

# BIO 206: BIOSTATISTICS 

## COURSE GUIDE



| Course Code | BIO 206 |
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| Course Title | Biostatistics |
| Course Writer | DR ILIYA S. NDAMS <br> AHMADU BELLO UNIVERSITY, <br> ZARIA |
|  | OLATUNJI AROWOLO <br> School of Science and Tech |
|  | Lagos State Polytechnic |
| Ikorodu- Lagos |  |$\quad$| ABIODUN ADAMS |
| :--- |
| Course Coordinator | | School of Science and Tech |
| :--- |
| National Open University of Nigeria |
| Victorial Island-Lagos |

## NATIONAL OPEN UNIVERSITY OF NIGERIA

Headquarters
14/16 Ahmadu Bello Way
Victoria Island
Lagos
Abuja Annex
245 Samuel Adesujo Ademulegun Street
Central Business District
Opposite Arewa Suites
Abuja
e-mail: centralinfo @ nou.edu.ng

URL www.nou.edu.ng

Statistics for Agriculture and Biology sciences (206) is a second semester course. It is a two credit unit compulsory course which all students offering Bachelor of Science (BSc) in Biology must take.
Statistics is a familiar and accepted part of modern world that is concern with obtaining an insight into the real word by means of the analysis of numerical relationships. It is used in amost all fields of human endevour. It is applied in sports, public health, education, surveys, operations research, quality control, estimation and prediction.
Since this course Statistics for Agriculture and Biology sciences entails analysis of numerical relationships, we will focus on the meaning of statistics and biostatistics(collections of quantitative information and method of handling such data,drawing inferences on the basis of observation), also frequency of distribution,probability, hypothesis,correlation and regression,covariance, ANOVA and the use of statistical package.

## What you will learn in this course

In this course, you have the course units and a course guide. The course guide will tell you briefly what the course is all about. It is a general overview of the course materials you will be using and how to use those materials. It also helps you to allocate the appropriate time to each unit so that you can successfully complete the course within the stipulated time limit.

The course guide also helps you to know how to go about your Tutor-Marked-Assignment which will form part of your overall assessment at the end of the course. Also, there will be tutorial classes that are related to this course, where you can interact with your facilitators and other students. Please I encourage you to attend these tutorial classes.

This course exposes you data collection, management and analysis, the knowledge will be helpful during your project data collection and analysis, it is indeed very interesting field of Biology.

## Course Aims

This course aims to enable you to know/understand the use of different statistical and biostatistical analysis and packages for biological sciences and agricultural data interpretations and inference.

## Course Objectives

To achieve the aim set above, there are objectives. Each unit has a set of objectives presented at the beginning of the unit. These objectives will give you what to concentrate and focus on while studying the unit and during your study to check your progress.

The Comprehensive Objectives of the Course are given below. At the end of the course/after going through this course, you should be able to:

- Discuss the use of statistics in area of biology, agriculture and medicine
- Discuss the different sampling methods and understand the purpose and importance of sampling
- Mention type of frequency distribution.
- Organise data using frequency distribution.
- Explain the normal, poisson and binomial distributions
- Computethe probabilies in poisson and binomial probability distributions
- State the null and alternative statistical hypothesis
- Determine the level of confidence in a biological data
- Explain the relationship between type I and II errors
- Explain the purpose of goodnessof fit test
- Compute correlation and regression
- Explain types of correlation and Regression
- Explain the principle of experimental design
- Define Anova and test statistical hypothesis using Anova
- Compute the spear rank correlation coefficient
- Give the difference between non-parametric and parametric test
- Describe the applications of SPSS and MINITAB in different statistical procedures
- Mention example of statistical tools in the statistical packages.


## Working through the Course

To successfully complete this course. You are required to read each study unit, read the textbooks and other materials provided by the National Open University.

Reading the reference materials can also be of great assistance.
Each unit has self -assessment exercise which you are advised to do. At certain periods during the course you will be required to submit your assignments for the purpose of assessment.

There will be a final examination at the end of the course. The course should take you about 17 weeks to complete.

This course guide provides you with all the components of the course, how to go about studying and how you should allocate your time to each unit so as to finish on time and successfully.

## The Course Materials

The main components of the course are:
1 The Study Guide
2 Study Units
3 Reference/ Further Readings
4 Assignments
5 Presentation Schedule

## Study Units

The study units in this course are given below:
BIO 206 Statistics for Agriculture and Biology sciences (2 UNITS)
Unit 1: Use of Statistics in Biology and Agriculture
Unit 2: Frequency distribution
Unit 3: Probability distributions
Unit 4: Estimation and hypothesis testing
Unit 5: Contigency tables
Unit 6: Correlation, Regression and covariance
Unit 7: Simple Experimental design and Analysis of Variance (ANOVA)
Unit 8: Non-Parametric Tests
Unit 9: Use of statistical packages

In unit one, the mening of statistics and biostatistics, the application of statistics in biology-related fields and the limitation of such applications

Unit two and three explain frequency distribution types, and how it can be use to organise data.Also types of probability distribution; normal, poisson and binomial.

Unit four and five are concerned with estimation and hypothesis testing; null and alternative hypotheses,level of confidence in biological data,Test hypotheses involving means using z and t tests.Also, contigency tables;chi-square distribution and goodness of fit test.

Unit six deals with correlation and regression and covariance; bivariate distribution and scatter diagram.

Unit seven and eight explain simple experimental design and analysis of variance (ANOVA);principle of experimentation and Anova and test statistical hypothesis. Also, Non-parametric tests; Sign test and Kruskal Wallis test.

Unit nine discusses the SPSS(Statistical Package for Social Sciences) and MINITAB

Each unit will take a week or two lectures, will include an introduction, objectives, reading materials, self assessment question(s), conclusion, summary, tutor-marked assignments (TMAs), references and other reading resources.

There are activities related to the lecture in each unit which will help your progress and comprehension of the unit. You are required to work on these exercises which together with the TMAs will enable you to achieve the objective of each unit.

## Presentation Schedule

There is a time-table prepared for the early and timely completion and submissions of your TMAs as well as attending the tutorial classes. You are required to submit all your assignments by the stipulated date and time. Avoid falling behind the schedule time.


#### Abstract

Assessment There are three aspects to the assessment of this course. The first one is the self-assessment exercises. The second is the tutormarked assignments and the third is the written examination or the examination to be taken at the end of the course.

Do the exercises or activities in the unit applying the information and knowledge you acquired during the course. The tutor-marked assignments must be submitted to your facilitator for formal assessment in accordance


with the deadlines stated in the presentation schedule and the assignment file.

The work submitted to your tutor for assessment will account for $30 \%$ of your total work.

At the end of this course you have to sit for a final or end of course examination of about a three hour duration which will account for $70 \%$ of your total course mark.

## Tutor Marked Assignment

This is the continuous assessment component of this course and it accounts for $30 \%$ of the total score. You will be given four (4) TMAs by your facilitator to answer. Three of which must be answered before you are allowed to sit for the end of the course examination.

These answered assignments must be returned to your facilitator.
You are expected to complete the assignments by using the information and material in your reading references and study units.

Reading and researching into the references will give you a wider view point and give you a deeper understanding of the subject.

1 Make sure that each assignment reaches your facilitator on or before the deadline given in the presentation schedule and assignment file. If for any reason you are not able to complete your assignment, make sure you contact your facilitator before the assignment is due to discuss the possibility of an extension. Request for extension will not be granted after the due date unless there is an exceptional circumstance.
2 Make sure you revise the whole course content before sitting for examination. The self-assessment activities and TMAs will be useful for this purposes and if you have any comments please do before the examination. The end of course examination covers information from all parts of the course.

## Course Marking Scheme

| Assignment | Marks |
| :--- | :--- |
| Assignment 1-4 | Four assignments, best three <br> marks of the four count at 10\% <br> each - 30\% of course marks. |
| End of course examination | $70 \%$ of overall course marks |
| Total | $100 \%$ of course materials |

## Facilitators/ Tutors and Tutorials

Sixteen (16) hours are provided for tutorials for this course. You will be notified of the dates, times and location for these tutorial classes.

As soon as you are allocated a tutorial group, the name and phone number of your facilitator will be given to you.

These are the duties of your facilitator:

- He or she will mark and comment on your assignment
- He will monitor your progress and provide any necessary assistance you need.
- He or she will mark your TMAs and return to you as soon as possible.
(You are expected to mail your tutored assignment to your facilitators at least two days before the schedule date).

Do not delay to contact your facilitator by telephone or e-mail for necessary assistance if

- You do not understand any part of the study in the course material.
- You have difficulty with the self assessment activities.
- You have a problem or question with an assignment or with the grading of the assignment.

It is important and necessary you attend the tutorial classes because this is the only chance to have face to face contact with your facilitator and to ask questions which will be answered instantly. It is also a period where you can point out any problem encountered in the course of your study.

## Summary

Statistics for Agriculture and Biology sciences (206) deals with the ways of collecting, organizing,summarizing and describing quantifiable data, and methods of drawing inferences and generalizing upon them.

Also, this course has been able to explain the data collection and analysis using statistical tools derived from the fields of biological sciences; medicine, pharmacy, Biochemistry, Microbiology, agricultural sciences and other biology-related areas

On the completion of this course, you will how and when a statistical package is used for biological data. In addition you will be able to answer the following questions:

- What is sample size?
- What is considered the goal of sampling?
- List three incorrect methods that are often used to obtain a sample
- Names and three of frequency distributions
- Outline the stages involved in the construction of a frequency
- Ouline five characteristics of a normal curve
- Construct a probability distribution for three patients given a head ache relief tablet. The probabilities for 0.1 .2 ro 3 success are $0,18,0.52,0,21$, and 0.09 , respecrively.
- If the hypothesis of a population mean is 151 (i.e $H_{\mathrm{O}}$ : $\mu=151$ ).List the three possible hypotheses
- What is implication of committing a Type II error?
- What is a right tailled test with $\alpha=50$. If the sample of 36 had $6, x=$ 475. Conduct a two tailed test with $\alpha=0.01$.
- Distinguished between Observed and Expected frequencies
- Give two examples in nature of two variables that are positively correlated and two that are negatively correlated
- Outline and briefly discuss the principles involved in experimental

The list of questions you are expected to answer is not limited to the above list.

I believe you will agree with me that Statistics for Agriculture and Biology sciences is a very interesting field of biology.

I wish you success in this course.

## BIO 206: BIOSTATISTICS

## COURSE MATERIAL



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|  | OLATUNJI AROWOLO <br> Content Editor |
|  | School of Science and Tech <br> Lagos State Polytechnic <br> Ikorodu- Lagos |
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Headquarters
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Victoria Island
Lagos

## Abuja Annex

245 Samuel Adesujo Ademulegun Street

## Central Business District

Opposite Arewa Suites
Abuja
e-mail: centralinfo @ nou.edu.ng
URL www.nou.edu.ng

NATIONAL OPEN UNIVERSITY OF NIGERIA
STATISTICS FOR BIOLOGY AND AGRICULTURE
BY
DR ILIYA S. NDAMS
AHMADU BELLO UNIVERSITY, ZARIA

UNIT ONE:
USE OF STATISTICS IN BIOLOGY \& AGRICULTURE

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1.3.4 SAMPLING
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1.3.8 RANDOM SAMPLING
1.3.9 STRATIFIED SAMPLING
1.3.10 CLUSTER SAMPLING
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| 2.3 .5 | OTHER FORMS OF DATA REPRESENTATIONS |
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## MODULE 1

## UNIT ONE: USE OF STATISTICS IN BIOLOGY \& AGRICULTURE

### 1.0 INTRODUCTION:

Statistics is a familiar and accepted part of modern world that is concern with obtaining an insight into the real world by means of the analysis of numerical relationships. It is used in almost all fields of human endeavour. It is applied in sports, public health, education, surveys, operations research, quality control, estimation and prediction.

This unit discusses the meaning of Statistics and Biostatistics, the application of statistics in biology-related fields and the limitation of such applications.

### 1.2 OBJECTIVES:

At the end of this unit, you should be able to:
i) Define the terms Statistics and Biostatistics.
ii) Discuss the use of statistics in areas of biology, agriculture and medicine.
iii) Discuss discrete and continuous variables
iv) Define a sample and sampling
v) Discuss the different sampling methods and understand the purpose and importance of sampling and the advantages made possible by sampling
vi). Explain sampling distribution

### 1.3.1 STATISTICS AND BIOSTATISTICS

The word statistics is used in two senses. It refers to collections of quantitative information, and to methods of handling that sort of data i.e. descriptive statistics. It also refers to the drawing of inferences about large groups on the basis of observations made on smaller one i.e. inferential statistics.

Statistics, then, is to do with ways of collecting, organizing, summarizing and describing quantifiable data, and methods of drawing inferences and generalizing upon them. While the term Biostatistics is used when the data that are being analysed using statistical tools, are derived from the fields of biological sciences: Medicine, Pharmacy, Biochemistry, Microbiology, Agricultural Sciences and other biology-related areas.

### 1.3.2 USE OF STATISTICS IN BIOLOGY, AGRICULTURE AND MEDICINE.

Unlike other fields of science such as the physical sciences of chemistry and physics, variation is regarded as a fundamental feature in natural sciences of biology, agriculture and medicine. Biostatistics helps to explain this natural variation inherent in these fields of natural sciences. For example, variation may occur due to age of the population or may occur among individuals of a population due to diseases or their genetic makeup. Experimental design is an important aspect of biostatistics that describe on how to collect, organize, summarize and analyze data such that valid and objective conclusions or decision about the population can be drawn.

Therefore before applying statistics in research, a research must know:

- What technique to use for an investigation
- What to be achieved
- Rules of using the technique, using correctly the statistical techniques for analysis of biological data
- Statistical significance test for comparing one set of data with another.
- Determination of relationship between two variables either the use of correlation or fitting the best straight line or curve on a graph.


### 1.3.3 DISCRETE AND CONTINUOUS VARIABLES

To gain knowledge about secondly haphazard events, statistician collect information for variables which describe the event. Therefore, a variable is a characteristics attribute that can assume different value.

Variables can be classified into two broad categories.

1. Qualitative variables
2. Quantitative variables

Qualitative variables are variables that can be placed into distinct categories, according to some characteristic or attribute. For example, if subjects are classified according to gender (i.e. male or female), then the variable gender is qualitative.

Quantitative variables are numerical and can be ordered or ranked. For example, the variable age is numerical, and people can be ranked according to the value of their ages. Quantitative variables can be grouped into two:

1. Discrete Variables - can be assigned values such as $0,1,2,3$ (integers) and are said to be variables that assume values that can be counted. Examples include number of children in a family, number of birds in a pen, number of trees in a garden, number of animals per litter etc.
2. Continuous variables - can assume all values between any specific values. They are obtained by measuring. This applies to variables such as length, weight, height, yield, temperature and time, that can be thought of as capable of assuming any value in some interval of values.

Figure1.1 : Summary of classification of variables.


