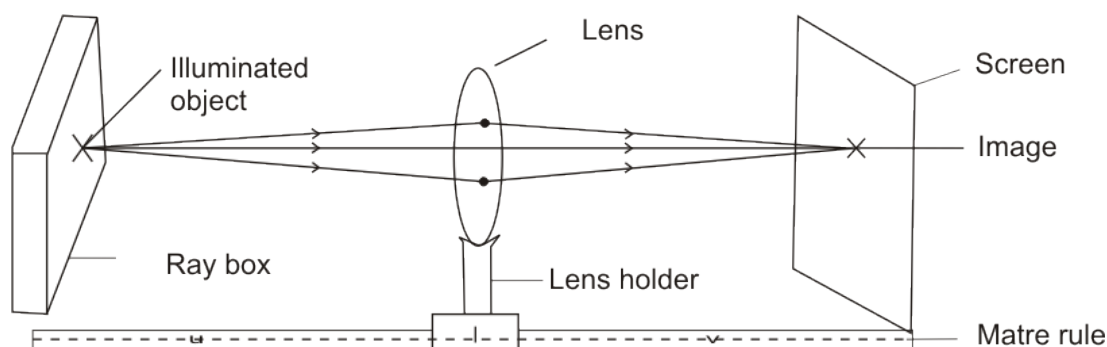


Fig. 4.6(b): Focal length of a converging lens by  $u$ ,  $v$  measurements

You can use the lens formula to obtain the focal length of a converging lens, by measuring the object and image distances. You will first obtain the approximate value of  $f$  by the Quick but approximate method (Fig. 4.6(a)). You will do this by focusing the image of a distant object on a screen using a lens. Place the lens in front of the screen and adjust it until a sharp image of a distant object (Fig. 4.6a) is obtained in the screen. Measure the distance between the lens and the screen which is the approximate value of the focal length. Rays coming from the distant object are parallel rays and therefore converge to the principal focus of the lens after refraction.

Now, having obtained the approximate focal length,  $f$ , of the lens, set up the apparatus as shown in Fig. 4.6(b).

Place an illuminated object at a known distance,  $u$ , greater than  $f$  in front of the lens. Adjust the position of the screen on the other side of the lens until you obtain a sharp image of the object on the screen. Measure the



distances  $u$  and  $v$  from the ray box to the lens ( $u$ ) and the screen to the lens ( $v$ ). Repeat the procedure for five other values of  $u$  and obtain the corresponding values of  $v$ . Tabulate your readings as in Table 4.1.

Table 4.1: Values of  $u$  and the corresponding values of  $v$

	$u$	$v$	$uv$	$u+v$	$uv / (u+v)$
1					
2					
.					
.					
.					
6					

Table A

	$u$	$v$	$1/u$	$1/v$	$1/u + 1/v$
1					
2					
.					
.					
.					
6					

Table B

You can obtain the value of  $f$  from your readings in Table 4.1 in the following ways:

- Add the values of  $uv / (u + v)$  in table A and find the average. The result is the experimental value of  $f$ .
- Add the values of  $1/u + 1/v$  in table A and find the average. The result is  $1/f$  from which you will then determine the values of  $f$ .  $f$  is the reciprocal of the value obtained.

Plot a graph of  $w$  against  $(u + v)$ . You will obtain a straight line graph. The slope of this graph is equivalent to  $f$ . Obtain it (Fig. 4.7<sub>(a)</sub>).



Plot a graph of  $1/v$  against  $1/u$ . You will obtain a graph with intercepts on both axes. Each of the intercepts gives you a value of  $1/f$ . From these values; find the reciprocal to obtain the value of  $f$  (Fig. 4.7(b)).

Fig. 4.7(a) Graph of  $uv$  against  $(u+v)$

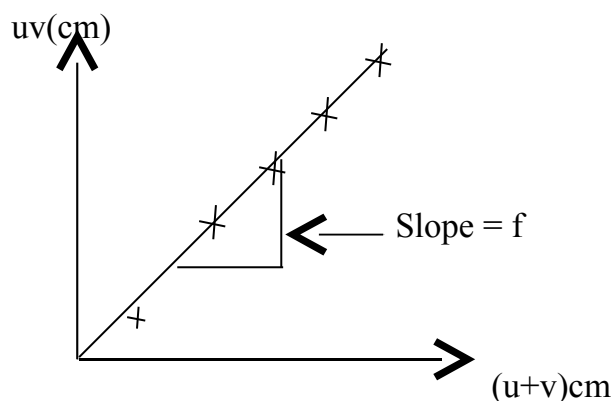


Fig. 4.7(a): Graph of  $uv$  against  $(u + V)$

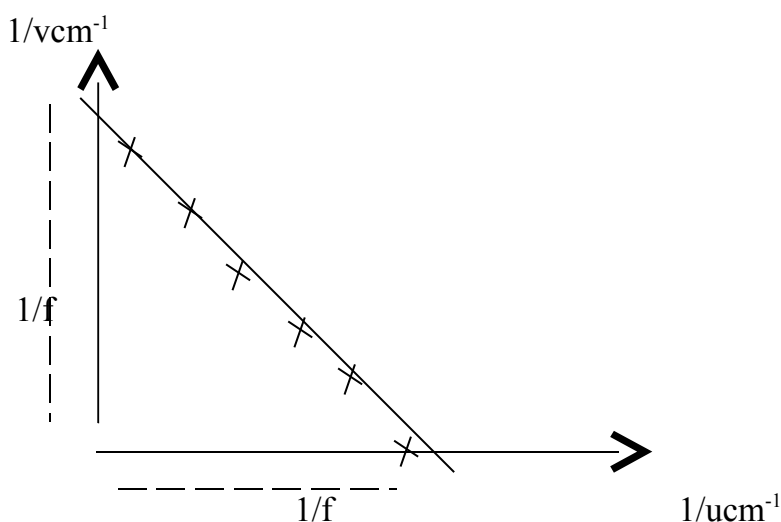


Fig. 4.7(b): Graph of  $1/v$  against  $1/u$ .

You will always take the following precautions when you deal with light rays and optical experiments:

- Use sharp pencil in drawing ray diagrams to obtain neat traces.
- Avoid parallax when measuring with the metre rule.
- Ensure proper alignment of object, lens or mirror and the screen.
- Ensure that a sharp image is properly focused on the screen.
- Ensure that optical pins (if used) are vertical and well spaced.

### Worked examples



1. Find the nature and position of the image of an object placed 15 cm in front of a converging lens of focal length 30 cm which produces a magnified image 16 cm high. What is the size of the object.

### Solution

Using the real – is – positive convention

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{15} = \frac{1}{30}$$

$$\frac{1}{v} = \frac{1}{30} + \frac{1}{15} = \frac{1+2}{30} = \frac{3}{30}$$

$$\frac{1}{v} = \frac{3}{30}$$

$$\Rightarrow v = 10 \text{ cm}$$

The image is virtual and is 10 cm from the lens.

Also,

$$m = \frac{v}{u}$$

$$\frac{\text{height of image}}{\text{height of object}}$$

$$\Rightarrow \frac{v}{u} = \frac{\text{height of image}}{\text{height of object}}$$

Substituting gives us

$$\frac{10}{15} = \frac{16}{\text{height of object}}$$

$$\therefore \text{height of object} = \frac{15 \times 16}{10}$$

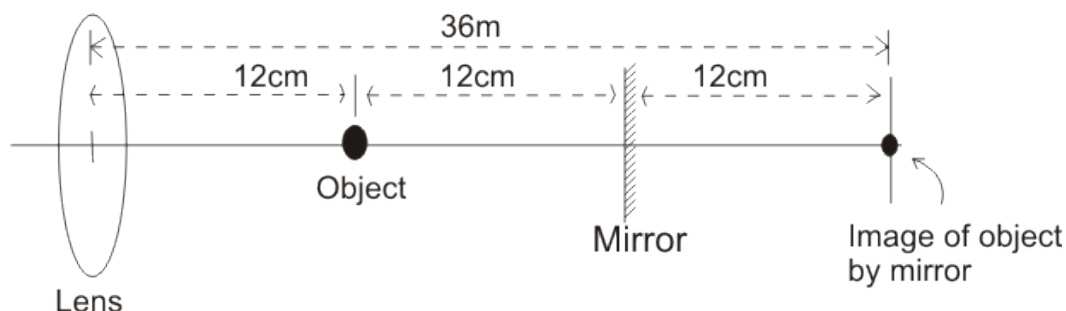
$$= \frac{240}{10} = 24 \text{ cm}$$

The object is 24 cm high



2. An object is placed between a converging lens of focal length 10 cm and a plane mirror, being 12 cm distant from each. Find the positions of two real images formed by the lens on the opposite side from the object.

### Solution



The two real images formed by the lens are:

- (i) The one formed by the object directly,
- (ii) The image of the object formed by the mirror which now becomes a second object for the lens.

- (i) Image formed by the lens from the object directly.  
Using the real – is – positive sign convention

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{12} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{12} = \frac{6-5}{60} = \frac{1}{60}$$

$$\frac{1}{v} = \frac{1}{60}$$

$$\Rightarrow v = 60 \text{ cm}$$

The image distance is 60 cm from the lens on the opposite side of the object.

- (ii) The image of the object formed by the mirror which now becomes a second object for the lens.

Using the real – is – positive sign convention



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{36} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{36} = \frac{36-10}{360} = \frac{26}{360}$$

$$\frac{1}{v} = \frac{26}{360}$$

$$26v = 360$$

$$\Rightarrow v = \frac{360}{26} = 13.85\text{cm}$$

The second image distance is 13.85 cm from the lens on the opposite side of the object.

## SELF ASSESSMENT EXERCISE 2

1. Why is it advisable not to look directly at the sun? Give scientific reason for your answer.
2. A convex lens forms a real image of an object magnified two times.  
The distance between the object and image is 60cm. Find the focal length of the lens.
3. Describe how you would determine the focal length of a converging lens by means of;
  - (i) Quick but approximate method
  - (ii) The measurement of object and image distances.

## 4.0 CONCLUSION

In this unit you learned about refraction of light through lenses. You also learned how to locate images formed by convex (converging) and concave (diverging) lens and their applications. Further more; you learned how to determine the focal length of a convex lens using the quick but approximate method and by the measurement of object distance (u) and image distance (v). Finally, you learned the derivation and use of the lens formula.