NOTE: Data are the values (measurements and observations) that the variables can assume

### 1.3.4 SAMPLING

When a set of observations is collected from a population, the population mean $(\mu)$, population variance $\left(\sigma^{2}\right)$ and population standard deviation $(\sigma)$ can be
computed from it as the properties of the population. In the case of a sample, the parameters that describe it are the sample mean (x), sample variance $\left(\mathrm{s}^{2}\right)$ and sample standard deviation (s). Since the sample is a portion of the population, the parameters of the sample represent an estimate of the true parameters of the population. Therefore, sampling is a random process of selecting a sample from a population selected for study.

### 1.3.5 SAMPLE

A sample is a subgroup of the population selected for study. When a sample is chosen at random from a population, it is said to be an unbiased sample. That is, the sample for the most part, is representative of the population. But if a sample is selected incorrectly, it may be a biased sample when some type of systematic error has been made in the selection of the subjects. However, the sample must be random in order to make valid inferences about the population.

### 1.3.6 IMPORTANCE OF SAMPLE / SAMPLING

A sample is used to get information about a population for several reasons:

1. It saves the researcher time and money.
2. It enables the researcher to get information that he or she might not be able to obtain otherwise.
3. It enables the researcher to get more detailed information about a particular subject.

### 1.3.7 SAMPLING METHODS

In order to obtain unbiased samples, several sampling methods have been developed. The most common methods are random, systematic, stratified, and clustered sampling.

### 1.3.8 RANDOM SAMPLING

For a sample to be a random sample, every member of the population must have an equal chance of being selected. Therefore, a random sample is one that has the same chance as any other of being selected.

Randomness assists in avoiding various forms of conscious and unconscious bias and can be achieved by these two ways:

1. Number each element of the population and then place the numbers on cards. Place the cards in a hat or bowl, mix them, and then select the sample by drawing the cards. You must ensure that the numbers are well mixed.
2. The second and most preferred way of selecting a random sample is to use random numbers e.g. Table of random numbers by Fisher and Yates. The table comprises of a series of digits $0,1,2 \ldots$ up to 9 arranged as such that each number had the same chance of appearing in any given position.

### 1.3.9 STRATIFIED SAMPLING

A stratified sample is a sample obtained by dividing the population into subgroups, called strata, according to various homogenous (alike) characteristics and then selecting members from each stratum for the sample. For example, you
can group the items on basis of their age, size, colour etc. The advantage of stratified sampling is that it increases precision because all types of groups are represented through stratification and a heterogeneous population is made into a homogenous one.

### 1.3.10 CLUSTER SAMPLING

A cluster sample is a sample obtained by selecting a preexisting or natural group, called a Cluster and using the members in the cluster for the sample. For example a habitat, or a large area or field is divided into smaller units and a number of such units are randomly selected and used as a sample.

There are three advantages to using a cluster sample instead of other types of sample:

1. A cluster sample can reduce cost
2. It can simplify field work.
3. It is convenient.

The major disadvantage of cluster sampling is that the elements in a cluster may not have the same variations in characteristics as elements selected individually from a population.

### 1.3.11 SYSTEMATIC OR SKIP SAMPLING

This method involves taking an item as a sample from a larger population at regular intervals. For example, when sampling from a poultry farm, every third
or fifth or tenth chick coming out of the cage is taken and included in the sample. This is done after the first number is selected at random for counting to start.

### 1.3.12 PROPORTIONATE SAMPLING

This type of sampling involves selecting a sample in proportion to the different groups in the population under study. For example: Assuming in a given terrestrial habitat there are the following different proportions of organism:
Trees - 100

Shrubs - 150

Vertebrates - 60

Invertebrates - 250

In sampling such a population, you may wish to pick 15 trees, 10 shrubs, 5 vertebrates, 20 invertebrates. These will form your sample.

In addition to the above sampling methods, other methods are sometimes used.

- In sequence sampling - successive units taken from production lines are sampled to ensure that products meet certain standards set by the manufacturing company. This is used in quality control.
- In double sample, a very large population is given a questionnaire to determine those who meet the qualifications for a study. After the questionnaires are reviewed, a second, smaller population is defined. Then a sample is selected from this group.
- In multistage sampling, the researcher uses a combination of sampling methods.


### 1.3.13 SAMPLING DISTRIBUTION

If a sample of n observation is taken at random from a population, the sample is expected to have a mean $x$. Suppose another sample also of $n$ observations is taken from the same population, it will similarly have a mean $x$ (as the first one). The numerical values of these means will differ slightly because even though the samples are taken from the same population, the representative members of the two samples differ. As you take samples from the same population so will you get different numerical values for their means. This set of numerical values is called the Sampling distribution of the mean and it is determined by the nature of the population and the sample size. Therefore, a sampling distribution is the distribution of values from a mass of samples, one value per sample. If the sample size is large then the sampling distribution of the mean approximates very closely to a normal curve. This implies that the mean of the sampling distribution of means is equal to the population mean.

## Examples

1. Identify the following as either discrete or continuous data:
(a) Number of patients coming to the hospital each day
(b) Lifetimes of micro organism in a certain pond
(c) Heights of 200 birds in a certain farm
(d) Yearly salary of an agronomist
(e) Temperatures recorded every 30 minutes of a little boy suffering from malaria fever

Answer
(a) Discrete (b) continuous (c) continuous (d) Discrete (e) continuous
2. Identify each of the following sampling methods.
(a) Every fish in a fish pond has equal chances of being included in a sample
(b) Every corn plant that is 2 m apart in a particular farm will be included in a sample
(c) In surveying piggeries within a region, we choose to select 50 pig farms blocks and then investigate every pigs within the selected blocks.

Ans
(a) Random (b) Systematic (c) Cluster

### 1.4 TUTOR MARKED ASSIGNMENT

1. Why do you need statistics in science based disciplines?
2. What is a sample size?
3. What is the purpose of sampling?
4. What is considered the goal of sampling?
5. In your opinion, which sampling method(s) provided the best sample to represent a population of trees in a forest?
6. List three incorrect methods that are often used to obtain a sample.

### 1.5 REFERENCES

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## UNIT TWO: FREQUENCY DISTRIBUTION

### 2.1 INTRODUCTION:

Measurements or counting gives rise to raw data. Raw data itself is difficult to comprehend because it lacks organization, summarization, which renders it meaningless. Thus, the raw data has to be put in some order through classification and tabulation so as to reduce its volume and heterogeneity. To describe situations, draw conclusions or make inferences about events, the researcher must organize the data in some meaningful way. The most convenient method of organizing data is to construct a frequency distribution.

### 2.2 OBJECTIVES:

At the end of this unit, you should be able to
i) Define frequency and frequency distribution.
ii) Mention the types of frequency distribution
iii) organize data using frequency distributions
iv) Give reasons for constructing distribution.
v) Represent data in methods other than frequency distribution.

### 2.3 MAIN CONTENTS

### 2.3.1 The Frequency distribution

Frequency is the number of occurrences of an element in a sample and is symbolized by $f$. A frequency distribution is the organization of raw data in table form, using classes and frequencies. When data are collected in original form, that is as observed or recorded they are called raw data.

### 2.3.2 Types of Frequency Distribution

Two types of frequency distributions that are most often used are the:

## Categorical Frequency

This is used for data that can be placed in specific categories, such as nominal or ordinal-level data. It is useful to know the proportion of values that fall within a group, category or observation rather than the number of values or frequencies. To get the relative frequency, the frequency of occurrence of each number is divided by the total number of values and multiplied by hundred. This can be expressed as follows:
$\frac{\mathrm{f}}{\mathrm{n}} \times 100 \%$

Where $\mathrm{f}=$ Frequency of the category class and $\mathrm{n}=$ total number of values.

## For example:

The data below represents the blood groups of 40 students in a Biostatistics class.
Construct a frequency distribution for the data.

| $\mathbf{A}$ | $\mathbf{A B}$ | $\mathbf{B}$ | $\mathbf{O}$ | $\mathbf{O}$ | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{A B}$ | $\mathbf{A}$ | $\mathbf{B}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{O}$ | $\mathbf{O}$ | $\mathbf{O}$ | $\mathbf{A}$ | $\mathbf{A B}$ | $\mathbf{B}$ | $\mathbf{B}$ | $\mathbf{A}$ | $\mathbf{O}$ | $\mathbf{A B}$ |
| $\mathbf{A}$ | $\mathbf{O}$ | $\mathbf{O}$ | $\mathbf{A}$ | $\mathbf{A B}$ | $\mathbf{B}$ | $\mathbf{B}$ | $\mathbf{A}$ | $\mathbf{A}$ | $\mathbf{B}$ |
| $\mathbf{A B}$ | $\mathbf{A}$ | $\mathbf{O}$ | $\mathbf{B}$ | $\mathbf{A B}$ | $\mathbf{O}$ | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{A}$ | $\mathbf{B}$ |

## SOLUTION:

Since the data are categorical, the blood groups: A, B, O and AB can be used as the classes for the distribution.

| Class | Tally | Frequency | Percent |
| :--- | :--- | :--- | :--- |
| A | $/ / / / / / / / / / / / / / / / /$ | 12 | 30 |
| B | $/ / / / / / / / / / / / / / / / / / / /$ | 11 | 27.5 |
| O | 10 | 25 |  |
| AB | $/ / / / / / / /$ | 7 | 17.5 |
| TOTAL |  |  | $\mathbf{1 0 0}$ |

Therefore, it can be concluded that in the sample more students have type A blood group because its frequency is the highest.

### 2.3.3 UNGROUPED FREQUENCY DISTRIBUTION

This is a list of the figures in array form, occurring in the raw data, together with the frequency of each figure, i.e. a frequency is constructed for a data based on a single data values for each class.
For example: Given below, are the wing length measurements (to the nearest whole millimeter) of 50 laughing doves.

| 76 | 73 | 75 | 73 | 74 | 74 | 72 | 75 | 76 | 73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 68 | 72 | 78 | 74 | 75 | 72 | 76 | 76 | 77 | 70 |
| 78 | 72 | 70 | 74 | 76 | 75 | 75 | 79 | 75 | 74 |
| 75 | 70 | 73 | 75 | 70 | 74 | 76 | 74 | 75 | 74 |
| 78 | 74 | 75 | 74 | 73 | 74 | 71 | 72 | 71 | 79 |

Construct the frequency distribution of the above data.

## SOLUTION:

The measurements above are presented in the order in which the observations were recorded. This can be represented in an ordered array so that the minimum and maximum values can easily be read.

| 68 | 70 | 70 | 70 | 70 | 71 | 71 | 72 | 72 | 72 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 72 | 72 | 73 | 73 | 73 | 73 | 73 | 74 | 74 | 74 |
| 74 | 74 | 74 | 74 | 74 | 74 | 74 | 75 | 75 | 75 |
| 75 | 75 | 75 | 75 | 75 | 75 | 75 | 76 | 76 | 76 |
| 76 | 76 | 76 | 77 | 78 | 78 | 78 | 78 | 79 | 79 |

Find the range of the data: Highest value - lowest value $(79-68=11)$. Since the range of the data is small, classes of single data values can be used.

Table 2.1: A tally of frequency of the wing length (mm) of 50 laughing doves.

| Class limits | Tally | Frequency | Cumulative <br> frequency | Relative <br> frequency <br> (\%) |
| :--- | :--- | :--- | :--- | :--- |
| 68 | $/$ | 1 | 1 | 2 |
| 70 | $/ / / /$ | 4 | 5 | 8 |
| 71 | $/ /$ | 2 | 7 | 4 |
| 72 | $/ / / / /$ | 5 | 12 | 10 |
| 73 | $/ / / / /$ | 5 | 17 | 10 |
| 74 | $/ / / / / / / / / / / / / / / / / /$ | 10 | 27 | 20 |
| 75 | $/ / / / / /$ | 6 | 37 | 20 |
| 76 | $/$ | 1 | 43 | 12 |
| 77 | $/ / / / /$ | 4 | 44 | 2 |
| 78 | $/ /$ | 2 | 48 | 8 |
| 79 |  |  | 50 | 4 |
|  |  |  | $\mathbf{1 0 0}$ |  |

Figures 2.1 (A-C): The frequency distribution of the wing length of 50 laughing doves
(A)

(B)

(C)


### 2.34 GROUPED FREQUENCY DISTRIBUTION

The heights in inches of commonly grown herbs are shown below. Organize the data into a frequency distribution with six classes, and make useful suggestions.

| 18 | 20 | 18 | 18 | 24 | 10 | 15 | 12 | 29 | 36 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 13 | 20 | 18 | 24 | 18 | 16 | 16 | 20 | 7 |  |

## Solution:

Find the range of the data: Highest value - lowest value ( $36-7=29$ )
The class width is given by $\frac{\text { Range }}{\text { Number of classes }}=\frac{29}{6}=4.8$ (round it up to the nearest whole number $=5$ )

Table 2.2: Frequency distribution for grouped data

| Class <br> limits | Class <br> boundaries | Tally | Frequency | Cumulative <br> frequency | Relative <br> frequency <br> $(\%)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $5-10$ | $4.5-10.5$ | $/ /$ | 2 | 2 | 10.5 |
| $11-16$ | $10.5-16.5$ | $/ / / / / / / / / /$ | 5 | 7 | 26.3 |
| $17-22$ | $16.5-22.5$ | $/ / / / / / /$ | 2 | 15 | 42.1 |
| $23-28$ | $22.5-28.5$ | $/ /$ | 2 | 17 | 10.5 |
| $29-34$ | $28.5-34.5$ | $/$ | 1 | 18 | 5.3 |
| $35-40$ | $34.5-40.5$ | $/$ | 1 | 19 | 5.3 |
| Total |  |  |  |  | $\mathbf{1 0 0}$ |

## Rules to be followed in the construction of a frequency

1. There should be enough classes to clearly represent the data. Classes between 5 and 20 are mostly suggested.
2. The class width should be an odd number. The class midpoint $\left(X_{m}\right)$ is given by

$$
\mathrm{X}_{\mathrm{m}}=\underline{\text { Lower boundary }+ \text { Upper boundary }} \text { OR Lower limit + Upper limit }
$$

$$
2
$$

$$
2
$$

Midpoint is the numeric location of the center of the class
3. The classes must not have overlapping class limits e.g.

Class But should be: Class
500-100
50-100
100-150
101-151
150-200
200-250
152-202
203-253
4. The classes must be continuous, even if there are no values in a class ie there should be no gaps in the classes for lack of values.
5. Enough classes should be created to accommodate the whole data. i.e. every value in the data must belong to a class. Note: If zero frequency is the first or last, then it can be ignored.
6. The classes must be of equal width. In rule number 3 above, the class width is 50 . Here, it is important to note that some times in open ended distribution i.e. distribution that has no specific beginning or ending value as:

## Class

50-100
101-151
152-202
203-253
254 and above

## Temperature ${ }^{\circ} \mathbf{C}$

## Below 5

6-10
11-15
16-20
21-25

In the class distribution above any value above 254 is tallied in the last class while in the distribution for temperature, simply means that any value below $5^{\circ} \mathrm{C}$ will be tallied in the first class.

## Effect of grouping

As a result of grouping, it is possible to detect a pattern in the figures but grouping results in the loss of information i.e. calculations made from a grouped frequency distribution can never be exact, and consequently excessive accuracy can only result in spurious accuracy.

## The reasons for constructing a frequency distribution are:

1. To organize the data in a meaningful, intelligible way.
2. To enable the reader to determine the nature and shape of the distribution.
3. To facilitate computational procedures for measures of average and spread.
4. To enable the researcher to draw charts and graphs for the presentation of data.
5. To enable the reader to make comparisons among different data sets.

### 2.3.5 OTHER FORMS OF DATA REPRESENTATIONS INCLUDE:

Figure 2.3: A bar chart of the nutrient content of a seed


Figure 2.4: A scatter plot or dot diagram of the wing length of 50 laughing doves


Figure 2.5: A cummulative frequency of the wing length of 50 laughing doves


Figure 2.6: A Pie chart showing the distribution of fruits in an orchard


### 2.4 TUTOR MARKED ASSIGNMENT

1. Name the three types of frequency distributions.
2. Outline the stages involved in the construction of a frequency.
3. The data below represents the number of bats trapped in mist net in 30 trials.

| 2 | 9 | 4 | 3 | 6 |
| :--- | :--- | :--- | :--- | :--- |
| 6 | 2 | 8 | 6 | 5 |
| 7 | 5 | 3 | 8 | 6 |
| 6 | 2 | 3 | 2 | 4 |
| 6 | 9 | 9 | 8 | 9 |
| 4 | 2 | 1 | 7 | 4 |

(a) Construct an ungrouped frequency distribution for the data.
(b) Construct a histogram for the data.

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## UNIT THREE: PROBABILITY DISTRIBUTIONS

### 3.1 INTRODUCTION

Probability is a branch of mathematics which as a general concept can be defined as the chance of an event occurring. It is the basis of inferential statistics. This unit looks at three particular distributions, the Normal, the Poisson and the Binomial - all of which are important in sampling theory.

### 3.2 OBJECTIVES:

At the end of this unit, you should be able to
i) Distinguish between frequency and probability distributions
ii) Explain the Normal, Poisson and Binomial distributions
iii) Compute the probabilities in Poisson and binomial probability distributions.

### 3.3 MAIN CONTENTS

### 3.3.1 PROBABILITY DISTRIBUTION

A distribution is a scatter of related values, such as the assortment of weights in a group of cattle. A frequency distribution shows us how many times given values in a range of values occur. A Probability distribution is very similar because it shows us how probable given random variable values in a range of such values are. For example: if we toss two coins we can obtain 0,1 or 2 'heads'. If we prepare a table showing the probabilities of all the random variable values we will have the probability distribution as shown below.

| Number of 'heads' | Sequential event | Probability |
| :---: | :---: | :---: |
| 0 | TT | $0.5 \times 0.5=0.25$ |
| 1 | \{HT | $0.5 \times 0.5\}=0.50$ |
|  | \{TH | $0.5 \times 0.5\}$ |
| 2 | HH | $0.5 \times 0.5=0.25$ |
| Total |  | 1.00 |

(Note that the sum of a probability distribution must be equals to 1 )
Therefore a probability distribution is simply a complete listing of all possible outcomes of an experiment, together with their probabilities.

### 3.3.2 THE NORMAL DISTRIBUTION

This is the most important distribution in statistics. It is also known as the Gaussian distribution named after Gauss, a German astronomer who showed its use in statistics. The normal distribution is defined by just two statistics, the mean and the standard deviation. Normal distribution is concerned with results obtained by taking measurements on continuous random variable (i.e the quantified value of a random event) like weight, yield etc. The normal distribution is a particular pattern of variation of numbers around the mean. It is symmetrical (hence we express the standard deviation as $\pm$ ) and the frequency of individual numbers falls off equally away from the mean in both directions. In terms of human height, progressively larger and smaller people than the average occur symmetrically with decreasing frequency towards respectively giants or dwarfs. What is important
about this distribution is not only that this kind of natural variation often occurs, but also that it is the distribution which comes with the best statistical reference for data analysis and testing of hypotheses. It so happens that the curve given by this probabilities distribution approximates very closely to a Mathematical_curve. This curve is called the Normal curve.

In checking for normality, it is important to know whether an experimental data is an approximate fit to a normal distribution. This is easily checked with large samples. There should be roughly equal numbers of observations on either side of the mean. Things are more difficult when we have only a few samples. In experiments, it is not uncommon to have no more than three data per treatment. However, even here we can get clues. If the distribution is normal, there should be no relationship between the magnitude of the mean and its standard deviation.

## Figure 3.1: Normal curve from a normal distribution



## Properties of a Normal Curve

- It is a Unimodal symmetrical curve
- The mean, mode \& median all coincide, thereby dividing the curve into two equal parts
- Most items on the curve are clustered around the mean
- No kurtosis or skewness in the curve
- The area beneath the curve is proportional to the observation associate with the part.

Figure 3.2: Normal curve with standard deviations


## Important Aspect or characteristics of the curve

The important aspect of the curve is the area in relationship with probability, if perpendiculars are erected at a distance of $1 \sigma$ from the mean and in both directions, the area covered by these perpendiculars and the curve will be about $68.26 \%$ of the total area. (It means that $68.26 \%$ of all the frequencies are formed within one standard deviation of the mean). The total probability encompassed by the area under the curve is $1(100 \%)$.

Normal distribution is defined by; $y=\frac{1}{\sigma \sqrt{2 \pi}} \ell^{-(x-u)^{2} /\left(2 \sigma^{2}\right)} \quad-\infty<x<\infty$

The measures of central tendency ( $\mu=$ Population mean) and dispersion ( $\sigma=$ Population standard deviation) are the parameters of the distribution and once they have been estimated for a particular population, the shape of its distribution curve can be worked out using the normal curve formula. Usually we do not know the values of $\mu$ and $\sigma$ and have to estimate them from a sample as $x$ and $s$. If the number of observations in the sample exceeds about 30 , then $x$ and $s$ are considered to be reliable estimates of the parameters.

### 3.3.3 STANDARDIZING THE NORMAL CURVE

Any value of an observation $X$ on the baseline of a normal curve can be standardized as a number of standard deviation units, the observation is away from the population mean, $\mu$. This is called a $z$-score. To transform $x$ into $z$ the formula is given by: $\quad z=\frac{(x-\mu)}{\sigma}$

If the population mean $\mu$ is larger than the sample mean $x$, the $z$ is negative. But if the sample size is more than about 30 observations, the sample mean (x) and standard deviation (s) are considered to be good estimates of $\mu$ and $\sigma$, and z is given by:

$$
z=\frac{(x-\mu)}{s}
$$

If the calculated value of z is larger than 1.96 (i.e. $\mathrm{P}<0.05$ or $95 \%$ confidence coefficient) then this is regarded as unlikely or statistically significant.

### 3.3.4 POISSON DISTRIBUTION

A Poisson distribution is a discrete probability distribution that is useful when $\mathbf{n}$ is larger and $\mathbf{p}$ is small and when the independent variables occur over a period of time. It can be used when a density of items is distributed over a given area or volume, such as the number of plants growing per acre. It can also be used to discover whether organisms are randomly distributed. For example, in ecological studies, Poisson distribution is used to describe the spread of organisms like insects, trees, and snails' etc. by the following:

1. Divide the large area into small squares of equal size
2. Count the particular animal or plant species under study in each square
3. You can also randomly select a number of squares, if the area is two large. The probability of $X$ occurrences in an interval of time, volume, area etc. for a variable where $\lambda$ (lambda) is the mean number of occurrences per unit (time, volume, area etc) is given by:

$$
\begin{aligned}
& P(x, \lambda)=\frac{\ell^{-\lambda} \lambda^{x}}{x!} \\
& \text { where } x=0,1,2,3, \ldots . . . . . . . . . . . . . . . . . . . . . . . . ~
\end{aligned}
$$

$e=$ constant, approximately equal to 2.7183

## For Example:

In a study on the distribution of tree-roosting birds, if there are 200 birds randomly distributed on 500 trees, find the probability that a given tree contains exactly three birds.

## Solution:

First, find the mean number $\lambda$ of birds on each tree

$$
\lambda=\frac{200}{500}=\frac{2}{5}=0.4
$$

That is 0.4 birds per tree. Therefore $\lambda=0.4$, while $\mathrm{x}=3$.
We can then substitute in the formular above.

$$
P(3 ; 0.4)=\frac{(2.7183)^{-0.4}(0.4)^{3}}{3!}=0.0072
$$

Thus, there is less than a $1 \%$ probability that any given tree will contain exactly three birds.

### 3.3.5 BINOMIAL DISTRIBUTION

A binomial experiment is a probability experiment that satisfies the following four requirements:

1. Each trial can have only two outcomes or outcomes that can be reduced to two outcomes. i.e these outcomes can either be success or failure. No two events can occur simultaneously.
2. There must be a fixed number of trails.
3. The outcomes of each trial must be independent of each other.
4. The probability of a success must remain the same for each trial.

A binomial distribution is a special probability distribution that describes the distribution of probabilities when there are only two possible outcomes for each trial of an experiment.

## Examples:

1. The answer to a multiple choice question (even though there are four or five answer choices) can be classified as correct or incorrect.
2. When tossing a coin, you get either a head or a tail.
3. In selecting individuals from human population you select either a male or a female, a boy or a girl etc.

The binomial probability formula is given by:

$$
P(x)=\frac{n!}{(n-x)!x!} p^{n} q^{n-x}
$$

Where: $\quad \mathrm{P}=\quad$ Numerical probability of a success $=\mathrm{P}(\mathrm{s})$
$q=\quad$ Numerical probability of a failure $=P(F)$
$\mathrm{n}=\quad$ Number of trials
$x=\quad$ The number of successes in $n$ trials
$!=\quad$ Mathematical symbol called 'factorial'. So $n!$ means multiple all the numbers in a count down from the total number in the sample. For example:

$$
7!=7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1, \text { and } 4!=4 \times 3 \times 2 \times 1
$$

## For Example:

1. A survey on birds showed that one out of five fire finch was trapped, using mist net, in a given season. If 10 birds are selected at random, find the probability that 3 of the birds were trapped in the previous season.

## Solution:

$$
\mathrm{n}=10, \mathrm{x}=3, \mathrm{P}=1 / 5 \text { and } \mathrm{q}=4 / 5
$$

Substituting these values in the formula above, we have

$$
P(3)=\frac{10!.}{(10-3)!3!}(1 / 5)^{3}(4 / 5)^{7}=0.201
$$

The mean $(\mu)$, variance $\left(\sigma^{2}\right)$ and standard deviation $(\sigma)$ of a variable that has the binomial distribution can be found by using the formular:
$\mu=$ n.p $; \quad \sigma^{2}=$ n.p.q $; \quad \sigma=\sqrt{ }$ n.p.q

From our example above:
The mean, $\mu=10 \times 1 / 5=2$
The variance, $\sigma^{2}=10 \times 1 / 2 \times 4 / 5=1.6$
The Standard deviation, $\sigma^{2}=\sqrt{ } 1.6=1.265$
2. $40 \%$ of the snails breed by a certain farm are of the type reticulata, determine the probability that, out of 12 snails chosen at random, (a) 2 (b) at most 3 will be of the type reticulate

## Solution

0.4 is the probability of chosen reticulata breed, and 0.6 is the probability of chosen a non reticulata breed.
(a) $P(2$ reticulata breed of 12 snails $)=\binom{12}{2}(0.4)^{2}(0.6)^{10}$
(b) $\mathrm{P}($ at most 3 reticulata snails breed $)=\mathrm{P}(0$ reticulata $)+\mathrm{P}(1$ reticulata $)+$

$$
\begin{aligned}
& \mathrm{P}(2 \text { reticulata })+\mathrm{P}(3 \text { reticulata }) \\
= & \binom{12}{0}(0.4)^{0}(0.6)^{12}+\binom{12}{1}(0.4)^{1}(0.6)^{11}+ \\
& \binom{12}{2}(0.4)^{2}(0.6)^{10}+\binom{12}{3}(0.4)^{3}(0.6)^{9}
\end{aligned}
$$

### 3.3.6 TUTOR MARKED ASSIGNMENT

1. What is a Normal Curve?
2. Outline any five characteristics of a normal curve.
3. Construct a probability distribution for three patients given a headache relief tablet. The probabilities for $0,1,2$ or 3 success are $0.18,0.52,0.21$ and 0.09 , respectively.
4. In a certain rabbit farm in the southern part of Nigeria $25 \%$ of the rabbit breed are of the type bauscat, determine the probability that, out of 3 rabbits chosen at random, (a) 1 (b) at most 4 will be of the type bauscat

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## UNIT FOUR: ESTIMATION AND HYPOTHESIS TESTING

### 4.1 INTRODUCTION

Researchers are interested in answering many types of questions. For example, a scientist might want to know whether the earth is warming up. A physician might want to know whether a new medication will lower a person's blood pressure. An educator might wish to see whether a new teaching technique is better than a traditional one. These types of questions can be addressed through statistical hypothesis testing, which is a decision-making process for evaluating claims about a population.

### 4.2 OBJECTIVES:

At the end of this unit, you should be able to:

1. Define estimation and statistical hypothesis
2. State the null and alternative statistical hypotheses
3. Distinguish the possible outcomes of a hypothesis test
4. Determine the level of confidence in a biological data
5. State the steps used in hypothesis testing
6. Explain the relationship between type I and type II errors
7. Test hypotheses involving means using z and t tests.

## 4,3 MAIN CONTENTS

### 4.3.1 ESTIMATION

Estimation is the entire process of using an estimator to produce an estimate of a parameter. Estimation and hypothesis testing are interrelated. An estimate is any specific value of a statistic while an estimator is any statistic used to estimate a parameter. For example, the sample mean $x$ is used to estimate the populations mean $\boldsymbol{\mu}$. A Point estimate is obtained when a single number is used to estimate a population parameter. For example $\mathbf{s}=\mathbf{3 0}$. An Interval estimate is obtained when a range of values is used to estimate a population parameter. For example, a range of values between 20 and 30 allows evaluation of the estimate unlike the point estimate of a single value.

### 4.3.2 STATISTICAL HYPOTHESIS

A statistical hypothesis is a conjecture about a population parameter. This conjecture may or may not be true.

There are two types of statistical hypotheses for each situation: the null hypothesis and the alternative hypothesis.

## NULL AND ALTERNATIVE HYPOTHESES

The null hypothesis, symbolized by $\boldsymbol{H}_{\boldsymbol{o}}$, is a statistical hypothesis that states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters. It can be accepted or rejected as the case
may be. If it conforms sufficiently closely in a statistical sense, it is accepted, if it doesn't, it is rejected. If the sample results do not support the null hypothesis then the conclusion which is on the rejection of the null hypothesis is known as the alternative hypothesis.

Alternative Hypothesis, symbolized by $\boldsymbol{H}_{1}$, is the conclusion to be drawn contingent on the rejection of the null hypothesis i.e. it states that there is a difference between a parameter and a specific value under study.

For example supposing we want to test the hypothesis that a population mean $(\mu)$ is equal to 55. The hypothesis is that: $H_{o}: \mu=55$.