## 5.0 SUMMARY



- Convex lenses are converging lenses. They converge parallel rays of light to a real principal focus.
- Concave lenses are diverging lenses. They diverge parallel rays of light away from a virtual principal focus.
- The principal focus of a lens is the point on the principal axis to which rays parallel and close to the principal axis converge after refraction through a converging lens or appear to diverge from after refraction through a diverging lens.
- Focal length of a lens is the distance from the lens to the principal focus.
- The power of a lens is the reciprocal of the focal length expressed in metres.
- Convex lenses produce different images when the object is placed at different positions.
- Concave lenses always produce virtual diminished and erect image.
- Lens formulae

(i) Real – is – positive

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$m = \frac{v}{u} = \frac{\text{height of image}}{\text{height of object}}$$

(ii) New Cartesian

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$m = \frac{v}{u} = \frac{\text{height of image}}{\text{height of object}}$$

## 6.0 TUTOR -MARKED ASSIGNMENT



1. Define
  - (i) The principal focus; and
  - (ii) The focal length of a converging lens.
2.
  - (a) State the characteristics of images formed by concave lens
  - (b) What is the application of such a lens and why?
3. An object is placed on the principal axis of a lens and at a distance of half the focal length from the lens. Show by means of ray diagrams where the image is formed when the lens is;
  - (i) a diverging lens,
  - (ii) a converging lens
4. Find the nature and position of the image of an object placed 10 cm from a converging lens of focal length 15 cm.

## **7.0 REFERENCES/ FURTHER READINGS**

- Anyakoha, M.W. (2000). New School Physics for Senior Secondary Schools. Onitsha: Africana FEP.
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## **UNIT 5      APPLICATIONS OF LIGHT WAVES**

### **CONTENTS**

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
  - 3.1 The Simple Camera and Projector
    - 3.1.1 The Camera- Features and Operations
    - 3.1.2 The Projector- Features and Operations
  - 3.2 The Human Eye
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    - 3.2.2 The Action of the Eye
    - 3.2.3 The Defects of Vision and their Corrections
    - 3.2.4 Comparison of the Human Eye and the Camera
  - 3.3 Simple and Compound Microscopes
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- 7.0 References/Further Readings

### **1.0 INTRODUCTION**

You have learnt about the reflection and refraction of light in units 1 to 4 of this Module. In this unit you will learn to apply the knowledge you acquired from the earlier units to explain the working of cameras, human eyes, projectors, microscopes and telescope. You will also learn to describe these optical instruments with respect to the nature and functions of their various parts.



## 2.0 OBJECTIVES

By the time you finish this unit you will be able to:

- construct a model of a box camera
- explain the formation of images by the camera and the projector by tracing rays of light through them
- explain the roles played by some parts of the eyes in the formation of image on the retina
- compare and contrast the eye and the camera
- state the defects of the eye and their causes
- identify the type of lenses for correcting the various defects of the eyes
- trace the paths of light rays through simple and compound microscopes and telescopes.

## 3.0 MAIN CONTENT

### 3.1 The Simple Camera and Projector

#### 3.1.1 The Camera – Features and Operations

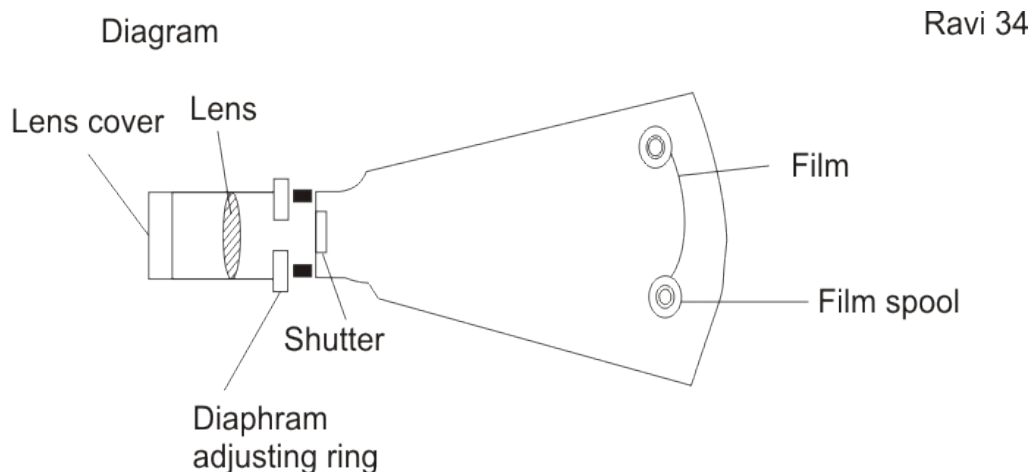


Fig. 5.1: A lens camera

The features and operations of the lens camera

- Consist of a light – tight box which contains a convex lens in front and a film at the opposite end
- The lens converges the incident rays from objects.
- the film serves as a screen on which the image is received



- characteristics of image formed
  - Real
  - Inverted
  - Diminished
- Focusing screw alters the distance of lens from film in order to always focus the image on the film
- Diaphragm controls the amount of light entering the camera. This is made possible by the variable aperture or hole.
- Shutter controls the amount of light entering the camera. It has variable speed and therefore can open for different lengths of time (exposure time).

The extent to which the diaphragm and the shutter combine to determine the amount of light entering the camera depends on:

- Sensitivity of the film;
- Brightness of the object; and
- Kind of effect that is wanted.

### 3.1.2 The Project –Features and Operations

The projector is an instrument for projecting on a screen an enlarged image of transparent object such as a slide. That is a projector is used for showing enlarged image of transparent slides and films on a white screen.

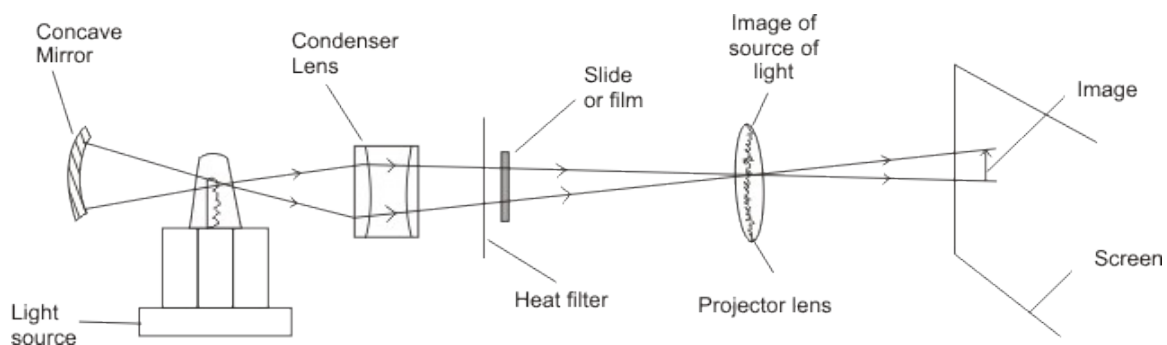


Fig. 5.2: The slide projector

- A powerful source of light is essential as the light on the screen has to spread over many times greater than that on the slide (film) in order to achieve the high magnification required. The camera projector magnification could be as high as 100 times. The powerful but small light source illuminates the object which is usually non-luminous. A converging mirror placed behind the light source helps to direct the light onto the film or slide. This source of light is at the



focus of the first lens aid the second lens converges the parallel beam upon the projection lens. The convergent beam illuminates the slide (film).

- The projection lens is adjusted so that the slide (film) is just beyond its principal focus.
- The slide (film) must be placed upside down so that the image on the screen appears the right way up.
- A device to absorb the intense heat from the source of light is usually interposed between the condenser and the slide ( film) or a fan is incorporated to send a cooling draught on the slide (film) to prevent it from over heating
- Characteristics of image formed
  - Real
  - Enlarged

The principle of the slide project is used in cine- projector, photographic enlargers and several other devises.

## 3.2 The Human Eye

### 3.2.1 The features and operations of the human eye

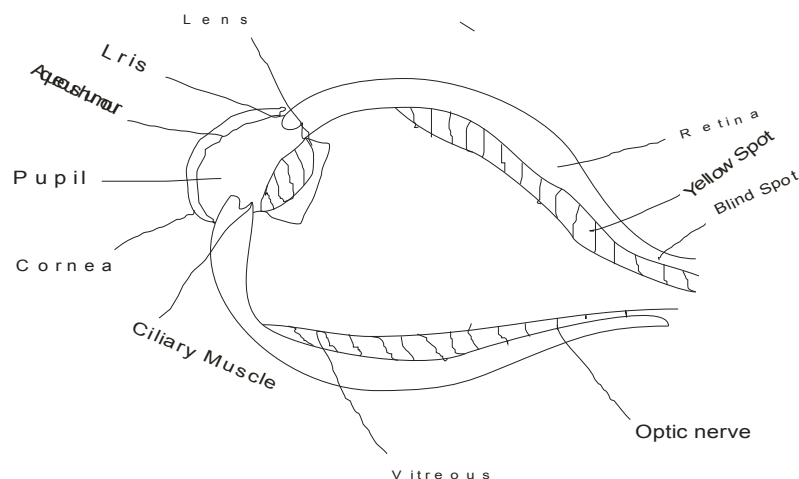


Fig. 5.3: Optical features of the human eye

- The cornea is transparent and allows light rays to pass through to meet the eye lens.
- The eye lens focuses the image of the object on the retina.

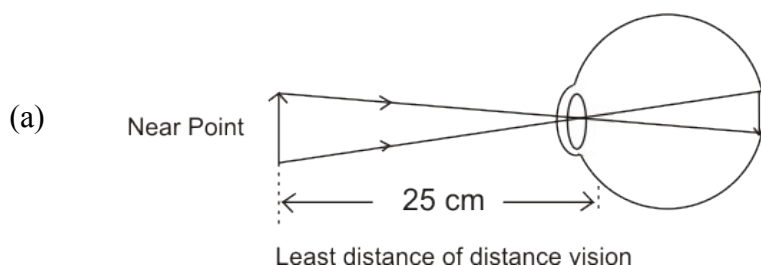


- The sensitive retina sends message to the brain by way of optic nerve.
- The pupil is a hole in the middle of the iris. It appears black because no light is reflected from it.
- The iris is the coloured portion of the eye. It adjusts the size of the pupil to vary the amount of light that enter the eye through the pupil
- The ciliary muscle alters the focal length of the eye lens in order to focus images on the retina (an action referred to as power of accommodation of the eye). The contraction and relaxation of the ciliary muscles vary the thickness of the lens (ie the focal length).
- The aqueous humour is a transparent liquid and fills the space between the cornea and the lens
- The vitreous humour is a jelly - like substance that fills the space between the lens and the retina.