Where: $\mu$ is the true value and 55 is the assumed value Therefore the three possible alternative hypotheses are

1. $H_{1}: \mu \neq 55 \Rightarrow$ expressed in Two-tailed test
2. $\quad H_{l}: \mu>55 \Rightarrow$ expressed in Right-tailed test
3. $H_{1}: \quad \mu<55 \Rightarrow$ expressed in Left-tailed test.

A statistical test uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.

A one-tailed test is either right- tailed when the inequality sign is $>$ or left-tailed when the inequality sign <. It indicates that the $H_{o}$ should be rejected when the test value is in the critical region on one side of the mean.

In a two-tailed test, the null hypothesis should be rejected when the test value (numerical value obtained from a statistical test) is in either of the two critical regions.

## THE LEVEL OF CONFIDENCE

The level of probability associated with an interval estimate is known as the confidence level or degree of confidence or the confidence coefficient.
'Confidence' is applied or used because the probability is an indicator of the degree of certainty that the particular method of estimation will produce an estimate which includes $\mu$. The most frequently used confidence levels employed in interval estimation are 90,95 and 99 percent as summarized below.

Table 4.1: Probability Levels and interval estimates

| Probability <br> levels | Confidence <br> Coefficient | $z$-value | Form of the interval estimate |
| :--- | :--- | :--- | :--- |
| 0.1 | 90 | 1.64 | $x-1.64 \sigma_{x}<\mu<x+1.64 \sigma_{x}$ |
| 0.05 | 95 | 1.96 | $x-1.96 \sigma_{x}<\mu<x+1.96 \sigma_{x}$ |
| 0.01 | 99 | 2.58 | $x-2.58 \sigma_{x}<\mu<x+2.58 \sigma_{x}$ |

Where: $\mathrm{x}=$ Sample mean
$\mu=$ Population mean
$\sigma_{x}=$ standard error of mean $x$.

Figure 4.1: Normal curve showing acceptance and rejection regions with a significance level ( $\alpha$ ) of 0.05


The significance difference is the degree of difference between sample mean (x) and population mean $\left(\mu_{H o}\right)$ that leads to the rejection of the null hypothesis. This is because it has only $5 \%$ or less chance of occurring.

In two-tailed test, once the $H_{o}$ cannot be accepted, it is concluded that the hypothesized value and the true value are not the same.

The critical or rejection region is the range of values of the test values that indicates that there is significant difference and that the null hypothesis should be rejected. This tells us that the degree of difference between the two means cannot wholly be explained by chance.

## Steps in Hypothesis Testing

> Every hypothesis testing situation begins with the statement of hypothesis
$>$ Determine the type of data, that is whether the data is continuous or discrete
$>$ State the hypotheses. Be sure to state both the null and alternative hypotheses
$>$ Design the study. This step involves:

- Selecting the correct statistical test
- Choosing a level of significance
- Formulating a plan to carry out the study
$>$ Conduct the study and collect the data
$>$ Evaluate the data. Make the decision to reject or not reject the null hypothesis
> Summarize the results.
Note:

1. It is important to establish a criterion for the rejection and acceptance of the null hypothesis. In that regard, the level of risk you desire rejecting a null hypothesis is the level of significance ( $\alpha$ ).
2. It is also worthy of note that theoretically a test never proves that a hypothesis is true but merely provides statistical evidence for not rejecting a hypothesis.

## When you reject the Null Hypothesis, the five possibilities are:

1. There is direct cause and effect between the variables. For example, increase in height of an Okro plant brings about increase in its yield.
2. There is a reverse cause and effect relationship between the variables.
3. The relationship between the variables may be caused by a third variable
4. There may be a complexity of interrelationships among many variables.
5. The relationship may be coincidental.

## TYPE I AND TYPE II ERRORS

In hypothesis testing situation, there are four possible outcomes. That is, two possibilities of incorrect decisions, together with the two possibilities for correct decisions. These are listed in the table 4.2 below.

## Table 4.2

|  | $\boldsymbol{H}_{\boldsymbol{o}}$ ACCEPTED | $\boldsymbol{H}_{\boldsymbol{I}}$ ACCEPTED |
| :--- | :--- | :--- |
| $H_{o}$ True | Correct Decision | Type I error |
| $H_{l}$ True | Type II error | Correct Decision |

A Type I error occurs if one rejects the null hypothesis when it is true or it is the risk that a true hypothesis will be rejected.

A Type II error occurs if you do not reject the null hypothesis when it is false or when a false hypothesis is erroneously accepted as true.

### 4.3.3 TESTING A HYPOTHESIS INVOLVING A MEAN

1. Find the sample mean and standard deviations .
2. State the null hypothesis: $H_{o}: \mathrm{x}=\mu$
3. Use sample standard deviation ( $s$ ) to estimate population standard deviation ( $\sigma$ ) and compute $\boldsymbol{\sigma}_{\mathbf{x}}=\mathbf{S} / \sqrt{\mathbf{n}}$
4. Find the range for $\boldsymbol{\mu} \pm \mathbf{z} \boldsymbol{\sigma}$
5. Check it if the value of $\mathbf{x}$ falls within the range or not
6. If it does, then accept the $H_{o}$, and if it doesn't, then reject the $H_{o}$.

## EXAMPLES:

1. Assuming the following values of a sample are given: $\mathrm{x}=454 . \mathrm{n}=120$, standard deviation $(S)=27, \mu=460, \alpha=0.05$ or 95 (confidence coeff.).

## Solution:

State the null hypothesis.
$H_{o}: \mathrm{x}=\mu$ (That is, there is no difference between the sample mean and the population mean).
$\boldsymbol{\sigma}_{\mathrm{x}}=27 / \sqrt{ } 120=2.46$
Then calculate the range: $\mu+1.96 \times 2.46$ TO $\mu-1.96 \times 2.46$

$$
460+1.96 \times 2.46 \text { TO } 460-1.96 \times 2.46
$$

The range is: 465 TO 455

So the sample mean, $x=454$ does not fall within the range of 465 to 455 . Therefore, the null hypothesis is rejected. This implies that the sample mean x and the population mean $\mu$ are significantly different.
2. The mean yield of tomato following fertilizer treatment from 10 plots was 176.10 kg with standard deviation 3.88. Estimate the $95 \%$ confidence limit for the mean yield of tomato.

## Solution:

$\mu=176.10 \mathrm{~kg}, \alpha=0.05, \mathrm{n}=10, \sigma=3.88$
since the sample size $(\mathrm{n}=10)$ is small i.e less than 30 , the t -distribution is used instead.

$$
\mu \pm \mathrm{t}_{\mathrm{n}-1}\left(\sigma /{ }^{n} \mathrm{n}\right)
$$

$t_{n-1}=t_{10-1}=t_{9}$; check the value on the t -distribution table that correspond to degrees of freedom (9) at 0.05 level. This value is 2,262 .

Then $\sigma / \sqrt{ } n=3.88 / \sqrt{ } 10=1.23$
To fix the limits: $\quad L_{1}=\mu-t_{n-1}\left(\sigma /{ }^{n}\right)=176.10-2.262(1.23)$

$$
\begin{aligned}
& \mathrm{L}_{1}=173.318 \\
& \mathrm{~L}_{2}=\mu+\mathrm{t}_{\mathrm{n}-1}\left(\sigma / V_{\mathrm{n}}\right)=176.10+2.262(1.23) \\
& \mathrm{L}_{2}=178.882
\end{aligned}
$$

At $95 \%$ confidence, the true population mean ( $\mu$ ), will lie between the limits, 173.318 and 178.882 . The mean yield of tomato is 176.10 kg and is within the interval. Therefore, it is the true mean.

### 4.3.4 TESTING A HYPOTHESIS INVOLVING TWO MEANS

A sample of 158 obese girls of average age 15 was analysed with respect to various physical characteristics during early childhood. A control group of 94 nonobese girls of similar age and socioeconomic background was also analysed. The following table gives the sample means and standard deviations for two characteristics of the two groups.

|  | Obese group | Non obese group |
| :--- | :--- | :--- |
| Birth weight | $\mathrm{x}_{1}=7.04, \mathrm{~S}_{1}=1.2$ | $\mathrm{x}_{2}=7.19, \mathrm{~S}_{2}=0.9$ |
| One year weight | $\mathrm{x}_{1}=23.3, \mathrm{~S}_{1}=2.8$ | $\mathrm{x}_{2}=21.9, \mathrm{~S}_{2}=3.0$ |

(a) Use the data to test the null hypothesis : $H_{o}: \mu_{1}=\mu_{2}$, at $\alpha=0.05$ for
i. Birth weight
ii. One year weight
(b) What conclusions can be drawn from the results in (a) i and ii?

## SOLUTION:

## State the null hypothesis:

There is no difference between obese and non-obese girls at birth weight and one year weight in the two populations.

The Z formula for testing two means is given by: $\mathrm{Z}=$

$$
\sqrt{\underline{\mathrm{X}}_{1}-\frac{\mathrm{X}}{2}} \underline{\mathrm{~S}}_{1}^{2} / \mathrm{n}_{1}+\mathrm{S}_{2}{ }^{2} / \mathrm{n}_{2}
$$

Where: $\mathrm{x}_{1}$ - mean value of the first group
$\mathrm{x}_{2}$ - mean value of the second group
$S_{1}-$ standard deviation of the first group
$S_{2}$ - standard deviation of the second group
$\mathrm{n}_{1}-$ number of the first sample $(=158)$
$\mathrm{n}_{2}-$ number of the second sample ( $=94$ )
Therefore,
(a) i. Birth weight: $\quad Z=\frac{7.04-719}{\sqrt{1.2^{2} / 158+0.9^{2} / 94}}=-0.15 / 0.133=-1.13$
(a) ii. One year weight: $\mathrm{Z}=\frac{23.3-21.9}{\sqrt{2.8^{2} / 158+3.0^{2} / 94}}=\frac{1.4}{\sqrt{0.145}}=3.67$

## Decision:

1. Since the Z - value ( -1.13 ) for birth weight is within the non critical region (i.e within the range of -1.96 to +1.96 ) (Figure 3.1), we accept the null hypothesis. This signifies that the difference between obese and non-obese girls at birth weight is statistically not significant.
2. The $z$ value (3.67) for one year weight falls within the critical region (i.e outside the range of -1.96 to +1.96 ). Therefore, we reject the null hypothesis because the difference is statistically significant. This signifies that there was difference between obese and non-obese girls at one year weight.

## (b) Conclusion.

The conclusion that can be drawn from these decisions above is that, one characteristic feature of obesity is large increase in weight during the first one year of life in girls.

### 4.3.5 TWO-TAILED TEST - When $\sigma_{x}$ is known (' $n$ ' more than 30)

Decision: The rejection of the null hypothesis is simply that the assumed value is not the true value.

For Instance: If the mean of a population is assumed to be 500 , with $\sigma=50$. If a sample of 36 had a mean of 475 , conduct a two-tailed test with $\alpha=0.01$.

## Solution:

The hypothesis are: Null Hypothesis, $H_{o}: \mu=500$
Alternative hypothesis : $\mu \neq 500$
With $\alpha=0.01$, the $z$ - value is 2.58 (Table 4.1).
The decision rule is: Accept the $H_{o}$ if the Critical ratio (CR) falls between $\pm 2.58$. OR Reject the $H_{o}$ and accept the $H_{l}$ if CR is greater than $+2.58(\mathrm{CR}>+2.58)$ or less than -$2.58(\mathrm{CR}<-2.58)$. Note that it is this two rejection regions that made this test a TWO - TAILED TEST.

## Compute the CR

$C R=\frac{\mathbf{x}-\boldsymbol{\mu}_{\hat{H}_{0}}}{\boldsymbol{\sigma} / \sqrt{\mathbf{n}}}=\frac{475-500}{50 / \sqrt{36}}=-3.00$

## CONCLUSION:

Since the CR value (-3.00) is less than -2.58 , we reject the $H_{o}$ and accept the $H_{l}$.
NOTE: z-distribution can only be used to determine the rejection regions when the sample size ( n ) is more than 30 . If the sample size is 30 or less, then the sampling distribution takes shape of t-distribution.

### 4.3.6 TWO-TAILED TEST - When $\sigma_{x}$ is unknown (' $n$ ' less than 30)

Assuming the following data is given: $\mu=612, x=608, S=50, n=13$ and $\alpha=0.05$. Conduct a two-tailed test.

## Solution:

The hypotheses are: Null Hypothesis, $H_{o}: \mu=612$
Alternative hypothesis: $\mu \neq 612$
With $\alpha=0.05$, since the n is less than 30 (i.e $\mathrm{n}=13$ ), we us t -distribution.
First, we calculate the degrees of freedom $(D F)=n-1$ i.e $\quad 13-1=12$.
Using the t -distribution table, the t -value at 2 DF is 2.179 \{expressed as $\left(\mathrm{t}_{0.05 / 2}=.025\right)$ t. $025=2.179\}$.

The decision rule is: Accept the $H_{o}$ if the Critical ratio (CR) falls between $\pm 2.179$. OR Reject the $H_{o}$ and accept the $H_{l}$, if CR is greater than +2.179 or less than -2.179 .

$$
\boldsymbol{\sigma}_{\mathbf{x}}=\mathbf{S} / \sqrt{ } \mathbf{n}-\mathbf{1} \quad 5 / \sqrt{ } 12=1.44
$$

Therefore, $\mathrm{CR}=\frac{\mathbf{x}-\boldsymbol{\mu}_{\mathbf{H o}_{-}}}{\mathbf{S} / \sqrt{\mathbf{n}-\mathbf{1}}}=\frac{608-612}{1.44}=-2.77$

## Conclusion:

Since CR is less than -2.179 , we reject the Null Hypothesis and accept the Alternative Hypothesis.

### 4.3.7 LEFT - TAILED TEST

Assuming that we are given the following null hypothesis; $H_{o}: \mu=100$. Conduct a lefttailed test with $\alpha=0.05, \sigma=15, \mathrm{n}=36, \mathrm{x}=88$.

## Solution:

State the hypothesis: $H_{o}: \mu=100$ and $H_{l}: \mu<100$
With $\alpha=0.05$, the z -value is -1.64 .
Decision rule: Accept the $H_{o}$, if $\mathrm{CR} \geq-1.64$ OR Reject $H_{o}$ and accept $H_{l}$ if $\mathrm{CR}<-1.64$.
$C R=\frac{88-100}{15 / \sqrt{ } 36}=-4.8$

## Conclusion:

Since the CR value (-4.8) is less than -1.64 , we reject the $H_{o}$ and accept the $H_{l}$.

### 4.3.8 STUDENT'S T-DISTRIBUTION (The $\boldsymbol{t}$-test)

The $t$-distribution is characterized by the following properties, which are similar to that of a standard normal distribution;

1. It is bell-shaped.
2. It is symmetric about the mean
3. The mean, median and mode are equal to 0 and are located at the center of the distribution.
4. The curve never touches the $x$-axis.

While the properties below differentiate it from the standard normal distribution.
5. The variance is greater than 1.
6. The t-distribution is a family of curves based on the sample size ( $n$ ), with the degrees of freedom $n-1$.
7. As the sample size increases, the t-distribution approaches the normal distribution.

## The $t$-test

When the population standard deviation is unknown and the sample size is less than 30, the $z$-test is inappropriate for testing hypotheses involving. The $t$-test is used instead.

The $t$-test is define as a statistical test for the mean of a population and is used when the population is normally or approximately normally distribution, $\sigma$ is unknown, and $n<30$.

Formula for $t$-test: $t_{n-1}=\frac{x-\mu}{s / \sqrt{n}}$
Where: $\mathrm{x}=$ Sample mean.
$\mu=$ Population mean.
$\mathrm{s}=$ Sample standard deviation.
$\mathrm{n}=$ Sample size.
Testing of hypotheses using the $t$-test follows the same procedure as for the z-test, except that you use the $t$ - table instead.

### 4.4 TUTOR MARKED ASSIGNMENT

1. If the hypothesis for a population mean is 151 (i.e $H_{o}: \mu=151$ ). List the three possible hypotheses.
2. A sample of 35 housefly wing lengths from a population has a mean of 44.8. Given that the population standard deviation is 3.90 . Compute the confidence limits at 95 and $99 \%$.
3. What is the implication of committing a Type II error?
4. What is a right-tailed test?
5. Assuming the population mean is 500 , with $\sigma=50$. If the sample of 36 had 6 , $x=475$. Conduct a two-tailed test with $\alpha=0.01$.

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UNIT FIVE: CONTINGENCY TABLES

### 5.1 INTRODUCTION

Contingency tables are usually constructed for the purpose of studying the relationship between two or more variables of classification. One may wish to know whether the two variables are independence or there is association between them. By means of chi square $\left(\chi^{2}\right)$ test it is possible to test the hypothesis that the two variables are independent (Independence test).

### 5.2 OBJECTIVES:

At the end of this unit, you should be able to:

1. Define chi-square distribution.
2. Mention the properties and steps in calculating chi-square.
3. Explain the purpose of goodness of fit test
4. Describe the steps in the goodness-of-fit test, and use them to arrive at statistical decisions about the population.

### 5.3 MAIN CONTENT

### 5.3.1 CHI-SQUARE DISTRIBUTION

Chi-square $\left(\chi^{2}\right)$ is the general method for testing compatibility based on a measure of the extent to which the observed and expected frequencies agree. Chisquare is also, referred to as test for homogeneity randomness, association, independence and goodness of fit. The assumptions for the chi-square goodness-of-fit test are:

1. The data are obtained from a random sample.
2. The expected frequency for each category must be 5 or more.

Chi-square is defined by the formula:

$$
\chi^{2}=\sum_{i=1}^{k} \frac{\left(o_{i}-e_{i}\right)^{2}}{e_{i}}
$$

Where $o_{i}$ and $e_{i}$ denote the observed and expected frequencies, respectively, for the ith cell, and k denotes the number of cells.

The frequencies we observe are compared to those we expect on the basis of some null hypotheses. If the differences between the observed and expected frequencies are great and exceed the critical value at appropriate degrees of freedom, we are then obliged to the reject the null hypothesis $\left(H_{o}\right)$ and accept the alternative hypothesis $\left(H_{l}\right)$.

## PROPERTIES OF CHI-SQUARE

1. It is concerned with the sequences of normally distribution observations, thereby estimating the variance.
2. Chi-square values tend to become normal as the sample size (n) increases. Its degree of freedom is $\mathrm{n}-1$.
3. It is non-symmetrical
4. $\quad \chi^{2}$ can take any value from zero to in infinity.
5. $\quad \chi^{2}$ is additive and always positive.
6. Chi-square can be:
(a) One-way classification as in testing the goodness of fit of a hypothesis, with two or more classes.
(b) Two-way classification as in determining association or differences between two different classes or the test may involve more than two classes.

## CHI-SQUARE TESTING

Chi-square distributions are used in a procedure that involves the comparison of the differences between the sample frequencies of occurrences or
percentages that are actually observed and the hypothetical or theoretical population frequencies of occurrences or percentages that are expected if the hypothesis is true.

## Steps in the general $\chi^{2}$ testing procedure

1. Formulate the null and alternative hypotheses.
2. Select the level of significance to be used in the particular testing situation.
3. Take random samples from the populations, and record the observed frequencies that are actually obtained.
4. Compute the frequencies of percentages that would be expected if the null hypothesis is true.
5. Use the observed and the expected frequencies to compute the $\chi^{2}$.
6. Compare the value of $\chi^{2}$ computed in step 5 with the $\chi^{2}$ table value at the specified level of significance (step 2).

## For Example:

In an experiment to test the effectiveness of three different traps for catching birds, the number of birds captured in each trap design over the study period was recorded as follows:

| Design A | Design B | Design C | Total |
| :--- | :--- | :--- | :--- |
| 10 | 27 | 15 | 52 |

The observed frequencies are, of course, 10, 27 and 15 . The question we are interested in is "Does the observation that more birds were caught in trap design $B$ really reflect a genuine difference, or could the difference be due to chance scatter or sampling error?" Another question could be "Is the distribution of frequencies between the traps homogenous (i.e evenly spread)?" From these, the hypotheses are:
$H_{o}$ : The observed frequencies are homogenous and any departure can be accounted for by chance scatter or sampling error;
$H_{1}$ : The observed frequencies depart from those expected of a homogenous (even) distribution by an amount that cannot be explained by sampling error.

Then, what frequencies would have been expected if $H_{o}$ is indeed true. That means if the frequencies reflect a homogenous distribution, we would expect the 52 birds to be equally distributed in all the three trap designs. That is $52 / 3$ $=17.33$ birds in each design is our expected frequency. To calculate $\chi^{2}$, we can summarize our frequencies as shown below.

| Design | Observed <br> frequency | Expected <br> frequency | Obs. - Exp. | (Obs. - Exp. $)^{2} /$ Exp. |
| :--- | :--- | :--- | :--- | :--- |
| A | 10 | 17.33 | -7.33 | 3.10 |
| B | 27 | 17.33 | 9.67 | 5.40 |
| C | 15 | 17.33 | -2.33 | 0.313 |
|  |  |  | $\chi^{2}=8.813$ |  |

The calculated $\chi^{2}=8.813$ can then be compared with the critical or table $\chi^{2}$ value at 0.05 or 0.01 levels of significance.

The degrees of freedom $=\mathrm{n}-1 \Rightarrow 3-1=\mathbf{2}$. From the $\chi^{2}$ table at 2 d.f, we have 5.99 under the 0.05 (5\%) level of significance and 9.21 under the $0.01(1 \%)$. Our calculated $\chi^{2}$ value of 8.813 is bigger than the first but smaller than the second. We conclude that the difference between the observed and expected frequencies is statistically 'significant' but not 'highly significant'. This simply means that "the trap of design $B$ was shown in a trial to be more effective in catching birds than the other two traps tested; $\chi_{(2 \mathrm{~d} . \mathrm{f})}^{2}=$ 8.813, $P<0.05 \prime$ ".

Chi-square tests that require classification of observed frequencies in two ways i.e two rows and two columns are presented in $2 \times 2$ contingency tables as shown below:

Table 5.1: A general $2 \times 2$ contingency table.

| Classification | Characteristics <br> Present | Characteristics Absent | Total |
| :---: | :---: | :---: | :---: |
| Sample A | A | B | $\mathrm{a}+\mathrm{b}$ |
| Sample B | B | D | $\mathrm{c}+\mathrm{d}$ |
| Total | $\mathrm{a}+\mathrm{b}$ | $\mathrm{b}+\mathrm{d}$ | N |

$$
\chi^{2}=\frac{\mathrm{N}(\mathrm{ad}-\mathrm{bc})^{2}}{(\mathrm{a}+\mathrm{b})(\mathrm{a}+\mathrm{c})(\mathrm{b}+\mathrm{d})(\mathrm{c}+\mathrm{d})}
$$

Where: $\mathrm{N}=\mathrm{a}+\mathrm{b}+\mathrm{c}+\mathrm{d}$

The degrees of freedom $=\mathrm{n}-1 \Rightarrow 2-1=1$

## Worked Examples:

## 1. Test for Goodness-of-Fit

The offspring of a certain cross gave the following colours: Red, Black or white in the ratio 9:3:4. Assuming the experiment gave 74,32 , and 38 offsprings respectively in those categories, is the theory substantiated?

| Red | Black | White | Total |
| :---: | :---: | :---: | :---: |
| 74 | 32 | 38 | $\mathbf{1 4 4}$ |
| 9 | 3 | 4 | $\mathbf{1 6}$ |

The expected frequencies are calculated as follows:
Red: $\quad 9 / 16 \times 144=54$
Black: $\quad 3 / 16 \times 144=27$

White: $\quad 4 / 16 \times 144=36$

$$
\begin{gathered}
\chi^{2}=\frac{(74-54)^{2}}{54}+\frac{(32-27)^{2}}{27}+\frac{(38-36)^{2}}{36}=8.45 \\
\chi^{2}=8.45
\end{gathered}
$$

The D.F $\quad \mathrm{n}-1$

$$
3-1
$$

$$
=2
$$

The calculated $\chi 2$ of 8.45 is higher than the table $\chi^{2}$ value of 5.99 at $\alpha=$ 0.05 and d.f 2. Therefore, we conclude that the number of offsprings in the 3 colours is not compatible with the given ratios. i.e we reject the null hypothesis.

## 2. Test for Independence

Two plant extracts are claimed to be effective in curing stomach ulcer were tested on patients. The patients' reactions to treatment were recorded in the table below:

| EXTRACTS | EFFICACY |  |  |
| :--- | :---: | :---: | :---: |
|  | HELPED | HARMED | NO EFFECT |
| Anona senegalensis | 62 | 84 | 24 |
| Bauhinia monandra | 34 | 44 | 22 |

Test the data whether the two extracts have the same effects.

## Solution:

Our null hypothesis $\left(H_{o}\right)$ : The two plant extracts have the same effect on the patients.

First calculate the expected frequencies.

|  | Helped | Harmed | No effect | Total |
| :--- | :---: | :---: | :---: | :---: |
| A. senegalensis | 62 | 84 | 24 | 170 |
| B. monandra | 34 | 44 | 22 | 100 |
| Total | $\mathbf{9 6}$ | $\mathbf{1 2 8}$ | $\mathbf{4 6}$ | $\mathbf{2 7 0}$ |

## Frequencies of each category:

Helped: $\quad$ A. senegalensis: $\quad 170 \times 96 \div 270=60.4$
B. monandra: $\quad 100 \times 96 \div 270=35.6$

Harmed: A. senegalensis: $\quad 170 \times 128 \div 270=80.4$
B. monandra: $\quad 100 \times 128 \div 270=47.4$

No effect: A. senegalensis: $\quad 170 \times 46 \div 270=29.0$
B. monandra: $\quad 100 \times 46 \div 270=17.0$

## Form a table of expected frequencies.

Helped Harmed No effect

| A. senegalensis | 60.4 | 80.6 | 29.0 |
| :--- | :---: | :---: | :---: |
| B. monandra | $\frac{35.6}{\mathbf{9 6}}$ | $\frac{47.4}{\mathbf{1 2 8}}$ | $\frac{17.0}{\mathbf{4 6}}$ |
| $\left.\chi^{2}=\frac{(62-60.4)^{2}}{60.4}+\frac{(84-80.6}{80.6}\right)^{2}+\ldots \ldots \ldots \ldots\left(\frac{(22-17.0)^{2}}{17.0}\right.$ | $=2.83$ |  |  |

Therefore, $\chi^{2}=2.83$
Find the degrees of freedom (DF): Rows $(\mathrm{r})=2 ;$ Columns $(\mathrm{c})=3$
$\mathrm{DF}=(\mathrm{r}-1)(\mathrm{c}-1)$
$=(2-1)(3-1)=2$
The table $\chi^{2}$ value at DF 2 is 5.99 , at $\alpha=0.05$
Since our calculated $\chi^{2}$ value (2.83) is less than the $\chi^{2}$ table value (5.99), it shows that the effect of the extracts is not significant. Therefore, we accept our $H_{o}$ i.e the two extracts have the same effect on the patients.

## Additional Example:

The table below is an outcome of a survey of Ahmadu Bello University Zaria graduates working in Abuja. They were divided into four groups on the basis of their classes of degree and their income in practice, ten years after graduation.

| Class of degree | Income |  |  |
| :--- | :--- | :--- | :--- |
|  | High | Medium | Low |
|  | 22 | 10 | 10 |
| Second | 10 | 13 | 7 |
| Third | 20 | 6 | 6 |
| Pass | 5 | 9 | 15 |

Determine the relationship between the class of degree and their income.
First, we state the hypotheses.

1. Null hypothesis $\left(H_{o}\right)$ : There is no significant relationship between the class of degree and income of A.B.U. Zaria graduates in Abuja.
2. Alternative hypothesis $\left(H_{l}\right)$ : There is significant relationship between the class of degree and income of A.B.U. Zaria graduates in Abuja.

Compute the totals of the observed values.

|  | Income |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Class of degree | High | Medium | Low | Total |
| First | 22 | 10 | 10 | 42 |
| Second | 10 | 13 | 7 | 30 |
| Third | 20 | 6 | 6 | 32 |
| Pass | 5 | 9 | 15 | 29 |
| Total | $\mathbf{5 7}$ | $\mathbf{3 8}$ | $\mathbf{3 8}$ | $\mathbf{1 3 3}$ |

## Compute the expected frequencies.

First - High $\quad 42 \times 57 / 133=18$
First - Medium $\quad 42 \times 38 / 133=12$

First - Low $\quad 42 \times 38 / 133=12$
Second - High $\quad 30 \times 57 / 133=12.9$
Second - Medium $30 \times 38 / 133=8.6$

Second - Low $\quad 30 \times 38 / 133=8.6$
Third - High $\quad 32 \times 57 / 133=13.7$
Third - Medium $\quad 32 \times 38 / 133=9.1$

Third - Low $\quad 32 \times 38 / 133=9.1$

Pass - High $\quad 29 \times 57 / 133=12.4$
Pass - Medium $\quad 29 \times 38 / 133=8.3$
First - Low $\quad 29 \times 38 / 133=8.3$
Place the computed expected values in the table of frequencies.

|  | Income |  |  |
| :--- | :--- | :--- | :--- |
|  | High | Medium | Low |
| First | 18 | 12 | 12 |
| Second | 12.9 | 8.6 | 8.6 |
| Third | 13.7 | 9.1 | 9.1 |
| Pass | 12.4 | 8.3 | 8.3 |

We can compute the chi square value using the $\chi^{2}$ formular.

$$
\chi^{2}=\frac{(22-18)^{2}}{18}+\frac{(10-12)^{2}}{12}+\frac{(10-12.9)^{2}}{12.9} \cdots \cdots \cdot \frac{(15-8.3)^{2}}{8.3}=19.65
$$

Then determine the degrees of freedom (d.f).

$$
\begin{aligned}
\text { d.f } & =(\text { column }-1)(\text { row }-1) \\
& =(3-1)(4-1) \\
& =6
\end{aligned}
$$

Check the value in the chi square table at d.f 6
At d.f 6, the value is 12.59

## Conclusion:

Since the calculated value of 19.65 in higher than the table value of 12.59 , it shows that the relationship between the class of degree and income of the A.B.U.