### 3.2.2 The action of the eye

The normal eye in the relaxed state is focused on infinity known as the far point .

- Near point and far point: the nearest point at which an object can be clearly seen is called the near point. The farthest point at which an object can be clearly seen is called the far point.
- To see near objects clearly, the ciliary muscles increase the curvature of the eye lens and so enable it to focus the divergent light. Accommodation is effective upon the near point.
- For young children, the near point may be as close as 7.5 cm.
- The least distance of distinct vision for the normal eye is 25 cm.





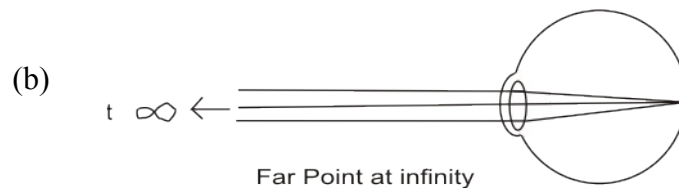


Fig. 5.4: Near point and far point

- Accommodation: This is the ability of the eye to focus both near and far objects on the retina through the action of the ciliary muscle by which the focal length of the lens is altered.
- Binocular vision: This is the combination of the two overlapping views of an object seen by both eyes in such a way as to create the right sense of distance and depth. By binocular vision we get a proper perspective of an object.
- Persistence of vision: A clear image formed on the retina gives a sensation of vision which remains for a short but definite time. The retention of the vision of the image is known as the persistence of vision. If a light flickers at a very short interval of about ten times per second, it appears to the eye as a continuous source of light owing to the persistence of vision. This phenomenon is made use of in the production and showing of motion pictures. A rapid succession of separate pictures cast on the screen appears to have continuity and can depict motion. If the speed of the successive pictures is reduced, the continuity is broken and the motion in the sequence becomes jerky or a thickening effect observed.

### 3.2.3 The Defects of Vision and their Corrections

The principal defects of vision which can be corrected with lenses are short sightedness ( myopia) , long sightedness ( hypermetropia), loss of accommodation (presbyopia) and astigmatism.. These eye defects are corrected by the use of suitable spectacles or by contact lenses which are placed to fit closely on the cornea. We are going to consider these eye defects and their corrections under these headings: meaning, cause and corrections.

#### i. Short Sightedness (Myopia)

Meaning: This is the ability of a person to see near objects clearly but not distant objects. This eye defect is called short- sightedness.



Cause: it is caused by a person's eye ball being too long and so focus distant object in front of the retina. The far point for the eye is much nearer than the normal eye.

Correction: A short – sighted person should wear a concave (diverging) spectacle lens. This spectacle lens (concave) is used to form a virtual image of a distant object at the person's far point as illustrated in Fig. 5.5.

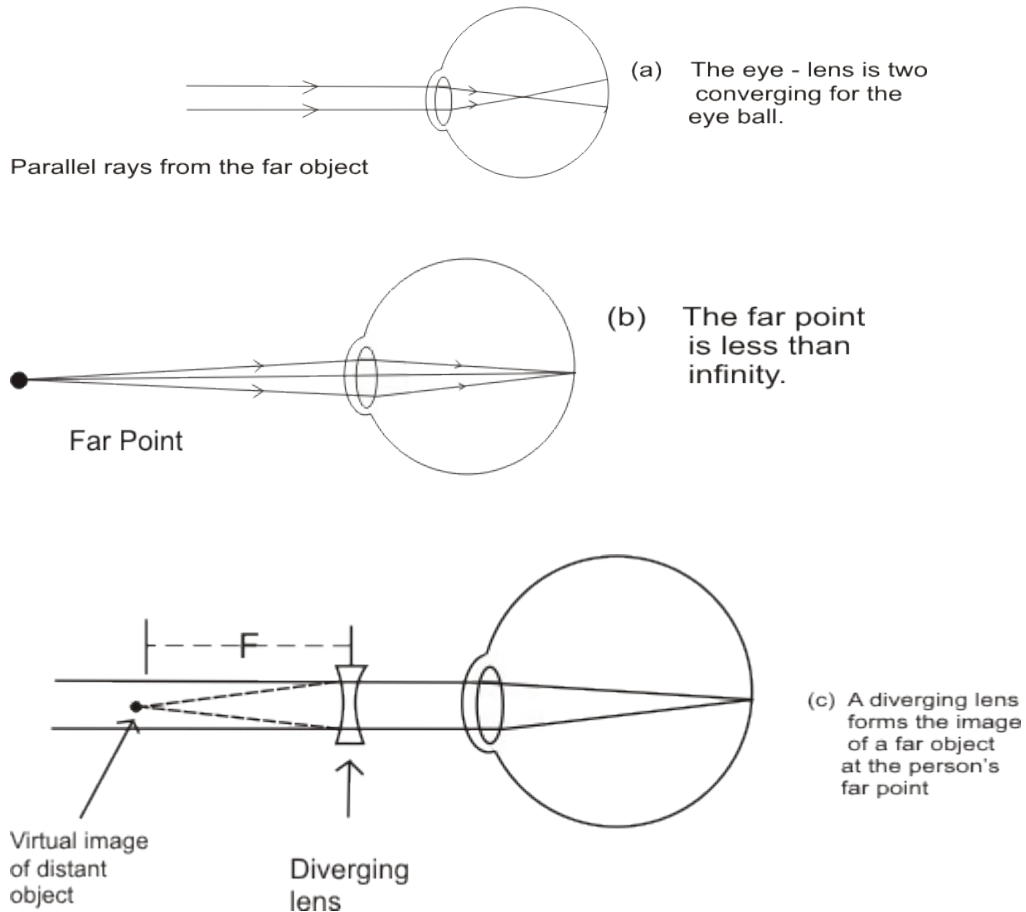


Fig. 5.5: Short – sightedness and its correction

Parallel rays from a distant object appear to diverge from a far point (the principal focus of the spectacle lens) and make them appear to come from a near distance. The short – sighted person has no problem of seeing near object closer to the eye than 25 cm because with the spectacle, the near point is slightly further away from the eye.

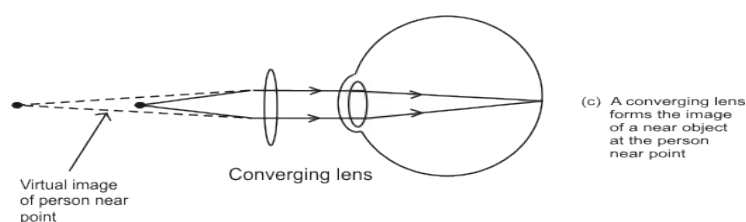
## ii. Long – Sight (Hypermetropia)

Meaning: This is the ability of a person to see distant object clearly but not near object. Common example is those who require glasses to read but not to see distant object.



**Cause:** it is caused by the person's eye ball being too short or eye lens being too thin or the optical system of the eye being too weak so that rays from the normal near point (25 cm) are brought to focus beyond, instead of on, the retina. That is, rays from near objects focus behind the retina.

**Correction:** A convex (converging) lens is used for the correction of long-sight. The focal length should be such that it would produce a virtual image of an object placed 25 cm away at the person's near point (Fig. 5.6).



### iii. Loss of Accommodation (Presbyopia)

**Meaning:** This is the loss of elasticity of the eye-lens. That is, the eye – lens tends to become inelastic and unable to accommodate. The far point usually approaches the eye while the near point recedes.

**Cause:** It is caused by increasing age which makes the eye-lens to become inelastic and unable to accommodate.

**Correction:** It requires two pairs for its correction. A concave pair for viewing distant object and a convex pair for viewing near objects. The two pairs may be combined in one frame with the top and bottom parts having different powers (bifocal spectacles).

### iv. Astigmatism

**Meaning:** this is when the focusing power of the eye is different in different planes. The horizontal lines may appear in focus while the vertical lines are not seen distinctly: This is caused by the distortion of the curvatures of the cornea or of the eye-lens.

**Correction:** It is corrected by shaping the spectacle lens so as to have different curvatures in different directions.



### 3.2.4 Comparison of the Human Eye and the Camera

Human Eye	Camera
The Eye	The Camera
Iris	Diaphragm
Pupil	Aperture ( in the diaphragm)
Lens	Lens
Retina	Screen
Eyelid	Shutter
Cilliary Muscle	Focusing ring

#### i. The similarities

- The human eye is impregnated with black pigments within
- The camera has light tight box painted black inside
- The pupil of the eye serves the same function as the aperture of the camera
- Both the human eye and the camera have converging lens to focus rays from an external object.
- Both have light sensitive materials on which the rays are focused the retina in the eye and the film in the camera.
- The iris in the human eye serves the same function as the diaphragm in the Camera, to regulate the amount of light entering the eye (camera).

#### iii. The differences

- The human eye has the power of accommodation. That is, can vary its lens focal length. While the focal length of the camera is fixed.
- The distance between the lens and the retina is fixed in the human eye. In the camera, the distance between the lens and the film can be varied
- The eye can suffer from the defect of vision unlike the camera.
- The eye is a biological organ while the camera is mechanical device, leading to other differences.

### SELF ASSESSMENT EXERCISE 1



1. In this exercise you would need the following materials:

Two cardboard boxes of about 25 by 15 cm such that one box can fit into the other, a convex lens of about 10 cm, tissues paper or any transparent paper, an illuminated object of height equal to half the height of the box and plasticize.