Zaria graduates in Abuja is significant. Therefore, we reject our null hypothesis $\left(H_{o}\right)$ and accept our alternative hypothesis $\left(H_{l}\right)$.

### 5.4 TUTOR MARKED ASSIGNMENT

1. Distinguish between Observed and Expected frequencies.
2. A scientist who claimed that eye colour is independent of coat colour in rabbits obtained the following data. Analyse the data statistically and draw suitable conclusion.

| Eye colour/ | Coat colour |  |  |
| :--- | :--- | :--- | :--- |
|  | White | Brown | Black |
| Red | 15 | 5 | 15 |
| Grey | 20 | 10 | 30 |
| Brown | 25 | 15 | 25 |

3. In a cross involving two cowpea varieties (smooth white and rough brown), a sample of 550 gave the following frequencies: 305 smooth white; 110 smooth brown; 105 rough white and 30 rough brown, in a ratio of 9:3:3:1. Test whether the frequencies agree with the given ratio.
4. The offspring of a certain cross gave the following colours, red, black or white in the ratio 9:3;4. If the experiment gave 74,32 and 38 offsprings in the categories respectively, can the theory be substantiated?

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## UNIT SIX: CORRELATION AND REGRESSION AND COVARIANCE

### 6.1 INTRODUCTION

Correlation and regression are other areas of inferential statistics which involve determining whether a relationship between two or more numerical or quantitative variables exists. This is when two characteristics are studied simultaneously on each member of a population in order to examine whether they are related. For instance, a researcher may be interested in finding out the relationship between height and yield in okro plant or a zoologist may not to know whether the birth weight of a certain animal is related to the life span. Therefore, correlation and regression analyses are used to measure association between two variables of a bivariate data.

### 6.2 OBJECTIVES:

At the end of this Unit, you should be able to:

1. Define correlation and regression and bivariate distribution.
2. Explain the types of correlation and regression
3. State possible relationships between variables
4. Draw a scatter diagram for a bivariate data
5. Compute correlation and regression.

### 6.3 MAIN CONTENT

### 6.3.1 CORRELATION

Correlation is a statistical technique that measures the degree/strength of linear relationship in a bivariate normal distribution where two continuous variables drawn from the same population are compared.

The correlation coefficient computed from a sample data measures the strength and direction of a linear relationship between two variables. The symbol, $\mathbf{r}$, represents samples correlation coefficient. While the Greek letter $\rho$ (rho) is the population correlation coefficient.

$$
r=\frac{N \sum X Y-\left(\sum X\right)\left(\sum Y\right)}{\sqrt{\left.N \sum X^{2}-\left(\sum X\right)^{2}\right]\left[N \sum Y^{2}-\left(\sum Y\right)^{2}\right]}}
$$

Where n is the number of data pairs ( $\mathrm{x}, \mathrm{y}$ )
OR

$$
r=\frac{\sum x y}{\sqrt{\left(\sum x^{2}\right)\left(\sum y^{2}\right)}}
$$

## Procedure for calculating simple linear correlation

1. Compute the mean x and y , the corrected sum of squares, $\sum x^{2}$ and $\sum y^{2}$, and the corrected sum of cross products $\sum x y$, of the variables x and y .
2. Compute the $\mathbf{r}$ value.
3. Compare the absolute value of the computed $r$ value to the tabular $\mathbf{r}$ values with $\mathrm{n}-2$ degrees of freedom at $5 \%$ and $1 \%$ levels of significance.
4. If the computed $\mathbf{r}$ value is greater than the tabular $\mathbf{r}$ value at $5 \%$ level but smaller than the tabular $\mathbf{r}$ value at the $1 \%$ level, then the simple linear correlation is significant at the $5 \%$ level of significance.

## The range of ' $r$ ' value

The range of values for correlation coefficient is from -1 to +1 that is if there is

1. Strong positive linear relationship between the variables, the value or $\mathbf{r}$ will be close to +1 .
2. Strong negative linear relationship between the variables, the value of $\mathbf{r}$ will be close to -1 .
3. No linear relationship between the variables or only a weak relationship, the value of $\mathbf{r}$ will be close to 0 .

Figure 6.1: Strength of linear relationships


### 6.3.2 REGRESSION

Regression is a statistical method used to describe the nature of the relationships between variables, that is, positive or negative linear or non linear. It tells us at what rate the two variables that are significantly correlated are associated by means of a regression line.

A regression line (sometimes called the least-square line) is a line that best fits the point in a scatter diagram, and it always passes through the point $(\mathrm{X}, \mathrm{Y})$. it enables one to estimate or protect the value of one variable from a given value of the other variable. The general equation for a fitted regression line is given as:

$$
Y=a+b X
$$

Where:
$\mathrm{Y}=$ Dependent variable on the vertical axis
$\mathrm{X}=$ Independent variable on the horizontal axis
$\mathrm{a}=$ Intercept
b $\quad=\quad$ Slope of the regression line or the correlation coefficient of Y on X.

The intercept (a) is estimated as:
$\mathrm{a}=\mathrm{y}-\mathrm{bx}$
where:

$$
\begin{aligned}
& y=\text { Mean of the dependent variable } \\
& x=\text { Mean of the independent variable }
\end{aligned}
$$

The regression coefficient is estimated as:

$$
b=\frac{n\left(\sum x y\right)-\left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^{2}\right)-\left(\sum x\right)^{2}}
$$

OR

$$
b=\frac{\sum x y}{\sum x^{2}}
$$

A dependent variable $(\mathrm{Y})$ is the variable in regression that cannot be controlled or manipulated.

An independent variable ( X ) is the variable in regression that can be controlled or manipulated.

NB: The determination of which variable is the dependent or independent is not always clear and is sometimes an arbitrary decision.

For example: A researcher may be interested in studying the relationship between amount of fertilizer applied and crop yield. Crop yield can be said to be the dependent variable while amount of fertilizer applied as independent variable.

## Procedure for calculating simple linear regression

1. Compute the mean $x$ and $y$, the corrected sum of squares, $\sum x^{2}$ and $\sum y^{2}$, and the corrected sum of cross products $\sum x y$, of the variables $x$ and $y$.
2. Compute the estimates of the regression parameters $\alpha$ and $\beta$ as $\alpha=y-\beta x$ ( $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are considered as the estimates of $\mathbf{a}$ and $\mathbf{b}$ rather than parameters).

Therefore $\beta=\sum x y / \sum x^{2}$
3. Then substitute the value of $\beta$ in the linear equation: $\alpha=y-\beta x$. Thus the estimated linear regression is $y=\alpha+\beta x$
4. Plot the observed points and draw a graphical representation of the estimated linear regression equation above.

- Plot the scatter diagram and compute: $y_{\text {minimum }}=\alpha+\beta x_{\text {minimum }}$ and

$$
y_{\text {maximum }}=\alpha+\beta x_{\text {maximum }}
$$

- Plot the two points $\left(\mathrm{x}_{\text {min }}, \mathrm{y}_{\text {min }}\right)$ and $\left(\mathrm{x}_{\max }, \mathrm{y}_{\max }\right)$ and draw the line between the two points.


## Note that:

- The drawn line must be within the range $x_{\min }$. and $x_{\max }$.
- The line must pass through the mean point $(x, y)$.
- The slope of the is $\beta$
- The line, if extended, must intersect the $y$-axis at the $y$ value of $\alpha$ (intercept).

5. Test the significance of $\beta$. First you calculate the residual mean square as:

$$
S_{y \cdot x}^{2}=\frac{\sum y^{2}-\left(\sum x y\right)^{2} / \sum x^{2}}{n-2}
$$

and compute the $t_{\beta}$ value as:

$$
t_{\beta}=\frac{\beta}{S_{y, x}^{2} / \sum x^{2}}
$$

## 6. The Conclusion.

The regression coefficient $\beta$ is said to be significantly different if the calculated t -value above is greater than the tabular value at the $5 \%$ and $1 \%$ levels of significance.

### 6.3.3 SCATTER DIAGRAM/PLOT

A scatter diagram is a graph of the ordered pairs ( $\mathrm{x}, \mathrm{y}$ ) of numbers consisting of the independent variable X and the dependent variable Y . It is a visual way to describe the nature of the relationship between x and y . i.e. it enable us to see whether there is any pattern among the points. The more distinct a pattern is, the more closely the two variables are related in some way. For example: Construct a scatter diagram for the data.

| Subject | Age | Pressure |
| :---: | :---: | :---: |
|  | $(\mathrm{x})$ | $(\mathrm{y})$ |
| A | 43 | 128 |
| B | 48 | 120 |
| C | 56 | 135 |
| D | 61 | 143 |
| E | 67 | 141 |
| F | 70 | 154 |
| G | 32 | 96 |

## Figure 6.2 Scatter diagram



### 6.3.4 TYPES OF CORRELATION AND REGRESSION

Correlation can be Simple or Multiple. In simple relationships, there are only two variables under study. For example, a researcher may wish to study the relationship between height and weight in a population of rats.

In Multiple relationships more than two variables are under study for example, a zoologist may wish to investigate the relationship between growth in snails and factors such as different feed protein levels, quantity of feed given per day and hours of lighting per day.

Regression can be linear or curvilinear regression. Linear could be either simple linear regression or multiple linear regressions. Curvilinear - could be exponential, quadratic, and logarithmic etc.

Simple relationships can also be Positive or Negative. A positive relationship exists when the two variables under study increase or decrease at the same time. In a negative relationship as one variable increases the other variable decreases, and vice versa.

Generally, simple correlation and simple linear regression may be:

1. Positive correlation - when an increase in one variable is associated to a greater or lesser extent with an increase in the other.
2. Negative correlation - when an increase in one variable is associated to a greater or lesser extent with a decrease in the other.
3. Perfect correlation - when a change in one variable is exactly matched by a change in the other variable. If both increase together, it is perfect positive correlation: if one decreases as the other increases, it is perfect negative correlation.
4. High correlation - When a change in one variable is almost exactly matched by a change in the other.
5. Low correlation - when a change in one variable is to a small extent matched by a change in the other.
6. Zero correlation - when the two variables are not in matched at all, and there is no relationship between changes in one variable and changes in the other.

Figure 6.3 A perfect positive correlation


Figure 6.4 A perfect negative correlation


Figure 6.5 A zero correlation


Figure 6.6 A curvilinear graph


### 6.3.5 SPURIOUS CORRELATION

When interpreting correlation, $r$, it is important to realize that, there may be no direct connection at all between highly correlated variables. When this is so, the correction is termed spurious or nonsense correlation. It can arise in two ways:
(a) There may be an indirect connection
(b) There may be a series of coincidences.

### 6.3.6 CONVARIANCE

When the association of two variables is assessed we can speak of the resulting assessment as the covariance ('Cov') of the variables. The use of analysis of covariance helps to eliminate variability. Covariance can be measured by finding the average of the products of the deviations of each of the paired variables from the overall mean of the relevant variable i.e.

$$
\operatorname{Cov}(x, y)=y\left(x_{i}-x\right)\left(y_{i}-y\right) / n
$$

## WORKED EXAMPLE.

Assuming the following are given values of the length $(\mathrm{X})$ and yield $(\mathrm{Y})$ of five Okro plants. Using the deviation method, estimate and interpret the linear association between the length and yield of the Okro plants.

| Length (X) | 3 | 4 | 6 | 5 | 2 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| Yield (Y) | 10 | 12 | 15 | 13 | 5 |

## Compute the means for $X$ and $Y$.

Mean of $\mathbf{X}=3+4+6+5+2 / 5=4$
Mean of $\mathbf{Y}=10+12+15+13+5 / 5=11$

## Compute the values of the deviates.

|  |  |  |  |  |  | Products <br> of deviates |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Length $(\mathbf{X})$ | Yield $(\mathbf{Y})$ | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{x}^{2}$ | $\mathbf{y}^{2}$ | $\mathbf{x y}$ |
| 3 | 10 | -1 | -1 | 1 | 1 | 1 |
| 4 | 12 | 0 | 1 | 0 | 1 | 0 |
| 6 | 15 | 2 | 4 | 4 | 16 | 8 |
| 5 | 13 | 1 | 2 | 1 | 4 | 2 |
| 2 | 5 | -2 | -6 | 4 | 36 | 12 |
| Total |  |  |  |  |  |  |

Then we can compute the correlation (r) value.

$$
\begin{aligned}
& r=\frac{\sum x y}{\sqrt{\left(\sum x^{2}\right)\left(\sum y^{2}\right)}} \\
& =\cdot \frac{23}{\sqrt{(10)(58)} \quad=\frac{23}{24.1}=\mathbf{0 . 9 5 4}}
\end{aligned}
$$

The $\mathbf{r}$ value of 0.954 is high and positive. That means the longer the Okro plant the more the yield.

The degrees of freedom (d.f) is given by:
d.f. $=\mathrm{n}-2$
where: $\mathbf{n}$ is the number of the bivariate data.
The d.f. $=5-2=3$
Then check the correlation table at d.f 3 and $95 \%$ level of significance $=0.878$.
Since the absolute value of $\mathbf{r}(0.954)$ is greater than correlation table value $(0.878)$ at $5 \%$ probability level, the linear association between length and yield is significant. That means that the longer the Okro plant, the more the yield in almost $95 \%$ of the population.

### 6.4 TUTOR MARKED ASSIGNMENT

1. What is meant by the statement that two variables are related?
2. Give examples in nature of two variables that are positively correlation and two that are negatively correlated.
3. The average normal daily temperature $\left({ }^{\circ} \mathrm{C}\right)$ and the corresponding average precipitate (inches) for the month of August for seven randomly selected Local Government Areas in Plateau State are shown in the data below.

| Av. daily temp. (x) | 30 | 27 | 28 | 32 | 27 | 23 | 18 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Av. mon. precip. (y) | 3.4 | 1.8 | 3.5 | 3.6 | 3.7 | 1.5 | 0.2 |

(a). Draw a scatter plot for the variables.
(b). State the hypothesis.
(c). Compute the value of the correlation coefficient.
(d). Test the significance of the correlation coefficient at $=0.05$.
(e). Give a brief explanation of the type of relationship.
4. The data of an experiment to apply sufficient quantities of fertilizer to optimize vegetation growth (grass yield) and avoid excessive application that could lead to runoff and nutrient enrichment of a nearby lake is shown in the table below.

| Weight of fertilizer (g/ <br> sq. m$)(\mathbf{x})$. | 25 | 50 | 75 | 100 | 125 | 150 | 175 | 200 | 225 | 250 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Yield of grass (g/sq. <br> $\mathrm{m})(\mathbf{y})$. | 84 | 90 | 90 | 154 | 148 | 169 | 206 | 244 | 212 | 248 |

(a). Plot the scattergram for the data.
(b). determine the regression of y on x .

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UNIT SEVEN: SIMPLE EXPERIMENTAL DESIGN AND ANALYSIS OF VARIANCE (ANOVA)

### 7.1 INTRODUCTION

Over the years, an enormous range of experimental patterns has been developed and designs are available to suit a wide variety of circumstances.