Use the above materials and follow the illustration of Fig. 5.7. Construct a box camera thus:

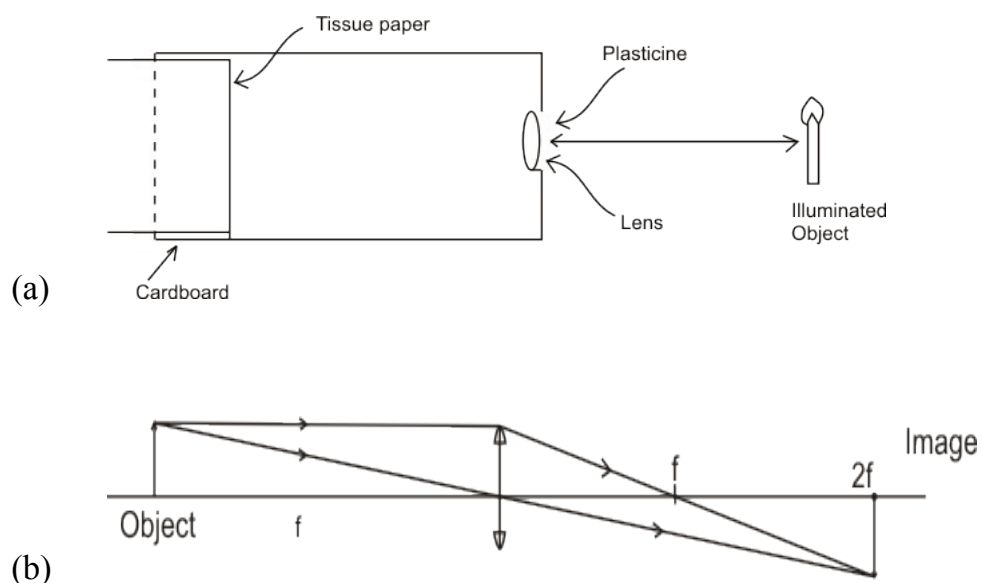


Fig. 5.7: Making a camera

Make a hole of diameter of about 9 cm on one face of the cardboard box. Remove the end of the box opposite the hole. Remove both ends of the smaller box and cover one end with tissue paper or transparent paper. Arrange the given lens to cover the hole (or opening) and hold it in position with plasticine. Fit the smaller box into the larger box with its covered end towards the lens. Place an illuminated object (burning candle stick) in front of the lens. Adjust the position of the small box until its sharp image appears on the tissue or transparent paper.

For you to obtain best result, you should carry out this exercise in a dark room or a fairly dark corner of a room

Describe the characteristic of the image you see on the film of the box camera which you have built.

2. With the aid of ray diagrams, illustrate



- i. The ability of a short- sighted person to see near object;
  - ii. his inability to see distant objects ; and
  - iii. the correction of the eye defect using diverging spectacle lens.
3. With the aid of ray diagrams, illustrate
  - i. the ability of a long- sighted persons to see distant objects;
  - ii. his inability to see near object, and
  - iii. the correction of the eye defect using converging spectacle lens
4. State the differences and the similarities between the human eye and the lens camera
5. With the aid of a labelled diagram illustrate the formation of image by the projector by tracing rays of light through it.

### 3.3 Simple and Compound Microscopes

#### 3.31 The Simple Microscope or Magnifying Glass

For viewing near object, the simple microscope or magnifying glass is a convex lens which is used to produce magnified images of small objects. The objects are usually placed nearer to the lens than the principal focus so that an erect virtual and magnified images seen clearly at 25 cm from the normal eye. Magnification of the object produced by the lens is given by the formula

$$\begin{aligned}
 M &= 1 + \frac{D}{f} \\
 &= 1 + \frac{D}{F} \quad (\text{image is virtual})
 \end{aligned}$$

Where

$M$  = angular magnification  
 $D$  = least distance of distinct vision (for a normal eye,  $D = 25$  cm)  
 $F$  = focal length of the lens.

Thus, the image subtends a larger visual angle than that produced when the object itself is placed 25 cm from the unaided eye and therefore looks bigger. This is illustrated in Fig. 5.8.



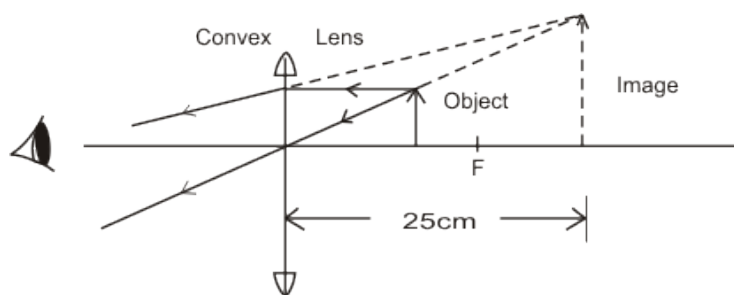


Fig. 5.8: Simple microscope or magnifying glass

The simple microscope is used for reading small print and for studying biological specimen in the laboratory

### 3.3.2 The Compound Microscope

For scientist to produce higher magnification than that produced from a simple microscope, a combination of two convex lenses is used in an arrangement called the compound microscope.

The compound microscope consists of two convex lenses fitted at the opposite ends of a tube. The tube is mounted on a stand so that it can be lowered or raised for focusing. The object is placed under and close to the lower lens which has a very short focal length and is called the objective lens. The final magnified image is viewed through the upper lens which has a longer focal length. This lens is called the eyepiece. The eyepiece has a comparatively larger focal length than the objective lens. The compound microscope generally has a horizontal platform under the objective lens on which the object to be viewed is placed. Usually there is a converging micro which serves to concentrate light on the object.

The object to be magnified is placed in front of the objective such that the object distance  $u$  is greater than the focal length of the objectives. This forms a magnified, inverted and real image of the object at  $I_1$ , as illustrated in Fig. 5.9

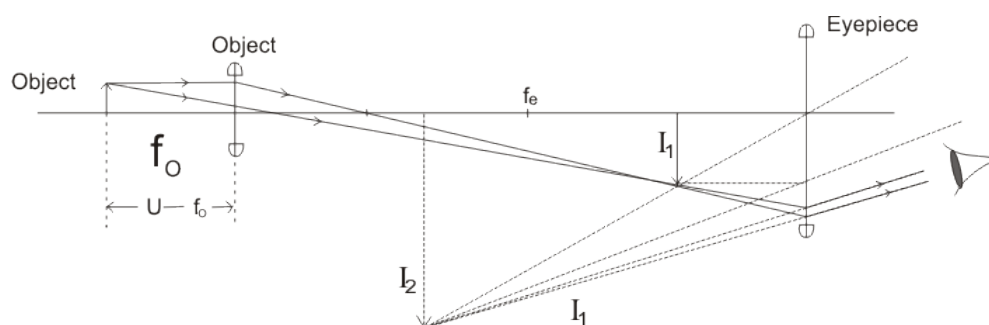


Fig. 5.9: A compound microscope



If the eye piece is adjusted correctly,  $I_1$ , will fall between the eyepiece and its principal focus  $f_e$  as shown. Then a magnified and virtual image of  $I_1$ , will be produced at  $I_2$ . Hence, the eye piece behaves just like a magnifying glass. The final image of an object produced by a compound microscope is highly magnified and inverted. The observer can adjust the eyepiece so as to obtain distance of about 25 cm (least distance of distinct vision for normal eye).

The magnifying power  $M$  of the compound microscope is the product of the magnification due to the two lenses. That is, magnification  $M = M_1 \times M_2$  where  $M_1$  is the magnification produced by the objective lens and  $M_2$  is the magnification produced by the eyepiece. Thus, a microscope with a magnifying power of 300 will show an object 300 times as large as it would appear to the naked eye from a distance of 25 cm.

### 3.4 The Telescope

The telescope is an instrument for viewing distant objects such as the stars and planets. There are many types of telescopes in use but, basically, a telescope is either designed to be used for observing objects on the earth's surface - the terrestrial telescope or for observing celestial objects- the astronomical telescope.

All telescopes consists of objective convex lens or concave mirror of long focal length because of the distance of the objects to be viewed and an eyepiece lens of short focal length for producing magnified image.

#### 3.4.1 The Astronomical Telescope

An astronomical telescope is used to view very distant objects like the planets and the stars. It makes use of two convex lenses O and E as its objectives and eyepiece respectively as earlier stated. The objective has a long focal length and the eyepiece a short focal length. Owing to the great distance of the object from the telescope, the light rays from any point on the object are very nearly parallel on reaching the objective.

Astronomical telescope can either be adjusted such that its image is formed at infinity (normal adjustment) or at a fairly near point. For instance

- i. When the final image is formed at infinity as illustrated in Fig. 5.10 the telescope is said to be in normal adjustment.



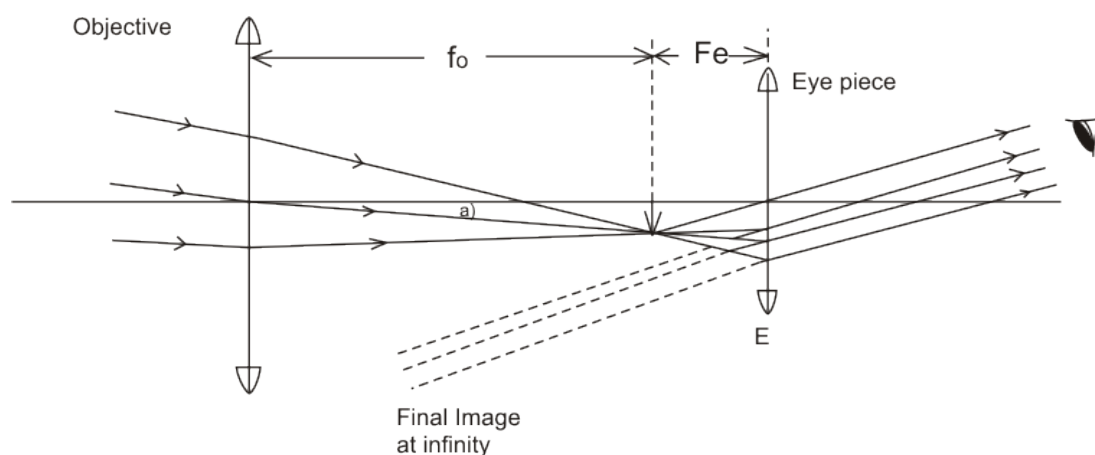


Fig. 5.10: An astronomical telescope in normal adjustment

In such an adjustment, the two lenses are arranged such that the distance between them is equal to the sum of their individual focal lengths ( $f_o + f_e$ ). In this case, the objective lens converges parallel rays from all points in the object and forms a real, inverted and diminished image  $I_1$  on its focal plane. The focus of the eyepiece coincides with that of the objective. Therefore, the eyepiece forms a virtual image of this image (which now serves as object) at the focal plane at infinity and this is the image that will be seen by the eye. Hence the total length of the telescope is the sum of the focal lengths of the objectives and the eyepiece.

In other words, if the position of the eyepiece is adjusted such that  $I_1$  (the real image) is made to fall on the principal focus of the eyepiece, the final image will be formed at infinity

- ii. On the other hand, an astronomical telescope can also be adjusted such that its image is formed at a fairly near point to the eyepiece. In this case, the position of the eyepiece is adjusted such that  $I_1$  (the normal image) falls between the eyepiece and its principal focus  $f_e$ , a magnified and virtual image of  $I_1$  is then formed at  $I_2$  as illustrated in Fig. 5.11.



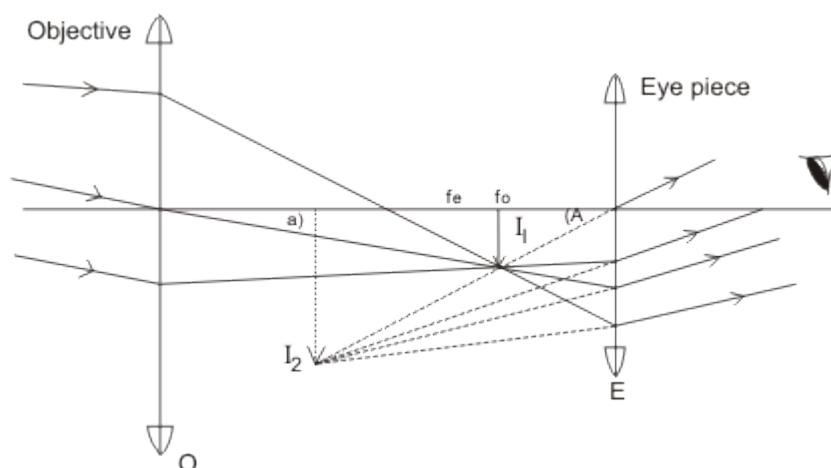


Fig. 5.11: Final image at near point astronomical telescope

Here, the eyepiece of the astronomical telescope performs exactly the same function as the eyepiece of the compound microscope. You can see from Figs. 5.10 and 5.11 that the final image of the object produced by an astronomical telescope is inverted.

The magnification produced by the astronomical telescope is given by the ratio of the angle subtended at the eye by the image of a distant object when a telescope is used to the angle subtended at the unaided eye by a distant object. That is, the angular magnification of the telescope. This angular magnification is given by

$$M = \frac{a}{a}$$

where

$$a = h/f_e$$

and

$$a = h/f_o$$

where

$f_e$  and  $f_o$  are the eyepiece and objective focal lengths respectively (Figs. 5.10 and 5.11) and  $h$  is the height of the real image  $I_1$ .

This implies that magnification

$$M = f_o / f_e.$$



### 3.4.2 Reflecting Astronomical Telescope

This is another form of astronomical telescope. It makes use of a large concave parabolic mirror with a polished coating of aluminum on the reflecting surface. The concave parabolic mirror serves as the objective of the telescope. It has a large diameter that enables it to gather a lot of light and also a long focal length which enhances increase in magnification of the image in order to reveal minor details in the object. The use of these two properties – large diameter and long focal length of concave mirrors are not easily obtained in convex lenses.

This telescope is very effective in the sense that it is able to collect as much light rays as possible from distant stars and planets thereby produces a fairly sharp image. Therefore, the most powerful telescopes are the reflecting ones.

Parallel rays from a distant object are reflected at the objective concave mirrors. These reflected rays are intercepted by a small plane mirror before they form a real image  $I_1$  (Fig. 5. 12). This image is magnified in the usual way by the eyepiece covering lens

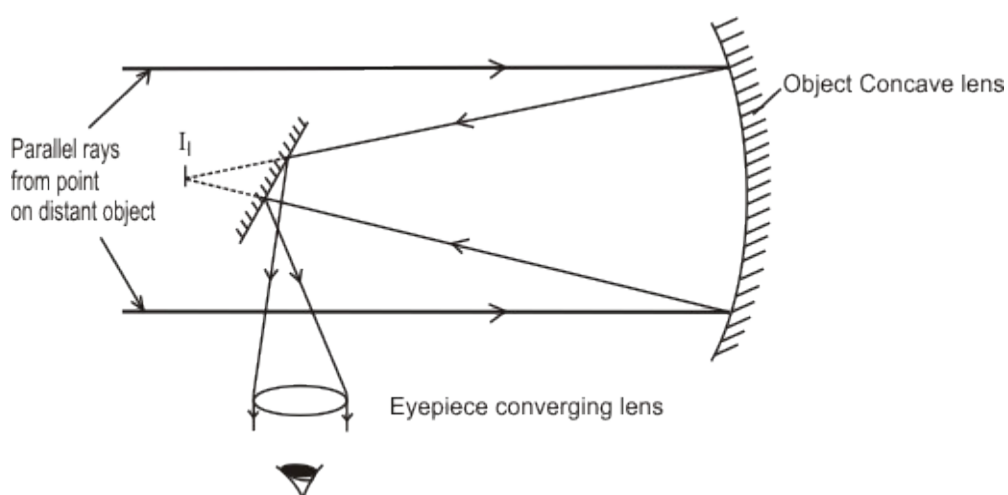


Fig .5.12: Reflecting astronomical telescope

Telescopes that use concave mirror as objectives are referred to as reflector or reflectory telescopes while those that use lens as objectives are called a refractor or refracting telescope.

### 3.4.3 The Terrestrial Telescope

This is a modified astronomical telescope for viewing distant objects on the earth surface. The difference between the terrestrial telescope and the astronomical telescope is that a convex lens is put in between the objective and the eyepiece so that a final image in erect position, but with the size unchanged is obtained.



For instance, if a convex lens C of a suitable short focal length  $f_c$  is placed at a distance  $d = f_o + 2f_c$  from the objective as illustrated in Fig. 5.13 a, magnified and erect image will be produced by the eyepiece.

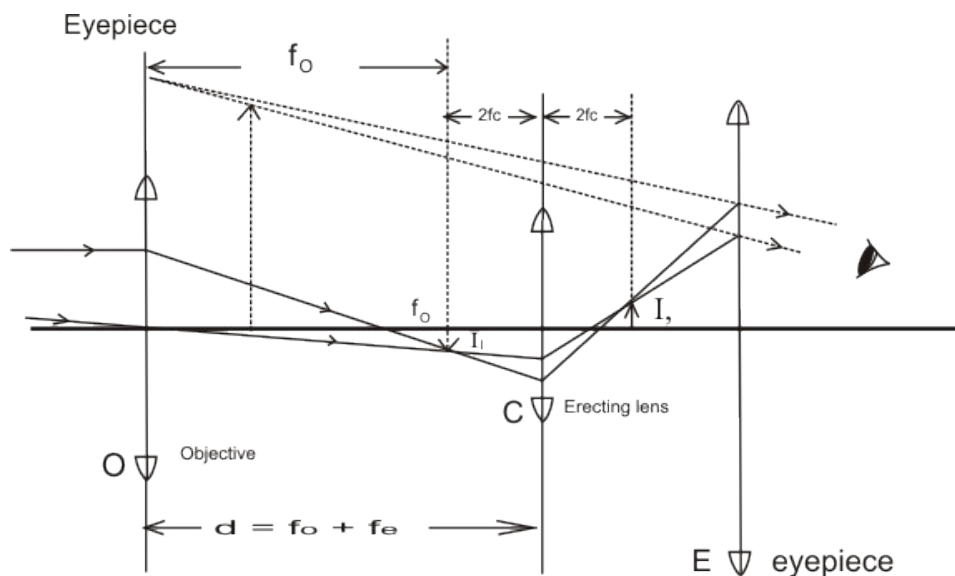


Fig. 5.13: Terrestrial telescope

In the terrestrial telescope, the lens C acts as an erecting lens and re-inverts the image  $I_1$ , formed by the objective. The re-inverted image  $I_2$  falls at the distance  $2f_c$  from C and is of the same size as  $I_1$ . The eyepiece is adjusted such that a magnified image of  $I_2$  is obtained. The final image  $I_3$  observed is an erect image of the object.

The terrestrial telescope is longer than the astronomical telescope by four times the focal length of the erecting lens. Also, distant objects appear near and in their natural orientation (that is, upright) when viewed through the terrestrial telescope.

### 3.4.4 The Galilean Telescope

The Galilean telescope uses convex lens of long focal length as the objective and a concave lens of short focal length as the eyepiece.

Parallel rays from distant object refracted by the objective lens are supposed to form a real image  $I_1$ , at its principle focus,  $f_o$ . But, before this image could be formed, the concave eyepiece intercepts the converging rays and makes them to diverge as illustrated in Fig. 5.14.



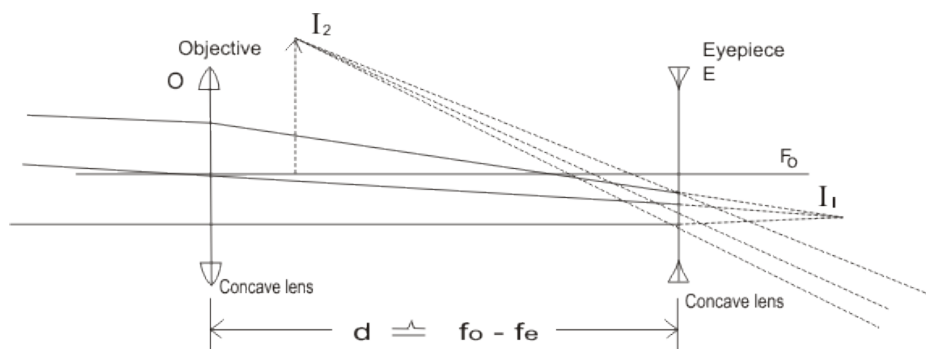


Fig. 5.14: The Galilean telescope

Thus, the eyepiece forms an erect, virtual and magnified image at  $I_2$ . In this situation, the distance between the two lenses, mounted on common axes is equal to the difference between their focal lengths (ie,  $d = f_o - f_e$ ). Thus, the distance between the objective and the eyepiece in a Galilean telescope is approximately the difference in their focal length ( $f_o - f_e$ ) this design is often mounted in pairs for low power binocular used as “opera glasses” and night glasses. They give a very small field of view except with very low magnifications, but as they only use two lenses to give an erect image, the light losses within them are relatively small.

### 3.4.5 The Prism Binoculars

The prism binoculars consist of a pair of astronomical telescopes each of which is fitted with two right – angled isosceles prisms. The arrangement of the two prisms serves a similar function to that of the erecting lens of a terrestrial telescope. That is, they revert the image formed by the objective and present an erect image of the object to the viewer.