Experimental design may be considered under two main headings. The first is the actual pattern of the experiment in which a correct of design depends
partly on knowledge of the experimental material and partly on the kind of question are wishes to ask. Secondly is the analysis of the data. As a rule, the results of the kind of experimental can be summarized in an analysis of variance (Anova) table. The design and analysis of an experiment obviously influence one another very strongly. That is for a given design, there really one satisfactory way of analyzing the data.

### 7.2 OBJECTIVES:

At the end of this unit, you should be able to:

1. Define experimental design.
2. Explain the principles of experimentation.
3. Mention and explain the different types of experimental design.
4. Define Anova and test statistical hypotheses using Anova.

### 7.3 MAIN CONTENT

### 7.3.1 EXPERIMENTAL DESIGN

Experimental design deals with the methods of constructing and analyzing comparative experiments such as comparing the effect of different factors or treatments. Planning experiments is made much more effective if you understand the advantages and disadvantages of different experimental designs and how they affect the "experimental error" against which we test our differences between treatments. There is the need to understand how experimental design as well as
treatment and replicate numbers impact on the "residual degrees of freedom" and whether you should be looking at one-tailed or two-tailed statistical tables.

Experimental design ensures that maximum precision is achieved for the amount of effort expended in an experiment. In every design, it is important that the roles of the treatments should be well defined and the objectives of the experiment properly understood.

An experiment is a systematic procedure for making observations under controlled conditions in such a way that they can be used for arriving at general conclusions regarding the population under study. An experimental unit is a plant or animal or group of plants or animals making up a single replicate of a single treatment. It is also called a plot.

### 7.3.2 PRINCIPLES INVOLVED IN EXPERIMENTATION

1. Randomization: This involves the allotting of treatments to the available material at random.
2. Replication / Reproducibility: is the need for an experiment to be planned in such a way that it can be replicated or reproduced. Replication is important in reducing error and enhancing accuracy. (Note: An error is the variation among varieties that cannot be accounted for by the variation due to treatments, blocks and other factors controlled in the experiment). Normally errors affecting any treatment tend to cancel out as the number of replications is increased.
3. Homogeneity/Sensitivity: A homogenous experimental setup is one that has uniformity of materials and therefore does not require the control of local variation. While sensitivity is to be able to estimate the effects of the treatments so that valid conclusion can be drawn.

### 7.3.3 TYPES OF EXPERIMENTAL DESIGN

1. Completely Randomized Design (CRD): is the simplest type of experimental design in which treatments are assigned at random to a set of plots. It is applicable when homogenous (same) experimental material is used on heterogeneous (different) experimental units or treatments, which is replicated more than once. CRD ensures that treatments allotment is completely random thereby avoiding biasness and minimizing inherent differences in experimental units or treatment. For example, the CRD for three replicates of six treatments is as follows.

| $A$ | $B$ | $A$ |
| :--- | :--- | :--- |
| $A$ | $E$ | $B$ |
| $D$ | $D$ | $E$ |
| $C$ | $C$ | $C$ |
| $F$ | $F$ | $E$ |
| $F$ | $B$ | $D$ |

A one-way analysis of variance is used in the analysis of CRD. Examples of such simple experiment include: Effects of different fertilizers (treatments) on yield of maize (experimental material), evaluating the effects of a feed of different protein levels (treatments (experimental material) on the growth of quails or fish of the same age, sex, size, etc.

## Advantages of CRD

a. The design is very flexible and can be used for any number of treatments.
b. The statistical analysis is comparatively easy and straightforward.
c. It is unaffected by missing observations for any treatment for some purely random accidental reason.

## Disadvantage of CRD

a. The design is inherently less informative than other more sophisticated layouts.
2. Randomized Block Design (RBD): is an experimental design in which the total area is divided into blocks and all of the treatments are arranged within each block in a random order. It is the probably the most widely used experimental design.


There are two sources of variation (i.e Treatment and Blocks) in RBD and therefore a two-way analysis of variance is used to analyze data obtained. Examples include experiments to determine the response of quails of different age
groups to different dietary protein levels, response of five variety of maize to three levels Indole-Acetic-Acid (IAA), etc.

## Advantages and Disadvantages of RBD

1. With heterogeneous material the residual variance can be reduced by choosing blocks of plots such that the plots within the blocks are fairly similar. i.e the design reduces the effect of heterogeneous material.
2. There is no restriction on the number of blocks or treatments, but in each block there must be the same number of plots, one to each treatment.
3. If some yields are accidentally lost, the analysis is again without due complications, although special modifications are required.

## Procedure of a randomized block experiment

Phase 1
$>$ Total rows and columns, and check Grand total by adding both row and column totals.
$>$ Calculate correction factor for experiment $=$ Grand total2/number plots.
$>$ Calculate TOTAL sum of squares (added squares - correction factor!).
YOU HAVE NOW NO FURTHER USE FOR THE PLOT RESULT DATA
Phase 2
$>$ Construct the analysis of variance table with the headings: Source of variation, d.f., Sum of squares, Mean square, $F$, and $P$.
> Insert sources of variation as Treatments, Blocks, Residual, and Total.
$>$ Allocate degrees of freedom as $n-1$ for number of Treatments and Replicates, and the product of these two d.f. for the Residual. Check that the three d.f.'s add up to $n-1$ for the Total (i.e. one less than the number of data in the experiment).
$>$ Square and add the TREATMENT totals to obtain the Treatments sum of squares of deviations.
$>$ Square and add the REPLICATE totals to obtain the Replicates sum of squares of deviations.
$>$ The "Residual" sum of squares of deviations is the remainder of the "total" sum of squares of deviations.

## YOU HAVE NOW USED ALL THE COLUMN AND ROW TOTALS

## End Phase

$>$ Calculate mean square for "treatments," "replicates," and "residual" by dividing each sum of squares of deviations by its own degrees of freedom.
3. Latin square: Is an experimental design in which the number of rows, columns, and treatments are equal and each treatment occurs just once in each row and column. As the name implies, a Latin square is a square design in that it consists of the same number of plots (though these don't have to be square) in two dimensions, i.e. $4 \times 4,5 \times 5,6 \times 6$, etc. The "dimension" of the square is the number of treatments in the experiment. We'll take as our example a Latin square with four treatments (A, B, C, and D) - i.e. a $4 \times 4(16$ plot) square.

| C | B | A | D |
| :--- | :--- | :--- | :--- |
| D | C | B | A |
| B | A | D | C |
| A | D | C | B |

## Simple Factorial Experiment

A factorial experiment is one where the treatments allocated to the experimental plots are combinations of two or more factors (hence the term "factorial"). For example, we might wish to test the effect on some measurement of a NUMBER of different fertilizers on MORE THAN ONE plant variety. Each plot would then be allocated the combination of one "level"of fertilizer (one of the fertilizers) applied to one "level" of variety (one of the varieties).

### 7.3.4 ANALYSIS OF VARIANCE (ANOVA)

Analysis of variance (abbreviated as ANOVA) is a systematic procedure for obtaining two or more estimate of variance and comparing them. Anova as a technique enables the comparison of means and variances in an experiment that involves more than two treatments. It estimates the value of the true variance of the population $\left(\sigma^{2}\right)$ from which the sample is drawn.

Anova permits us to conclude whether or not all means of the population under study are equal base upon the degree of variability in the sample data. Therefore, it is extremely efficient and powerful technique for investigating relationships between several groups of data.

## ASSUMPTIONS IN ANOVA

In Anova, the following assumptions must be made or else its appropriateness becomes questionable.

1. Samples are drawn randomly and each sample is independent of the other samples.
2. The populations under study have distribution which is approximately the normal curve.
3. The populations from which the sample values are obtained, all have the same population variance $\left(\sigma^{2}\right)$. That is $\left(\sigma_{1}{ }^{2}=\sigma_{2}{ }^{2}=\sigma_{3}{ }^{2}=\ldots \ldots \ldots \ldots . \sigma_{\mathrm{k}}{ }^{2}\right)$, the variances of all the populations are equal.

## HYPOTHESES IN ANOVA

1. The null hypothesis $\left(H_{o}\right)$ in Anova is that the independent samples are drawn from populations with the same means: $H_{o}: \mu_{1}=\mu_{2}=\mu_{3}=$ $\ldots \ldots \ldots \ldots \ldots . \mu_{\mathrm{k} .}$.

Where, k , is the number of populations under study.
2. The alternative hypothesis $\left(H_{l}\right)$ in Anova is that, not all population means are equal. That is, at least one mean is different from the others. $H_{1}: \mu_{1} \neq \mu_{2} \neq \mu_{3} \neq \ldots \ldots \ldots . \mu_{\mathrm{k}}$.

## THE CONCLUSIONS

Conclusions concerning $H_{o}$ in Anova test are based on computed variance ratio, sometimes called the F-ratio.

1. Accept the null hypothesis, $H_{0,}$, if the F-ratio value is less than the table value. That is, it is not significant.
2. Reject the null hypothesis, $H_{o}$, and accept the alternative hypothesis, $H_{l}$, if the F-ratio value is more than the table value. That is it is significant.

## EXAMPLES:

1. The table below shows the number of seeds for five varieties of garden egg to three level of Indo-acetic acid (IAA).

| IAAIVarieties | A | B | C | D | E |
| :--- | :--- | :--- | :--- | :--- | :--- |
| I | 3 | 5 | 10 | 7 | 8 |
| II | 2 | 4 | 7 | 4 | 5 |
| III | 4 | 5 | 8 | 6 | 7 |

## Solution:

State the null hypothesis: There is no significant difference between seed number in five varieties of garden egg and levels of IAA.

Calculate the totals from the table as below:

| IAAIVarieties | A | B | C | D | E | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| I | 3 | 5 | 10 | 7 | 8 | $\mathbf{3 3}$ |
| II | 2 | 4 | 7 | 4 | 5 | $\mathbf{2 2}$ |
| III | 4 | 5 | 8 | 6 | 7 | $\mathbf{3 0}$ |
| Total | $\mathbf{9}$ | $\mathbf{1 4}$ | $\mathbf{2 5}$ | $\mathbf{1 7}$ | $\mathbf{2 0}$ | GT=85 |

Then calculate the Correction Factor (CF) as:

$$
\mathrm{CF}=\mathrm{GT}^{2} / \mathrm{N}=85^{2} / 15=481.7
$$

Calculate the Mean Squares (SS)
BLOCKss $=\left(33^{2}+22^{2}+30^{2} / 5\right)-\mathrm{CF}=12.9$
VARIETIESss $=\left(9^{2}+14^{2}+\ldots \ldots \ldots . .+20^{2}\right) / 3-\mathrm{CF}=48.6$
TOTALss $=\left(3^{2}+5^{2}+\ldots \ldots \ldots . .+7^{2}\right)-\mathrm{CF}=65.3$
ERRORss $=$ TOTALss $-($ BLOCK + VARIETIESss $)=3.8$
Then calculate:
BLOCK $_{\text {MS }}=$ BLOCKss $/$ BLOCK $_{\text {DF }}=12.9 / 2=6.45$
VARIETIES $_{\text {MS }}=$ VARIETIESss/VARIETIES $_{\text {DF }}=48.6 / 4=12.15$
Block F-value $=$ Block $_{\text {MS }} /$ Error $_{\text {MS }}=6.45 / 0.475=13.58$
Varieties F-value $=$ Varieties $_{\text {MS }} /$ Error $_{\text {MS }}=12.15 / 0.475=25.58$

The calculated F-values are compared with the F-distribution table, using their respective degrees of freedoms.

| SOURCE | DF | SS | MS | F |
| :--- | :--- | :--- | :--- | :--- |
| Block | 2 | 12.9 | 6.45 | $13.58^{* *}$ |
| Varieties | 4 | 48.6 | 12.15 | $25.58^{* *}$ |
| Error | 8 | 3.8 | 0.475 |  |
| Total | 14 | 65.3 |  |  |

** indicates that the values are highly significant.

## Conclusion:

Since the F-values are highly significant, we reject the null hypothesis. It means that the three levels of IAA have effect on the seed number of the five varieties of garden egg.
2. Complete the Anova table below and draw your conclusions.

| Source | Sum of <br> squares (SS) | Degrees of <br> freedom <br> $(\mathrm{DF})$ | Mean <br> squares (MS) | F-ratio |
| :--- | :--- | :--- | :--- | :--- |
| Varieties | 123.44 | $*$ | $*$ | $*$ |
| Residual | $*$ | 15 | $*$ |  |
| Total | 210.21 | 18 |  |  |

RESIDUALss $=210.21-123.44=86.8$
VARIETIES $_{\text {DF }}=18-15=3$
VARIETIES $_{\text {MS }}=123.44 / 3=41.15$
RESIDUAL $_{\text {MS }}=86.8 / 15=5.79$
F- ratio $=41.15 / 5.79=7.11$

The complete table is as shown below.

| Source | Sum of | Degrees of | Mean squares | F-ratio |
| :--- | :--- | :--- | :--- | :--- |


|  | squares (SS) | freedom (DF) | (MS) |  |
| :--- | :--- | :--- | :--- | :--- |
| Varieties | 123.44 | 3 | 41.15 | 7.11 |
| Residual | 86.8 | 15 | 5.79 |  |
| Total | 210.21 | 18 |  |  |

## Conclusion:

The observed variance ratio of 7.12 is greater than the table values at both $5 \%$ (3.29) and $1 \%$ (5.42). That means there is high significant difference among the varieties. Therefore, we reject the null hypothesis that the varieties are the same.

### 7.4 TUTOR MARKED ASSIGNMENT

1. What do you understand by 'Experimental design'?
2. Which type of experimental design is most appropriate to use for the following Anova. (a) One-way classification. (b) Two-way classification.
3. Outline and briefly discuss the principles involved in experimental.
4. What is the importance of using homogenous materials in an experiment?
5. From the Anova table below

| Source of <br> variation | d.f. | Sum of <br> Squares | Mean <br> Square | F |
| :--- | :--- | :--- | :--- | :--- |
| Block | 2 | 16 | 8 | 11.94 |
| Treatment | 5 | 145.8 | 29.17 | 43.54 |
| Residual/Error | 10 | 6.67 | 0.67 |  |
| Total | $\mathbf{1 7}$ | $\mathbf{1 6 8 . 5}$ |  |  |

a. Name the type of design that was employed.
b. How many treatments were compared?
c. How many observations were analysed?
d. Draw your conclusions on the F-values.

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## UNIT EIGHT: NON - PARAMETRIC TESTS

### 8.1 INTRODUCTION

Statistical test, such as mean, standard deviation, variance, $Z, t$ and F - tests are called parametric tests. This is because these assumptions are governed by the distribution of the sampled population or populations that is/or are at least approximately normal. But if the population in a particular hypothesis-testing situation is not normally distributed, then the non-parametric or distribution - free tests is used.

Non parametric test can also be used to test hypothesis that do not involve specific population parameters such as $\mu, \sigma$ or $P$.