That is, in the prismatic binoculars, each side consists of an astronomical telescope effectively folded in three. One half of this arrangement of astronomical telescope is illustrated in Fig. 5.15.

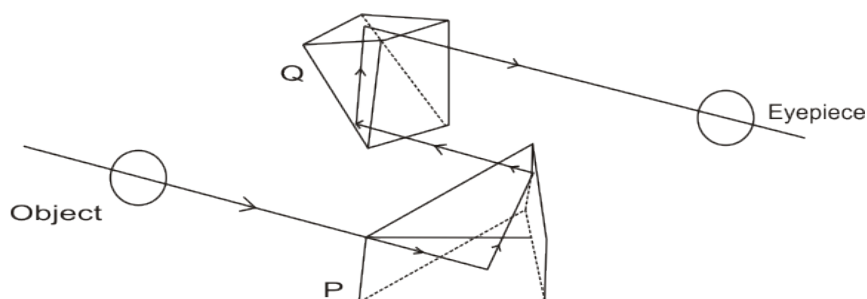


Fig. 5.15: Action of one half of prism binoculars



The re- inversion of the image is done in two stages, by two lateral inversions using  $45^\circ$  reflecting prisms which in addition to laterally inverting as a mirror would do, also displace the rays sideways so that they return along the next limb.

The two prisms are set so that the right - angle edges are horizontal and vertical in the figure- not parallel to one another

The first prism P turns the real inverted image formed by the objective in a horizontal direction (turns the image round and corrects for lateral inversion ) while the second prism Q turns this image the right way up). This erect image of the object is presented to the eyepiece which subsequently magnifies it. Without the prisms the image would be upside down and laterally inverted.

The major advantage of prism binoculars is that they can give wide field of view with a reasonable high magnification.

## SELF ASSESSMENT EXERCISE 2

1. With the aid of labeled diagrams trace the path of light rays through
  - a. Simple microscope
  - b. Compound microscope
  - c. The astronomical telescope
  - d. The reflecting astronomical telescope
  - e. Terrestrial telescope
  - f. The Galilean telescope
  - g. The prism binoculars
2. State for each of the optical instruments listed in '1' above
  - a. the advantage (s)
  - b. The disadvantage (s) and
  - c. The use (s)  
if any

## 4.0 CONCLUSION

In this unit you studied the applications of lenses. You applied the knowledge you acquired from earlier units in the study of the human eye and other optical instruments. You learnt the formation of images by the human eye, the camera, microscope and telescope. You also learned the similarities and the differences between the human eye and the camera, the defects of human eyes and the types of lenses for correcting the various defects. Finally, you learned how to describe the projectors,



microscopes and telescopes with respect to the nature and function of their various parts.

## 5.0 SUMMARY

- A camera consists of a converging lens which forms real images of objects on a film.
- In the film projector, a projector (convex) lens is used to project on a screen, a magnified image of a small picture on a film.
- The human eye is able to see objects at different distances because it has the ability for varying the focal length of its lens (Accommodation).
- The defects of the eye- long sight and short – sight can be corrected by using suitable lenses such as convex, and concave lenses respectively
- A single convex lens can act as a simple microscope (magnifier), if the object is placed within its focal length.
- The compound microscope consists of an objective lens of short focal length and an eyepiece lens of longer focal length.
- The astronomical telescope consists of a converging objective of long focal length and a converging eyepiece of short focal length.
- The terrestrial' telescope, a modified version of the astronomical telescope has a converging erecting lens between the objectives and the eyepiece.
- The Galilean telescope has a convex lens as its objectives and a concave lens as its eyepiece; it produces a final erect image.
- Prism binoculars consists of a pair of astronomical telescope each of which is fitted with two right –angled isosceles prisms; it is optically equivalent to a terrestrial telescope which is about three times longer in length.

## 6.0 TUTOR-MARKED ASSIGNMENT

1. Explain with a ray diagram how a converging lens of suitable focal length may be used as a simple magnifier
- 2
  - a. State two advantages of binocular vision.
  - b. How does the eye adjust itself to deal with
    - i. Light of different intensity?
    - ii. Objects at varying distances?
3. In using a slide projector, the image of the slide must be
  - i. the right way up
  - ii. magnifiedExplain briefly how this is done.



4. Sketch diagrams indicating the types of lenses used and the positions and nature of image formed by an astronomical telescope in
  - i. normal adjustment
  - ii. at near point
5. With the aid of ray diagrams illustrate
  - a. The ability of a long- sighted person to see distant objects;
  - b. His inability to see near objects;
  - c. The correction of the eye defect using converging spectacles lens

## **7.0 REFERENCES/ FURTHER READINGS**

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- Nelkon, M. (1999). Principles of Physics for Senior Secondary Schools. Ibadan Longman Group Limited
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## **UNIT 6      DISPERSION OF LIGHT AND COLOURS**

### **CONTENTS**

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
  - 3.1 The Concept of Dispersion of Light
  - 3.2 White Light Spectrum
  - 3.3 Production of Pure Spectrum
  - 3.4 Primary, Secondary and Complementary Colours
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    - 3.4.2 Secondary Colours
    - 3.4.3 Complementary Colours
  - 3.5 Mixing of Coloured Pigments (paints)
  - 3.6 Colours of Objects
  - 3.7 Light Filters
  - 3.8 Formation of Rainbow
  - 3.9 The Electromagnetic Spectrum
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Readings

### **1.0 INTRODUCTION**

In this unit you will learn about the dispersion of light and colours. Specifically, you will learn about the white light spectrum, its combination and production. You will also learn about the primary, secondary and complementary colours. In addition you will learn about mixing of colour pigments (paints), colours of object in white lights and light filters.

### **2.0 OBJECTIVES**

By the time you finish this unit, you will be able to:

- obtain the spectrum of white light
- show dispersion of white light by a triangular prism
- explain the formation of rainbow and why objects appear the way they do.



### 3.0 MAIN CONTENT

#### 3.1 The Concept of Dispersion of Light

Dispersion as you know is the breaking up of or separation of white light into its component colours when it is passed through a glass prism. It is due to the fact that different colours of white light travel at different speeds through the glass. As a result of which each colour is refracted in a slightly different direction or angle through the glass. That is each colour has its own wavelength, velocity and refractive index of refraction. Thus, white light is split up into seven colours: Red, Orange, Yellow, Green, Blue, Indigo and Violet. We remember these colours by picking the first letter of each colour to form ‘ROYGBIV.’

#### 3.2 White Light Spectrum

When a ray of light is inclined obliquely on a glass prism, it emerges deviated and separated into its component colours- Red, Orange, Yellow, Green, Blue, Indigo and Violet (ROYGBIV), due to refraction. These colours resulting from the separation (i.e. dispersion) of white light make up the spectrum which is generally referred to as the white light spectrum.

Sir Isaac Newton was the first to observe in an experiment that when white light is passed through a prism, an elongated coloured patch of light is obtained on a screen placed behind the prism. This coloured pattern now referred to as the spectrum of white light consists of the Red, Orange, yellow, Green, Blue, Indigo, Violet (ROYGBIV) earlier mentioned. An observation of the spectrum shows that the red light is deviated least while the violet light is deviated most. Because red is deviated least from its original direction in air, its change of speed on entering glass is least and as such its speed in glass is greatest. Violet is deviated most on entering glass and it has the least speed in glass. The white light spectrum is illustrated in Fig. 7.1.

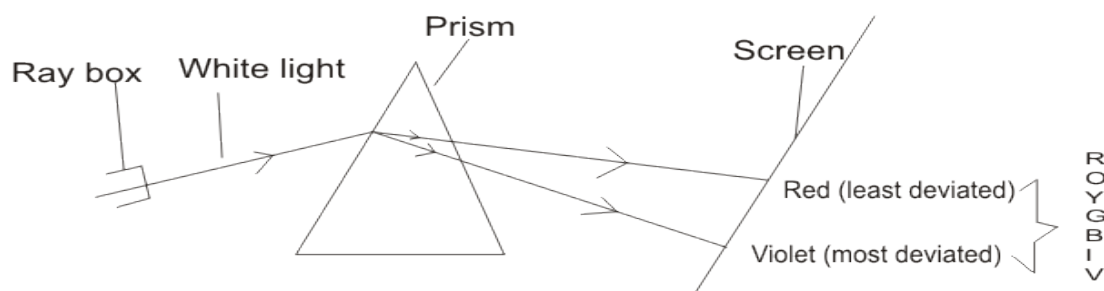


Fig. 7.1: Dispersion of white light resulting into the white light spectrum



If another identical prism is placed to intercept the refracted rays as shown in Fig. 7.2 a, the same arrangement of colours will energy on the screen. This time, however, the colours will be more widely separated. On the other hand, if the second prism is inverted as shown in Fig. 7.2b, the colours will disappear and only a patch of white light will be visible on the screen

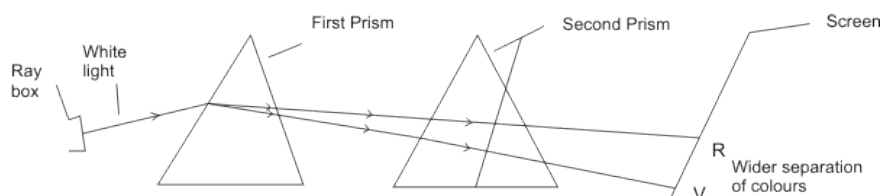


Fig. 7.2 a: Dispersion of white light spectrum

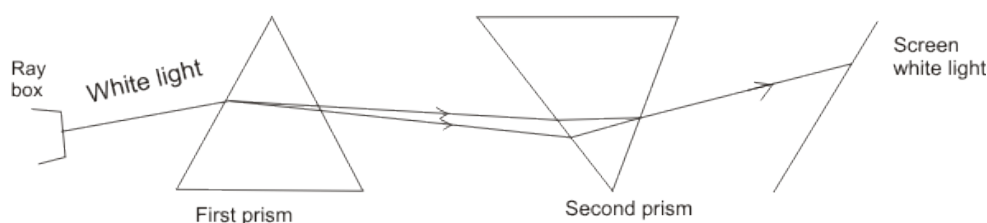


Fig. 7.2b: Recombination of white light spectrum

The disappearance of the colours was due to their recombination to reproduce white light. Hence white light spectrum is the band of colours formed when white light passes through a prism.

### SELF ASSESSMENT EXERCISE 1

Divide a circular disc into seven parts and paint each part with one of the

ROYGBIV colours (Fig. 7.3).

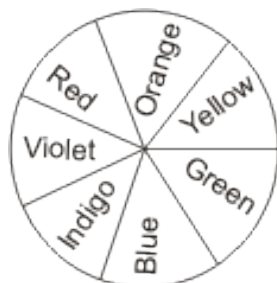


Fig. 7.3: Newton's colour disc

Spin the colour disc to make it rotate at high speed. You will observe that the colours are mixed. These colours you can see seem to be white.



Now slow down the rotation of the disc. As you do this, you will notice the individual colours again.

### Monochromatic light

You should note that if light of one wavelength ( $\alpha$ ) and colour (monochromatic) is passed through a prism refraction; occurs without dispersion. That is, if red light passes through a prism it will come out as red light (Fig. 7.4).

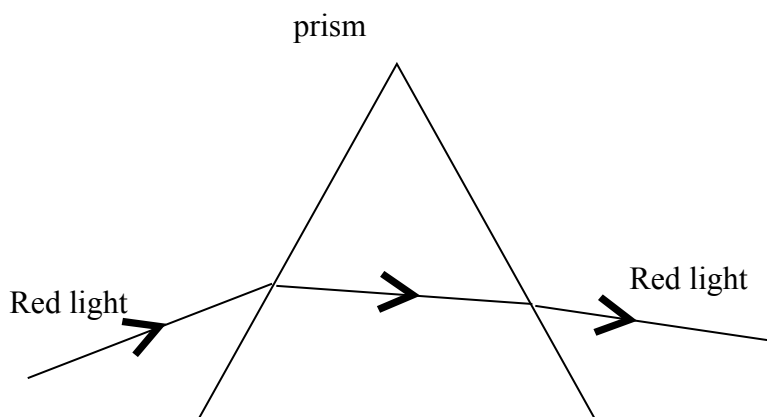


Fig. 7.4: No dispersion with monochromatic light

### 3.3 Production of Pure Spectrum

Have you noticed that the spectrum produced by a prism (Fig. 7.1) is an impure spectrum because the different colours overlap. If you have not, perform that simple experiment illustrated in Fig. 7.1 to observe the spectrum.

A pure spectrum is that in which the colours are clearly separated or distinct from each other. To produce such a pure spectrum, prism is used in combination with converging lenses of as shown in Fig. 7.5 below.

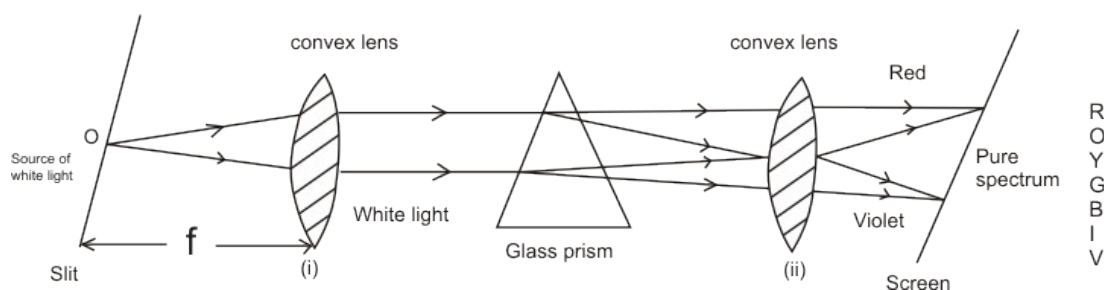


Fig. 7.5: Production of pure white light spectrum



The apparatus consists of these components – source of white light, slit, two convex lenses, one triangular prism and a screen.

The narrow slit illuminated by a bright source of white light produces a series of narrow coloured images which minimizes the chances of overlapping colours.

This source of white light is placed at the focal point of the first lens (i). This enables the converging lens (i) to produce a white parallel beam of light as the light incident on it is refracted. As you already know the prism disperses the beam of light into its component colours while the converging lens (ii) collects the different colours and converges them differently on the screen.

The screen is placed at the focal plane of the converging lens (ii) so that the coloured bands of the spectrum will appear well defined while the slit is placed at the principal focus,  $f$ , of the converging lens (i) to ensure that parallel rays are produced by the lens. Production of a clear spectrum is also ensured by using a very fine slit and a bright light source.

This pure spectrum consists of Red, Yellow, Green, Blue, Indigo and Violet. Red is least deviated while Violet is the most deviated.

### **3.4 Primary, Secondary and complementary Colours**

#### **3.4.1 Primary Colours**

Primary colours are red, green and blue, they are called primary colours because it is not possible to produce them by mixing other colours. When three primary coloured lights are mixed they produce white light.

#### **3.4.2 Secondary Colours**

Secondary colours are those colours that can be obtained by mixing any two primary colours. Examples are yellow, cyan and magenta. They are produced by mixing two primary colours such as

- Green + blue = cyan (secondary colour)
- Red + blue = magenta (secondary colour)
- Green + red = yellow (secondary colour)

This is illustrated in Fig. 7.6 below.



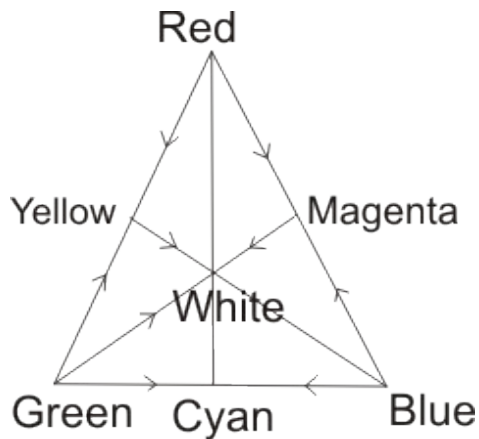


Fig. 7.6: Additive colour - mixing of lights

### 3. 4.3 Complementary Colours

If two colours, one primary and the other secondary are such that when mixed give white colour, then they are said to be complementary colours.

Examples are:

- Yellow + blue = white
- Cyan + red = white
- Magenta + green = white

This is also illustrated in Fig. 7.5 above.

### 3.5 Mixing of Coloured Pigments (paints)

In contrast to the mixing of colored light, when blue and yellow paints are mixed, the result is green not white. The colours of pigments (paints) are the result of the kinds of light they reflect. For instance, yellow paint reflects red, yellow and green light and absorbs the rest. On the other hand, blue paint reflects blue and green light and absorbs the rest. But, when yellow and blue paints are mixed the only colour reflected by both components of the mixture is green, which is thus the colour of the mixture.

This mixture of paints is known as colour mixing by subtraction (Fig. 7.7) it is different from mixing of coloured lights which are pure colours called mixing by addition



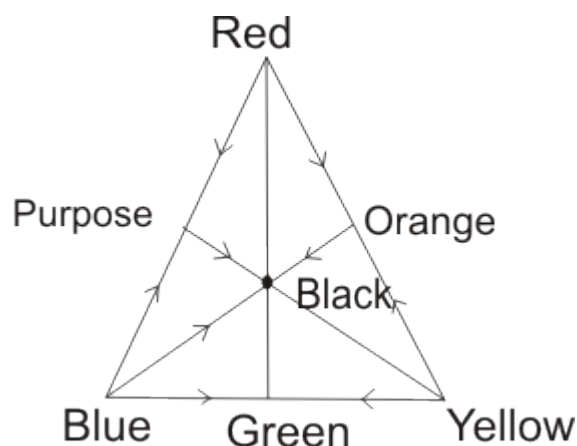


Fig. 7.7: Subtractive colour -mixing of paints

The primary colours for paints unlike lights are red, yellow and blue. They are the basis for all other colours of paint and cannot be produced by mixing. When a primary colour is mixed with a secondary colour on the opposite side of the triangle, the result will be black as shown in Fig. 7.7.

### 3.6 Colours of Objects

The colours of objects in white and coloured lights depend on the colours in the light incident or falling on them and also on the absorption and reflection of these lights by the objects.

A white object appears white in daylight because it reflects equally all the colours of the spectrum. Also the colours of opaque objects seen are the colours of light that are reflected. The following are some examples:

- Rose flower appears red because it reflects red light and absorbs the other colours
- White objects reflect all the colours of the spectrum and appear white.
- Black objects absorb all the colours of spectrum and appear black.

That is, objects appear to be of a certain colour because they reflect light of those colours and absorb the rest. Primary colours will reflect or transmit only their respective colours. Other colours will reflect or transmit other colours along with their own colours

Light filters are transparent- coloured sheets which allow light of the colour of the filter to pass through while absorbing all other colours in



the incident light they are made of gelatin and coloured with various dyes.

### 3.7 Light Filters

Filter papers are used to absorb light of various colours from the spectrum of white light. Generally, a filter paper absorbs all other colours except that of the filter. For instance;

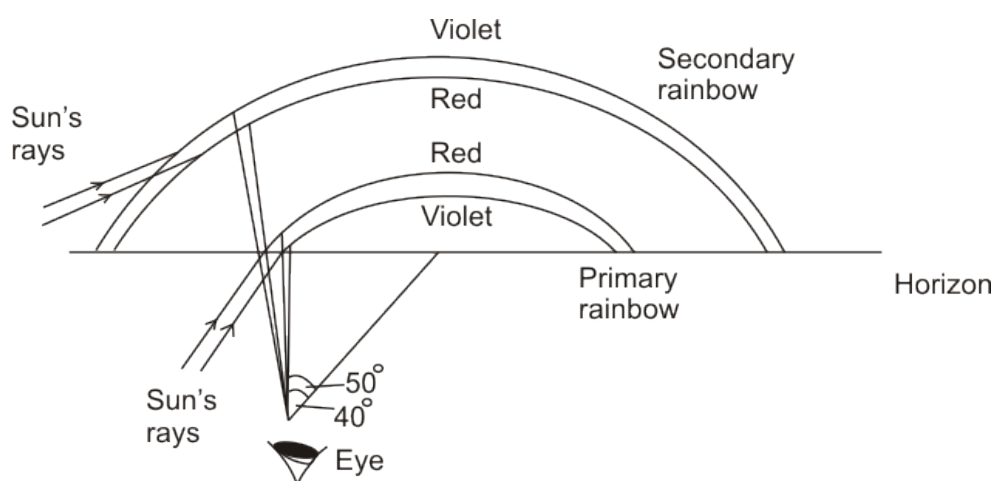
- When a red filter is placed on the path of white light, only red light passes through ;
- When a green filter is placed on the path of a red light, no light passes through. Hence, there is a black appearance.