### 8.2 OBJECTIVES:

At the end of this unit, you should be able to

1. State the advantages and disadvantages of non parametric method.
2. Give the differences between non-parametric and parametric test.
3. Test hypotheses using non parametric tests such as Sign test and KruskalWallis test.
4. Compute the Spear rank correlation coefficient.

### 8.3 MAIN CONTENT

### 8.3.1 ADVANTAGES OF NON-PARAMETRIC TEST

There are five advantages that non parametric methods have over parametric methods.

1. They can be used to test population when the variable is not normally distributed.
2. They can be used when the data are nominal or ordinal.
3. Can be used to test hypothesis that do not involve population parameters.
4. In most cases, computation is easier than in parametric.
5. They are easier to understand.

### 8.3.2 DISADVANTAGES OF NON-PARAMETRIC TEST

1. They are less sensitive than in parametric i.e larger differences are needed before the null hypothesis can be rejected.
2. They tend to use less information than the parametric tests.
3. They are less efficient than their parametric counterparts when the assumptions of the parametric are met.

## NOTE:

The researcher should use caution in deciding whether to use nonparametric or parametric methods. However, if the assumptions can be met, the parametric methods are preferred.

## DISTINCTION BETWEEN NON-PARAMETRIC AND PARAMETRIC TESTS

|  | NON - PARAMETRIC TESTS | PARAMETRIC TEST |
| :--- | :--- | :--- |
| 1 | May be used with actual observations, or <br> with observations converted to ranks. | Are used only with actual <br> observations |
| 2 | May be used with observations on nominal, <br> ordinal and interval scales | Generally restricted to observations <br> on interval scales. |
| 3 | Compare medians | Compare means and variances |
| 4 | Data are 'distribute in free' i.e must not be <br> normally distributed. | Data are required to be normally <br> distributed and to have similar <br> variances |


| 5 | Are suitable for data which are counts | Counts must be transformed |
| :--- | :--- | :--- |
| 6 | Are suitable for derived data e.g. <br> proportions, indices | Derived data may first have to be <br> transformed. |

### 8.3.3 THE SIGN TEST

The simplest non-parametric test is the sign test for single samples. It is used to test the value of a median for a specific sample. In using Sign test, you:

- Hypothesize the specific value for the median of a population.
- Select a sample of data and compare each value with the conjectured median.
- Assign plus sign if the data value is above the conjectured median.
- Assign minus sign if the data value is below the conjecture median.
- And zero (0) if it is the same as the conjecture median.
- Compare the number of plus and minus signs and ignore the zeros
- If the null hypothesis ( Ho ) is true, the number of plus signs should be approximately equal to the number of minus signs.
- But if the Ho is not true, there will be disproportionate number of plus or minus signs.


## NB:

The test value is the smaller number of plus or minus signs obtained in a sample. For example, if there are 12 plus signs and 5 minus signs, the test value is
5.

## For Example:

A chicken poultry owner hypothesizes that the median number of eggs he gets per day is 40 . A random sample of 20 days yields the following for the number of eggs laid each day.

| 18 | 30 | 39 | 36 | 43 |
| :--- | :--- | :--- | :--- | :--- |
| 29 | 34 | 40 | 40 | 32 |
| 39 | 34 | 16 | 37 | 45 |
| 39 | 22 | 36 | 28 | 52 |

At $\alpha=0.05$, test the owner's hypothesis

## Solution:

State the hypotheses:
$H_{o}:$ median $=40-$ that is the claim
$H_{l}:$ median $\neq 40$.
Find the critical value. Compare each value of data with the median

| - | + | 0 | - | - |
| :--- | :--- | :--- | :--- | :--- |
| - | - | - | - | - |
| - | - | - | + | - |
| - | 0 | - | - | + |

Then count the number of plus and minus signs, and ignore zero. This gives $\mathrm{n}=$ 18.

## Refer to Table of critical values for $\operatorname{Sign}$ test and $\alpha=\mathbf{0 . 0 5}$ for a two-tailed test: The critical value is 4 .

## Compute the test value.

Since there are 3 plus signs and 15 minutes sign, then the test value is 3. (Always use the smaller value as the test value). The test value 3 is less than the critical value $4(3<4)$. Therefore, the null hypothesis is rejected.

In summary, there is enough evidence to reject the claim that the median number of eggs layed per day is 40 .

## NB:

If the sample size is 25 or less, the table of critical values for Sign test is used. But when the sample size is 26 or more, the standard normal distribution table can be used to find the test value.

### 8.3.4 KRUSKAL - WALLIS TEST

The Kruskal - Wallis test, sometimes called the H - test is a nonparametric test which can be used to compare three or more means, where the assumptions for the ANOVA test: populations are normally distributed and the population variances are equal, cannot be met. In Kruskal - Wallis test, each sample size must be 5 or more with $\mathrm{M}-1$ degrees of freedom, where $\mathrm{m}=$ number of groups.

The distribution in kruskal - wallis can be approximated by the chi-square distribution.

## Formula for Kruskal-Wallis Test

$\mathrm{H}=\frac{12 .}{\dot{\mathrm{N}(\mathrm{N}+1)}} \times\left(\mathrm{R}_{\underline{1}}{ }^{2} / \mathrm{n}_{1}+\mathrm{R}_{2}{ }^{2} / \mathrm{n}_{2}+\ldots \ldots . .+\mathrm{R}_{\mathrm{k}}{ }^{2} / \mathrm{n}_{\mathrm{k}}\right)-3(\mathrm{~N}+1)$

Where:
$\mathrm{R}_{1} \quad=\quad$ Sum of ranks of sample 1
$\mathrm{n}_{1} \quad=\quad$ Size of sample 1
$\mathrm{R}_{2}=\quad$ Sum of ranks of sample 2
$\mathrm{n}_{2} \quad=\quad$ Size of sample 2
$\mathrm{R}_{\mathrm{k}} \quad=\quad$ Sum of ranks of sample K
$\mathrm{n}_{\mathrm{k}} \quad=\quad$ Size of sample k
$\mathrm{N}=\mathrm{n}_{1}+\mathrm{n}_{2}+\ldots . . \mathrm{n}_{\mathrm{k}}$
$\mathrm{k} \quad=\quad$ number of samples

## For Example:

A research was conducted to know the relationships of bill-length (mm) in three bird species. The data below were obtained.

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ |
| :---: | :---: | :---: |
| 33.5 | 38.0 | 32.0 |
| 37.5 | 31.5 | 33.0 |
| 35.0 | 34.0 | 36.5 |
| 39.1 | 35.1 | 34.9 |
| 31.1 | 33.4 | 32.5 |

${ }^{* *}$ At $\alpha=0.05$, is there enough evidence to reject the hypothesis that all the bird species have the same bill length?

## Solution:

## State the hypotheses

$H_{o}$ : There is no difference in the bill length of the 3 bird species in the forest reserve.
$H_{1}$ : There is a difference in the bill length of the 3 bird species in the forest reserve.

Find the critical value using the chi-square distribution table at degrees of freedom.
$\mathrm{Df}=\mathrm{m}-1$ at $\alpha=0.05$, where m is the number of groups
df $=3-1=2$ : The critical value is 5.991

Arrange all the data from lowest to highest and rank each value

| Bill - Length | Species | Rank |
| :---: | :---: | :---: |
| 31.1 | A | 1 |
| 31.5 | B | 2 |
| 32.0 | C | 3 |
| 32.5 | C | 4 |
| 33.0 | C | 5 |
| 33.4 | B | 6 |
| 33.5 | A | 7 |
| 34.0 | B | 8 |
| 34.9 | C | 9 |
| 35.1 | B | 10 |
| 35.0 | A | 11 |
| 36.5 | C | 12 |
| 37.5 | A | 13 |
| 38.0 | B | 14 |
| 39.1 | A | 15 |

Calculate the sum of the ranks of each bird species
Species A $=1+7+11+13+15=47$
Species B $=2+6+8+10+14=40$
Species C $=3+4+15+9+12=33$
Substitute the Kruskal - Wallis Test formula
H

$$
\left.=\frac{12}{\mathrm{~N}(\mathrm{~N}+1)} \frac{\left(\mathrm{R}^{2}\right.}{\mathrm{n}_{1}}+\underline{\mathrm{R}}_{2}^{2}{ }_{2}+\underline{\mathrm{R}}_{2}^{2}\right)-3(\mathrm{~N}+1)
$$

Where:
$\mathrm{N}=15, \quad \mathrm{R}_{1}=47, \quad \mathrm{R}_{2}=40, \quad \mathrm{R}_{3}=33$ and $\quad \mathrm{n}_{1}=\mathrm{n}_{2}=\mathrm{n}_{3}=5$
Therefore,
$\mathrm{H}=\frac{12}{15(15+1)} \quad \frac{\left(47^{2}\right.}{5}+\frac{40^{2}}{5}+\frac{\left.33^{2}\right)}{5}-3(15+1)$

$$
=0.05(979.6)-48
$$

$$
=48.98-48
$$

$\mathrm{H}=0.98$
Then deduce the decision.

## Decision:

Since the test value 0.98 is less than the critical value 5.991 , the decision is to accept the null hypothesis. This simply means that there is no difference in the bill length of the three bird species.

### 8.3.5 SPEARMAN RANK CORRELATION

The spearman rank correlation coefficient, denoted by $\mathbf{r}_{\mathbf{s}}$, is the nonparametric equivalent of the Pearson coefficient used for testing hypothesis when
samples obtained are not normally distributed. It can be used when the data are ranked. If the two sets of data have the same ranks, $\mathbf{r}_{\mathbf{s}}$ will be +1 . If the ranks are in exactly the opposite way, $\mathbf{r}_{\mathbf{s}}$ will be -1 . If there is no relationship between the rankings, then $\mathbf{r}_{\mathbf{s}}$ will be near 0 .

The formula for spear rank correlation is given as:
$r_{s}=1-\frac{\left(6 \sum d^{2}\right)}{n\left(n^{2}-1\right)}$
where: $d=$ difference in ranks
$\mathrm{n}=$ number of data pairs

## For Example:

The table below gives the number of horn bill nestlings ringed, and juveniles captured in the same season, over a 6 year study period at a woodland site. At $\alpha=0.05$, test the hypothesis that there is a linear correlation between the nestlings ringed and juveniles captured.

| Nestlings Ringed | 16 | 9 | 1 | 7 | 13 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Juveniles Captured | 16 | 8 | 5 | 4 | 10 | 3 |

## Solution:

- State the hypothesis

$$
H_{o}: \quad \rho=0 \quad \text { and } \quad H_{l}: \quad \rho \neq 0
$$

- Find the critical value using the table of critical values for the Rank correlation coefficient.
- And the value $\mathrm{n}=6$ and $\alpha=0.05$

The value is 0.886 .

- To find the test value, you first rank the data set.

| Nestlings ringed | Rank 1 | Juveniles <br> captured | Rank 2 |
| :---: | :---: | :---: | :---: |
| 16 | 1 | 16 | 1 |
| 9 | 4 | 8 | 3 |
| 1 | 6 | 5 | 4 |
| 7 | 5 | 4 | 5 |
| 13 | 3 | 10 | 2 |
| 15 | 2 | 3 | 6 |

Then subtract the rankings for each season $\left(R_{1}-R_{2}\right)$ and square the differences.

| $\mathbf{R}_{\mathbf{1}}$ | $\mathbf{R}_{\mathbf{2}}$ | $\mathbf{d}$ | $\mathbf{d}^{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 0 |
| 4 | 3 | -1 | 1 |
| 6 | 4 | 2 | 4 |
| 5 | 5 | 0 | 0 |
| 3 | 2 | 1 | 1 |
| 2 | 6 | -4 | 16 |

Substitute in the formula to find r .

$$
r_{s}=1-\frac{\left(6 \sum d^{2}\right)}{n\left(n^{2}-1\right)}
$$

where $n=$ the number of data pairs $(n=6)$

$$
\mathbf{r}_{\mathbf{s}}=1-\frac{6(22)}{6\left(6^{2}-1\right)}=0.371
$$

- Make a decision.

Since the $\mathbf{r}_{\mathbf{s}}=0.371$, which is less than the critical value of 0.886 , we accept the null hypothesis. Therefore, there is no enough evidence to say that there is a correlation between Nestlings ringed and Juveniles captured.

### 8.4 TUTOR MARKED ASSIGNMENT:

1. Why is the sign-test the simplest non-parametric test to use?
2. Ten patients suffering from severe muscle pains volunteered to try two new analgesics and the time of relief were recorded in minutes for each patient are as shown below.

| Patient number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Analgesic A <br> (min.) | 138 | 136 | 142 | 151 | 154 | 141 | 140 | 138 | 132 | 136 |
| Analgesic B <br> (min.) | 140 | 136 | 141 | 150 | 153 | 144 | 143 | 136 | 131 | 138 |

(a) State the hypotheses and identify the claim.
(b) Find the critical value and compute the test value.
(c) Make a decision based on your finding.
3. A study was conducted on 10 adults to determine the effect of alcohol on test scores. Each adult was given a test. Then for one week, each adult was required to consume a certain amount of alcohol and then he or she was retested. The results are shown below.

| Adult | Score before | Score after |
| :---: | :---: | :---: |
| 1 | 105 | 106 |
| 2 | 109 | 105 |
| 3 | 98 | 94 |
| 4 | 112 | 109 |
| 5 | 109 | 105 |
| 6 | 117 | 115 |
| 7 | 123 | 125 |
| 8 | 114 | 114 |
| 9 | 95 | 98 |
| 10 | 101 | 100 |

At $\alpha=0.10$, test the claim that alcohol does not affect a person's test score.

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## UNIT NINE: USE OF STATISTICAL PACKAGES

### 9.1 INTRODUCTION

In the past, statistical calculations were done with pencil and paper. However, with the advent of calculators, numerical computations have become much easier, especially where computers do all the numerical calculations. All one does is to enter the data into the computer and use the appropriate command; the computer will print the answer or display it on the screen. You should realize that you are responsible for understanding and interpreting each statistical concept. There are many statistical packages available. This unit discusses the SPSS (Statistic al package for Social Sciences) and MINITAB.

### 9.2 OBJECTIVES:

At the end of this unit, you should be able to:

1. Describe the applications of SPSS and MINITAB in different statistical procedures.
2. Mention examples of statistical tools in the statistical packages.

### 9.3 MAIN CONTENT

### 9.3.1 SPSS

This is a statistical package developed especially for the social sciences but it has wide applicability in the areas of biology and agriculture. It can be used to execute some statistical procedures such as summaries, custom tables, Anova (Analysis of variance), correlation and regression analyses, non-parametric tests,
time series, create charts (Bar, line, Pie, Area, Histogram, Scatter etc.) and lots more.

The SPSS is a comprehensive package that is command based but commands list can be generated from menus and interactive operations are possible.

### 9.3.2 MINITAB

The MINITAB statistical software provides a wide range of statistical analysis and graphical capabilities. Like the $\operatorname{SPSS}$, it is also a comprehensive command based statistical package. MINITAB can be used for various types of statistical analysis ranging from editing and manipulating data, basic statistics, arithmetic, regression, Anova, non parametric test, exploratory data analysis et.c.

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