An unusual situation occurs when a yellow filter is placed on the path of white light. This time all other colours are absorbed except yellow, green and red.

Subtractive colour- mixing also results from the super- position of two colour filters. A combination of two filters results in the transmission of only those colours which are common to both of them.

### 3.8 Formation of Rainbow

Have you seen a rainbow before? It is a natural phenomenon sometimes seen after a rain. It is formed in the same way as a spectrum is formed by a glass prism. Here, rain drops or drops of water suspended in the air after rainfall act as prisms which disperse the white beam of light into seven colours of the rainbow. The viewer can see the rainbow only if he is in a position to receive the refracted rays of the sunlight. This is illustrated in Fig. 7.8.





Sometimes, two rainbows are seen namely the primary and the secondary rainbows

### i. The Primary Rainbow

The primary rainbow is brighter than the secondary rainbow. It is caused by light rays which undergo one internal reflection inside the rain drop. When a horizontal light ray enters a water droplet at point A, dispersion takes place (Fig. 7.9 a) At  $R_1$ , the coloured rays undergo total internal reflection and they exit from the water droplet at B where further refraction occurs. The violet ray is refracted the most and the red ray the least. The red ray which emerges from each droplet makes a larger angle of about  $42^\circ$  with the horizontal than the violet ray that makes about  $40^\circ$  also with the horizontal.

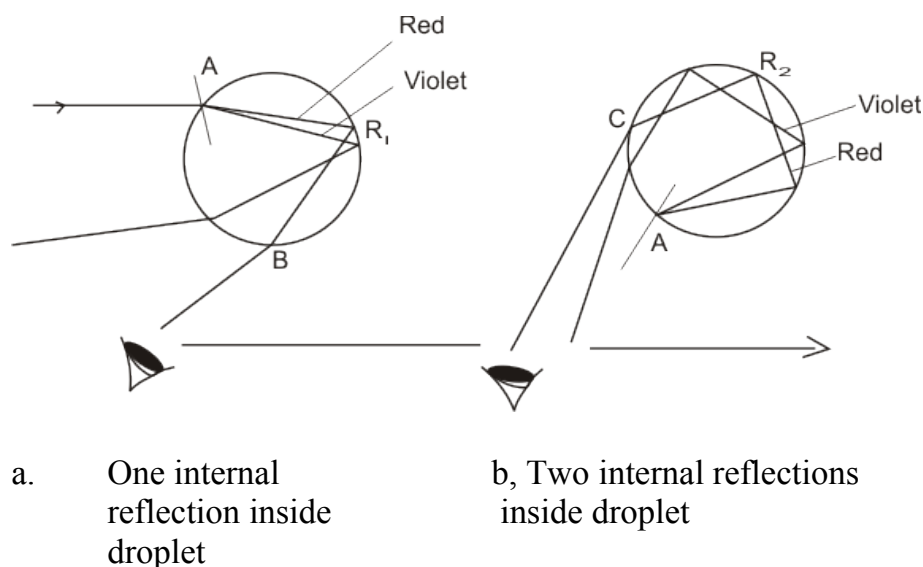


Fig. 7.9: Formation of rainbow- dispersion of sunlight by rain drops

Other droplets seen by the observer at the same angle with the horizontal produce the same colour. Hence, an arc of the colour, red is seen by the observer. Other coloured rays emerge at different angles to the horizontal. Thus, an arc of coloured bands- the rainbow – is seen in the sky. This band has red on its outer edge and violet on the inner edge (Fig. 7.10).



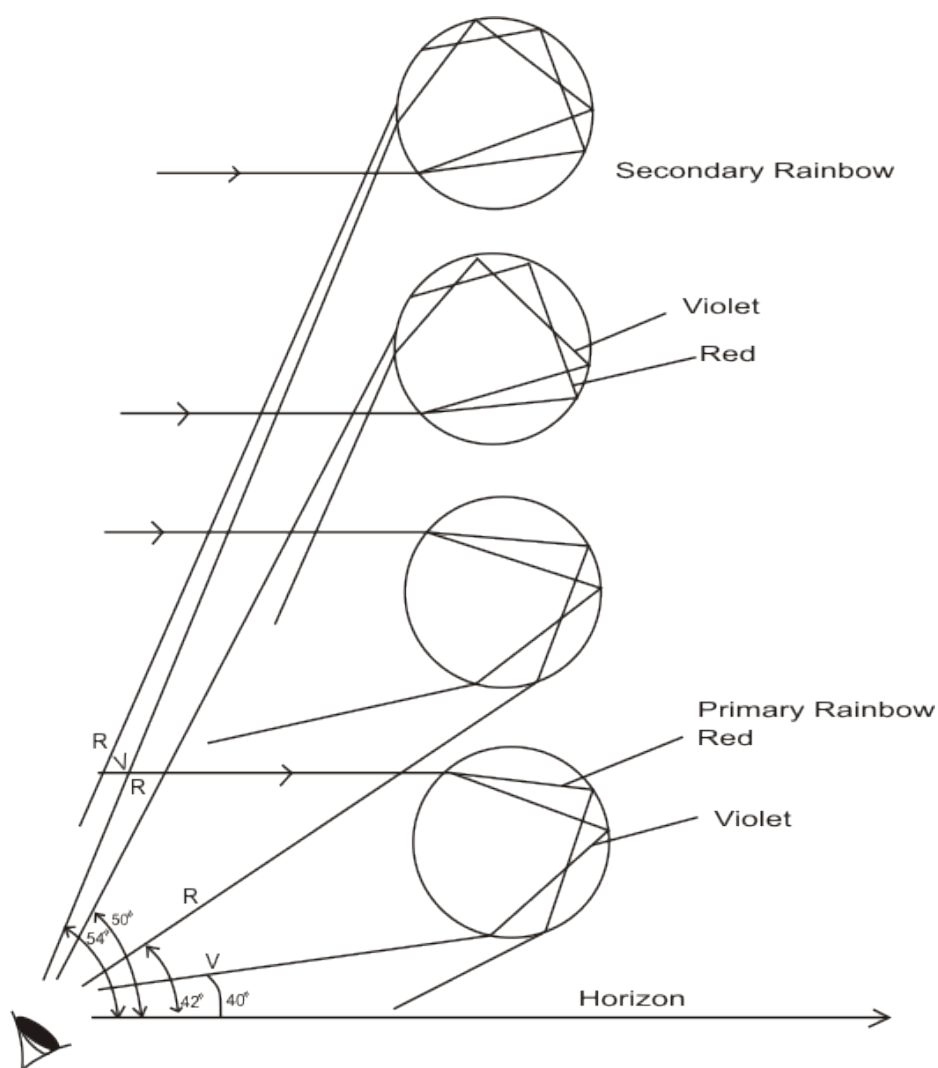


Fig. 7.10: Primary and secondary rainbows

## ii. The Secondary Rainbow

Secondary rainbows may occasionally be seen in addition to the primary ones. This is caused by light rays which undergo two internal reflections inside the rain drops. This causes the coloured bands of such rainbow to be in the reverse order. That is, it has violet on its outer edge and red on its inner edge ( Fig. 7.10) . In the secondary rainbow , the red rays are at angle of  $50^\circ$  and the violet rays at  $54^\circ$  to the horizon

## 3.9 The Electromagnetic Spectrum

You have seen that the spectrum of white light obtained from sun- light consists of seven colours. Beyond each end of this coloured spectrum (visible light) are the invisible rays (spectrum) of radiant energy called infra-red and ultraviolet rays. These rays are also emitted by the sun. These rays cannot be seen but they cause certain materials to



fluorescence and also affect photographic plates. They are known as ultra-violet rays.

Radio waves, infra-red rays, the visible rays (from red to violet) ultra-violet rays, x-rays and gamma rays are all electromagnetic waves. These waves make up the electromagnetic spectrum. They all travel through a vacuum with exactly the same speed and differ only in having different wavelengths.

## **SELF ASSESSMENT EXERCISE 2**

1. What do you understand by the dispersion of light?
2. Draw a clear diagram to show the dispersion of light through a prism
3. On what factors do the colours of objects depend?
4. Distinguish between primary and secondary colours. List the colours under their various groups.
5. What do you understand by white light spectrum? With the aid of a diagram describe how you can obtain a pure spectrum of white light.
6. Name the primary colour for paints
7. Explain how a rainbow is formed. With the aid of diagrams distinguish between primary and secondary rainbows.
8. List the radiations that make up the electromagnetic spectrum

## **4.0 CONCLUSION**

In this unit you learned about the dispersion of light and colours, you learned about the white light spectrum, its combination and production. You also learned about primary, secondary and complementary colours as well as light filters, colour pigments and the appearance of different colours of objects in white light. Finally, you learned about the formation of rainbow and radiation that made up the electromagnetic spectrum.



## 5.0 SUMMARY

- Dispersion is the separation of light into its constituent colours.
- White light is composed of seven colours red, orange, yellow, green, blue, indigo and violet. These colours make up the white light spectrum
- A pure spectrum of white light can be produced by using a bright source of white light, a narrow slit, two converging lenses, a prism and a screen.
- The three primary colours are red, green and blue. Secondary colours are obtained by additive combination (mixing) of any two of the secondary colours. The secondary colours are yellow, cyan and magenta
- Paint (pigments) are mixed by a process of subtractive colour-mixing. The primary colours for paint are red, yellow and blue.
- Light filters are transparent coloured sheets which allow light of the colour of the filter to pass through while absorbing all other colours in the incident light.
- The rainbow is caused by dispersion of sunlight by water droplets left suspended in the air by the rain
- The electromagnetic spectrum is made up of radio waves, infrared rays, the visible rays (from red to violet), ultra- violet rays, x-ray and gamma rays.

## 6.0 TUTOR -MARKED ASSIGNMENT

1. a. On what factors do the colours of an object depend?  
b. Why does a white object appear white in daylight?
2. With the aid of labelled diagram only illustrate the production of pure white light spectrum
3. What is a rainbow? What causes it? Name the two types.



## **7.0 REFERENCES/ FURTHER READINGS**

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